Recall basics of mathematical statistics

In your undergraduate education you have frequently been confronted with concepts of mathematical statistics already, for instance when doing an error analysis for experimental measurements or in quantum mechanics when discussing the measurement process. Remind yourself of the basic ideas, e.g. by consulting the collection of formulæ by Bronstein and Semendjajew, and then explain in one or two sentences. To be specific, consider an experimental series of measurements that comes about by redoing the same experiment \( N \) times.

Imagine that two observables \( X, Y \) have been measured simultaneously in every run, so an ensemble of measured pairs \( \{(X_i, Y_i), i = 1, \ldots N\} \) has been obtained.

a) How do you calculate the average (also “mean”) values \( \langle X \rangle \) and \( \langle Y \rangle \)? What motivates this definition?

b) How do you quantify the spreading of the measured values around the mean? Is your choice unique? What motivates it?

c) Perform a small recherche in the mathematical literature and figure out what is the difference between the “variance” and the “sample variance”. Explain.

d) What does it mean if the variables \( X \) and \( Y \) are statistically independent, intuitively and also in mathematical terms.

e) Under what conditions is the distribution of measurement results for the observables \( X \) and \( Y \) expected to be distributed according to a Gaussian? Under what conditions is a Gaussian distribution not expected. What alternatives do you know and when do they become relevant?

f) Suppose you calculate the mean of the observable \( X \) with \( N \) samples and later with \( N = \infty \) samples. How does the difference scale with \( N \)? (“error of the mean”).