Introduction to Spintronics

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I. Zutic, J. Fabian, and S. Das Sarma,
*Spintronics: Fundamentals and applications*,

J. Fabian, A. Matos-Abiague, C. Ertler, P. Stano, and I. Zutic,
*Semiconductor spintronics*,

J. Fabian and I. Zutic, The standard model of spin injection,

my group’s web site:
[www.physik.uni-regensburg.de/forschung/fabian/index_lecturenotes.html](http://www.physik.uni-regensburg.de/forschung/fabian/index_lecturenotes.html)
WHAT IS SPINTRONICS?
Prologue  The role of spin in solid state physics

Active (manipulatable)

- Ferromagnetism
- Magnetic resonance
- Nuclear NMR (MR)
- Electron paramagnetism
- ferromagnetic
- Conduction electron
- Spintronics

Passive

- Degeneracy
- Energy bands
- Cohesion in metals
- Normal transport
- Fermi liquid
- Paramagnetism

W.F. symmetry

- Exchange effects
- Superconductivity
- Strongly correlated systems
- (Fractional) quantum Hall systems

Spintronics

Time gap!!!

(2 reasons)

Quantum computing?

- 20xx?
Q: What is meant by spin in spintronics?

A: Ensemble spin (magnetization)

\[ \text{spin polarization} \quad \alpha = \frac{\text{spin}}{N_up + N_down} \]

Q: What is spin dynamics?

A: Spin phase evolution

Q: Does individual spin matter?

A: Yes, in spin relaxation, dephasing, decoherence

Most: Spin-based quantum computation (nanoscale spintronics)
Attempts to label fields rarely succeed, because names have a life of their own. ‘Nanotechnology’, when coined in 1974, had nothing like the meaning it has today — it referred to rather conventional micromechanical engineering. ‘Spintronics’, the field of quantum electronics that lies behind this year’s physics Nobel, is arguably a slightly ugly and brutal amalgam (of ‘electronics’ and the electron’s quantum property of ‘spin’), yet somehow it works.

what is spintronics?

narrow (device):
electronics with spin

broad:
umbrella for electron spin phenomena in solids
spintronics drive

technology

fundamental discoveries
The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics for 2007 jointly to

Albert Fert
Unité Mixte de Physique CNRS/THALES, Université Paris-Sud, Orsay, France

Peter Grünberg
Forschungszentrum Jülich, Germany,

"for the discovery of Giant Magnetoresistance".
**Giant MagnetoResistance**

P. Grunberg et al. (1988), A. Fert et al. (1988)

\[ GMR = \frac{R_P - R_{AP}}{R_P} \]

multilayers 30 - 40% at RT
GMR hard disk read heads

From: IBM web site
Tunneling Magneto Resistance
TMR non-volatile MRAM

- bit line (top)
- free layer, bit
- tunnel barrier AlO
- fixed (pinned) layer
- base electrode
- digit line (bottom)
- isolation transistor

Symbols:
- large current: up arrows
- small current: down arrows

Sense current direction:
- top to bottom
SPINTRONICS GOALS

spin control of electrical properties (I-V characteristics)

electrical control of spin (magnetization)


1. B. 1 Can the spin magnetic moment of a free electron be detected?
2. B. 2 Can Stern-Gerlach experiments be used to polarize electron beams?
SPINTRONICS’ 3 REQUIREMENTS

- EFFICIENT SPIN INJECTION

- SLOW SPIN RELAXATION @ SPIN CONTROL

- RELIABLE SPIN DETECTION

Silsbee-Johnson spin-charge coupling
materials issues: room temperature ferromagnetic semiconductors?

GaMnAs

- 1-15 % Mn
- p-doped (Mn replaces Ga)
- degenerate: $p = 10^{20} - 10^{21}/cm^3$
- $T_c$ up to 180 K
- fm and p-density coupled
- impurity band or not?

Fe/GaMnAs

- above room-temperature fm
- a few nm of GaMnAs involved
- bias control?
- anisotropies?


See also poster 234 for related work S. Mark et al.

There is no Si-based ferromagnetic semiconductor!
spin-orbit coupling in zincblende systems
what happens when inversion symmetry is broken?
(Bychkov-Rashba Hamiltonian)

Time reversal $+$ space inversion symmetry:

$$\varepsilon_{k\uparrow} = \varepsilon_{k\downarrow}$$

Time reversal symmetry only:

$$\varepsilon_{k\uparrow} = \varepsilon_{-k\downarrow}, \quad \varepsilon_{k\uparrow} \neq \varepsilon_{k\downarrow}$$

Effective spin-splitting magnetic field:

$$H_1(k) = \frac{\hbar}{2} \Omega(k) \cdot \sigma$$

Time reversal symmetry:

$$\Omega(-k) = -\Omega(k)$$

$$\Omega(k)_{BR} = \alpha_{BR}(k \times n)$$

$$\Omega(k)_D = \gamma_D(k_x, -k_y)$$

:spin transistors:

**Spin current**  
**basis for confusions**

N. Mott (1936): as long as spin-orbit coupling is weak, current is carried by spin up and spin down channels

$$H = \frac{p^2}{2m} + V(r) + \alpha(z \times \sigma) \cdot p$$

### Charge

- $\rho = e\psi^\dagger \psi$
- $\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0$
- $\mathbf{j} = eRe\psi^\dagger \mathbf{v} \psi$

### Spin

- $s = \frac{\hbar}{2} \psi^\dagger \sigma \psi$
- $\frac{\partial s}{\partial t} + \nabla \cdot \mathbf{j}_s = \xi$

\[
\mathbf{j}_s = \frac{\hbar}{4} \left[ \psi^\dagger \left\{ \mathbf{v}, \sigma \right\} \psi \right].
\]

- $\xi = \frac{1}{2} R e \left( \psi^\dagger i [H, \sigma] \psi \right)$

*Spin current is not conserved: no unique way of defining it microscopically*
Electrical Spin Injection
Johnson-Silsbee spin injection experiment

Silsbee: emf appears in the proximity of a ferromagnetic metal and spin-polarized nonmagnetic metal (inverse of spin injection)

Q: How much spin accumulates in $N$ if there is a charge current $j$ flowing?

Key assumptions:

- degenerate Fermi gases
- charge neutrality (no space charges)
- spin conservation at the contact
key concepts: electric transport

**Quasichemical potentials**

\[
f_0(\varepsilon) = \left[ \exp(\varepsilon - \eta) / k_B T \right] + 1 \rightarrow f(\varepsilon, x) = f_0 [\varepsilon - e\phi(x) - \eta - e\mu(x)]
\]

**Local charge neutrality**

\[
n(x) = \int d\varepsilon g(\varepsilon) f(\varepsilon, x) = n_0 = \int d\varepsilon g(\varepsilon) f_0(\varepsilon) \quad \Rightarrow \quad \mu(x) = -\phi(x)
\]

**Electric current**

\[
j = \sigma E + eD \nabla n = \left( -\sigma + e^2 D \frac{\partial n_0}{\partial \eta} \right) \nabla \phi + e^2 D g \frac{\partial n_0}{\partial \eta} \nabla \mu = \sigma \nabla \mu
\]

Einstein’s relation

**Contact resistance**

\[
j = \Sigma_c \Delta \mu \quad \mathcal{R}_c = 1 / \Sigma_c
\]

emf (electromotive force)

\[
\text{emf} = \mu(\infty) - \mu(\infty)
\]
Spin density and spin polarization

\[ n = n_{\uparrow} + n_{\downarrow} \]
\[ s = n_{\uparrow} - n_{\downarrow} \]
\[ P_n = \frac{n_{\uparrow} - n_{\downarrow}}{n_{\uparrow} + n_{\downarrow}} \]
\[ P_\chi = \frac{X_{\uparrow} - X_{\downarrow}}{X_{\uparrow} + X_{\downarrow}} \]

Spin accumulation

\[ n_{\uparrow}(x) = n_{0\uparrow}(\eta + e\mu_{\uparrow} + e\phi) \approx n_{0\uparrow} + g_{\uparrow}(e\mu_{\uparrow} + e\phi) \]
\[ n_{\downarrow}(x) = n_{0\downarrow}(\eta + e\mu_{\downarrow} + e\phi) \approx n_{0\downarrow} + g_{\downarrow}(e\mu_{\downarrow} + e\phi) \]

\[ \mu = (\mu_{\uparrow} + \mu_{\downarrow})/2 \]
\[ \mu_s = (\mu_{\uparrow} - \mu_{\downarrow})/2 \]

\[ s = s_0 + \delta s \]
\[ \delta s = 4e \frac{g_{\uparrow}g_{\downarrow}}{g} \mu_s \]
key concepts: spin-polarized transport

Charge and spin currents

\[ j = j_\uparrow + j_\downarrow \quad j_\uparrow = \sigma_\uparrow \nabla \mu_\uparrow \]
\[ j = j_\uparrow - j_\downarrow \quad j_\downarrow = \sigma_\downarrow \nabla \mu_\downarrow \]

Current spin polarization

\[ P_j = \frac{j_\uparrow - j_\downarrow}{j_\uparrow + j_\downarrow} = P_\sigma + \frac{1}{4} \nabla \mu_s \frac{\sigma_\uparrow \sigma_\downarrow}{\sigma} \]

Spin-polarized currents in contacts

\[ j_\uparrow = \Sigma_\uparrow \Delta \mu_\uparrow \]
\[ j_\downarrow = \Sigma_\downarrow \Delta \mu_\downarrow \]

\[ j = \Sigma \Delta \mu + \Sigma_s \Delta \mu_s \]
\[ j_s = \Sigma_s \Delta \mu + \Sigma \Delta \mu_s \]

\[ P_j = P_\Sigma + \Delta \mu_s / j R_c \quad R_c = \Sigma / 4 \Sigma_\uparrow \Sigma_\downarrow \neq R_c = 1 / \Sigma \]
key concepts: spin diffusion

Diffusion of spin accumulation

\[ \nabla j = 0 \quad \nabla j_s = e \delta s / \tau_s = \ldots = 4 e^2 \mu_s \frac{g^g g^\downarrow}{g^\downarrow g^g} \frac{1}{\tau_s} \]

Also,

\[ \nabla j_s = \nabla \left( P_\sigma j + \nabla \mu_s \frac{4 \sigma^\uparrow \sigma^\downarrow}{\sigma} \right) = 4 \frac{\sigma^\uparrow \sigma^\downarrow}{\sigma} \nabla^2 \mu_s \]

Compare,

\[ \nabla^2 \mu_s = \mu_s / L_s^2 \]

\[ L_s = (D \tau_s)^{1/2} \]
case study:
spin injection in FN junctions

Strategy:

1. study spin-polarized transport in each region
2. connect different regions by continuity equations
3. calculate spin injection efficiency

\[ P_{\sigma F} \approx 10 - 90\% \]
\[ L_{sN} \approx 10 \mu m \]
\[ L_{sF} \approx 10 \text{ nm} \]
**F region**

Spin diffusion

\[ \mu_{sF} = \mu_{sF}(0) e^{x/L_{sF}} \]

Current spin polarization

\[ P_{jF}(0) = P_{\sigma F} + \frac{1}{j} \frac{\mu_{sF}(0)}{R_F} \]

Effective resistance

\[ (R_F) = \frac{\sigma_F}{4\sigma_F \uparrow \sigma_F \downarrow} L_{sF} \neq R_F \]

**N region**

Spin diffusion

\[ \mu_{sN} = \mu_{sN}(0) e^{-x/L_{sN}} \]

Current spin polarization

\[ P_{jN}(0) = -\frac{1}{j} \frac{\mu_{sN}(0)}{R_N} \]

\[ R_N = \frac{L_{sN}}{\sigma_N} \neq R_N \]
\[ P_{jc} = P_{\Sigma} + \frac{1}{j} \frac{\mu_{SN}(0) - \mu_{SF}(0)}{R_c} \]

**Current spin polarization**

**Algebraic system**

\[ P_{jF}(0) = P_{\sigma F} + \frac{1}{j} \frac{\mu_{SF}(0)}{R_F} \]

\[ P_{jc} = P_{\Sigma} + \frac{1}{j} \frac{\mu_{SN}(0) - \mu_{SF}(0)}{R_c} \]

\[ P_{jN}(0) = -\frac{1}{j} \frac{\mu_{SN}(0)}{R_N} \]

**Continuity of spin current at \( C \):**

\[ P_j \equiv P_{jF}(0) = P_{jN}(0) = P_{jc} \]
Spin injection efficiency

central result of the standard model of spin injection:

\[ P_j = \frac{I_\uparrow - I_\downarrow}{I_\uparrow + I_\downarrow} = \frac{P_\Sigma R_C + P_{\sigma_F} R_F}{R_F + R_N + R_C} = \langle P_\sigma \rangle_R \]
Equivalent electrical circuit model of spin injection

\[ P_j = \frac{I_\uparrow - I_\downarrow}{I_\uparrow + I_\downarrow} = \frac{P_\Sigma R_C + P_{\sigma F} R_F}{R_F + R_N + R_C} = \langle P_\sigma \rangle_R \]
Transparent junction

\[ R_c \ll R_N, R_F \]

\[ P_j = \frac{R_F}{R_F + R_N} P_{\sigma F} \]

Spin injection efficiency is due to the spin-polarization of the ferromagnet

Q: What happens if we would inject spin into a semiconductor from a ferromagnetic metal?

\[ P_j \approx \frac{R_F}{R_N} P_{\sigma F} \ll P_{\sigma F} \]

the conductivity mismatch problem!
Tunnel junction

\[ R_c \gg R_N, R_F \]

\[ P_j \approx P_\Sigma \]

Spin injection efficiency is due to the spin-polarization of the contact

Q: Is there is the conductivity mismatch problem now?

No!
Spin injection into semiconductors can be done through tunnel junctions
Nonequilibrium resistance & spin bottleneck

\[ \mu_N(\infty) - \mu_F(-\infty) = (\mathcal{R}_F + \mathcal{R}_N + \mathcal{R}_c + \delta \mathcal{R}) \]

\[ \delta \mathcal{R} = \frac{R_N (P^2 \Sigma \mathcal{R}_c + P^2 F \mathcal{R}_F) + R_F \mathcal{R}_c (P_{\sigma F} - P_{\Sigma})^2}{R_F + R_c + R_N} > 0 \]

Q: Why is the extra resistance positive?

Spin bottleneck
Silsbee-Johnson spin-charge coupling

Q: Suppose there is a source of nonequilibrium spin at the far right of N. What is the emf due to the proximity with equilibrium spin polarization F?

\[ \text{emf} = P_j \mu_{SN}(\infty) \]
Spin injection in FNF junctions

$$\Delta R = \delta R_{\uparrow\downarrow} - \delta R_{\uparrow\uparrow}$$

**Transparent contacts**

$$\delta R \approx 2R_F P_{\sigma_F}^2$$
Tunnel contacts

\[ \Delta R \approx \frac{2R_c P_\sigma^2}{1 + (R_c/R_N)(d/2L_{SN})} \]

\[ \approx \frac{2R_c P_\sigma^2}{1 + \frac{\tau_{dwell}}{\tau_{SN}}} \]

Maximum signal if

\[ \tau_{dwell} \ll \tau_{SN} \]
Non-local spin injection geometry

Johnson-Silsbee experiment

Q: Suppose electric current drives spin injection in the spin injector circuit $F1/N$ as indicated below. What is the emf in the open detector $F2/N$ junction?

Non-local spin injection geometry

Johnson-Silsbee experiment

\[ \text{cmf} = \mu_N(\infty) - \mu_{F2}(\infty) \]

\[ R_{nl} = \frac{\text{emf}}{j} = \frac{R_N}{2} P^0_{j1} P^0_{j2} \frac{e^{-d/L_{SN}}}{\kappa} \]

Tunnel contacts

\[ R_{nl} = \frac{R_N}{2} P_{\Sigma 1} P_{\Sigma 2} e^{-d/L_{SN}} \]

Measurement of spin diffusion length
:experimental examples:
electrical spin injection into semiconductors

- Theoretically predicted by
  
  [Sov. Phys. Semicond. 10, 698-700 (1976)]

- Experimental realization in semiconductors:

visualizing spin injection

S. A. Crooker et al., JAP, 101, 081716 (2007)

spin injection into silicon

spin injection into graphene

single-layer on a SiO₂ substrate, room temperature

\[ \tau_s \approx 100 \text{ ps} \]

N. Tombros, C. Jozsa, M. Popinciuc, H. T. Jonkman, and B. J. van Wees

Optical Spin Injection
(usually called optical orientation)
Zincblende band structure (GaAs) optical orientation transitions

\[ P_{\sigma^+} \left( -\frac{3}{2} \rightarrow -\frac{1}{2} \right) = 3 \]
\[ P_{\sigma^+} \left( -\frac{1}{2} \rightarrow \frac{1}{2} \right) = 1 \]
\[ P_n = \frac{n_{\frac{1}{2}} - n_{-\frac{1}{2}}}{n_{\frac{1}{2}} + n_{-\frac{1}{2}}} = \frac{1 - 3}{1 + 3} = -\frac{1}{2} \]

Spin relaxation
Spin relaxation for pedestrians

Spin-orbit coupling

$\tau \sim 10^{-14}s$

$T_1 \sim 0.1 ns$

$L_s \sim 0.1 - 100 \mu m$

$L_s = \sqrt{DT_1}$
**key concepts:**

spin relaxation and dephasing

\[ B = B_0 z + B_1 \]

**Bloch eqs**

\[
\begin{align*}
\frac{\partial M_x}{\partial t} &= \gamma (M \times B)_x - \frac{M_x}{T_2} + D \nabla^2 M_x \\
\frac{\partial M_y}{\partial t} &= \gamma (M \times B)_y - \frac{M_y}{T_2} + D \nabla^2 M_y \\
\frac{\partial M_z}{\partial t} &= \gamma (M \times B)_z - \frac{M_z - M^0_z}{T_1} + D \nabla^2 M_z
\end{align*}
\]

*T_1* ... spin relaxation (longitudinal, spin lattice) time  

*T_2* ... spin dephasing (transverse, decoherence) time
Spin relaxation measurements

Spectral characteristics of magnetization depolarization

- CESR (resonant absorption of microwaves by Zeeman split electron system)
- Optical orientation

Time and space correlations of magnetization

- Johnson-Silsbee spin injection
- Time-resolved photoluminescence
- Faraday and (magneto-optic) Kerr effects (rotation of polarization plane of linearly polarized light transmitted—Faraday—or reflected---Kerr---by a magnetized sample; 100 fs resolution)
- spin-to-charge conversion
Time-resolved Faraday rotation

Source: web site of Awschalom’s group

ZnCdSe QW
mechanisms of spin relaxation

Elliott-Yafet mechanism
Elemental metals and semiconductors

Dyakonov-Perel mechanism
Semiconductors without center of inversion symmetry

Bir-Aronov-Pikus mechanism
Heavily p-doped semiconductors

Hyperfine interaction
Electrons bound on impurity sites or confined in quantum dots

J. Fabian, A. Matos-Abiague, C. Ertler, P. Stano, and I. Zutic,
spin relaxation:
GaAs, Si

GaAs (low temperature)

Si (non-degenerate densities)

spin relaxation time longest among semiconductors (+ graphene)


Elliott-Yafet mechanism
simple model of spin relaxation and dephasing
spin in a random fluctuating magnetic field
(almost Dyakonov-Perel mechanism)

J. Fabian, A. Matos-Abiague, C. Ertler, P. Stano, and I. Zutic,
\[ H = \frac{1}{2} \hbar \omega_0 \sigma_z + \frac{1}{2} \hbar \omega(t) \cdot \sigma \]

Zeeman $B = B_0 z$

fluctuating

\[ \omega_i(t) \omega_j(t') = \omega_i^2 \delta_{ij} e^{-|t-t'|/\tau_c} \]

Goal: get $s_x$, $s_y$, and $s_z$ and compare with Bloch to extract $1/T_1$ and $1/T_2$. 
General strategy

1. Hamiltonian

\[ H = H_0 + V(t) \]

2. Density matrix

\[ \dot{\rho} = \frac{1}{i\hbar} [H, \rho] \]

3. Interaction picture

\[
\begin{align*}
\rho_I(t) &= e^{iH_0t/\hbar} \rho(t) e^{-iH_0t/\hbar} \\
V_I(t) &= e^{iH_0t/\hbar} V(t) e^{-iH_0t/\hbar}
\end{align*}
\]

4. “Rotating frame”

\[ \dot{\rho}_I = \frac{1}{i\hbar} [V_I(t), \rho_I] \]
Iterative solution

5. Second order–exact

\[ \rho_I(t) = \rho_I(0) + \frac{1}{i\hbar} \int_0^t [V_I(t'), \rho_I(0)] dt' + \]
\[ + \left( \frac{1}{i\hbar} \right)^2 \int_0^t \int_0^{t'} [V_I(t'), [V_I(t''), \rho_I(t'')]] dt' dt'' \]

6. Nonperturbative–decay: equation of motion

\[ \dot{\rho}_I(t) = \frac{1}{i\hbar} [V_I(t), \rho_I(0)] + \left( \frac{1}{i\hbar} \right)^2 \int_0^t [V_I(t), [V_I(t'), \rho_I(t')]] dt' \]
7. **Linear term vanishes**

\[
\frac{1}{i\hbar} [V_I(t), \rho_I(0)] = 0, \quad \overline{V(t)} = 0
\]

8. **Quadratic term–perturbation:** \( \rho_I(t) \approx \rho_I(0) \)

\[
\dot{\rho}_I(t) = + \left( \frac{1}{i\hbar} \right)^2 \int_0^t \overline{[V_I(t), [V_I(t'), \rho_I(t')]]} dt'
\]

instead of

\[
\dot{\rho}_I(t) = + \left( \frac{1}{i\hbar} \right)^2 \int_0^t \overline{[V_I(t), [V_I(t'), \rho_I(t')]]} dt'
\]
Markov approximation
(coarse graining)

9. Future given by present, NOT past

$$\dot{\rho}_I(t) = + \left( \frac{1}{i\hbar} \right)^2 \int_{0}^{\infty} \Theta(t - t') \tau_c \left[ [V_I(t), [V_I(t'), \rho_I(t)]] \right] dt'$$

10. NEXT: specific Hamiltonian systems, bath degrees of freedom: fluctuating magnetic field, phonons, nuclear spins, ...
our system of fluctuating field:

\[ H_0 = \frac{1}{2} \hbar \omega_0 \sigma_z \]

\[ V(t) = \frac{1}{2} \hbar \omega(t) \cdot \sigma \]

\[ V_I(t) = \frac{1}{2} \hbar \omega(t) \cdot \sigma_I \]

\[ \rho(t) = \frac{1}{2} + s(t) \cdot \sigma \]

\[ \rho_I(t) = \frac{1}{2} + s(t) \cdot \sigma_I(t) \]
after “some” algebra ...

\[
\frac{1}{T_1} = \left( \frac{2}{\omega_x^2} + \frac{2}{\omega_y^2} \right) \frac{\tau_c}{\omega_0^2 \tau_c^2 + 1}
\]

\[
\frac{1}{T_2} = \omega_z^2 \tau_c + \omega_y^2 \frac{\tau_c}{\omega_0^2 \tau_c^2 + 1}
\]

In rotating frame

\[
\int_{0}^{t \gg \tau_c} dt' \omega_z(0) \omega_z(t') = \omega_z^2 \tau_c
\]

\[
\int_{0}^{t \gg \tau_c} dt' \omega_y(0) \omega_y(t') \cos \omega_0 t' = \omega_y^2 \frac{\tau_c}{\omega_0^2 \tau_c^2 + 1}
\]

effective correlation times
I. Precession frequency fluctuations---motional narrowing (see later): $1/T_2$—secular broadening

II. Lifetime broadening: $1/2T_1$

\[
\frac{1}{T_2} = \frac{1}{T_2'} + \frac{1}{2T_1}
\]
discussion 2

\[
\frac{1}{T_1} = \left( \frac{\omega_x^2 + \omega_y^2}{\omega_0^2 \tau_c^2} \right) \frac{\tau_c}{\omega_0^2 \tau_c^2 + 1}
\]

\[
\frac{1}{T_2} = \frac{\omega_z^2 \tau_c}{\omega_0^2 \tau_c^2} + \frac{\omega_y^2 \tau_c}{\omega_0^2 \tau_c^2 + 1}
\]

1. $\omega_0 \tau_c \ll 1$ and isotropic:

\[T_1 = T_2\]

2. $\omega_0 \tau_c \gg 1$:

\[T_1 \to \infty \quad T_2 \to \frac{\omega_z^2 \tau_c}{\omega_0^2 \tau_c^2} \]
motional narrowing

why $1/T_{1,2} \sim \tau_c$

Let $\Omega$ can flip randomly between two directions, up and down. Let $\tau_c$ be the corresponding correlation time and $\tau_c \ll 1/\Omega$. Then

$$\delta \varphi = \Omega \tau_c$$

is the phase change of the spin in one step. After time $t$ (that is, $N = t/\tau_c$ steps),

$$\Delta \varphi = \sqrt{\Delta \varphi^2} = \sqrt{N} \delta \phi = \sqrt{t/\tau_c \Omega \tau_c}$$

Spin dephasing:

$$\Delta \varphi(t = \tau_s) = 1 \rightarrow 1/\tau_s = \Omega^2 \tau_c$$
Spin as a qubit: quantum computing in quantum dots
:nanospintronics:

spin-based quantum information processing

D. Loss and D. P. DiVincenzo, PRA 57, 120 (1998)

\[ H(t) = \sum_{i,j} J_{ij}(t) S_i \cdot S_j + g^* \mu_B \sum_i S_i \cdot B_i(t) \]

- single and few spins manipulation and detection
- spin relaxation and decoherence
- entanglement control (EDAP: Fabian, Hohenester, PRB 71, 201304 (2005))
A. Wild et al., Electrostatically defined quantum dots in a Si/SiGe heterostructure, NJP 12, 113019 (2010).