Magneto-Gyrotropic Photogalvanic Effects in Semiconductor Quantum Wells

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(Dated: January 26, 2005)

We show that free-carrier (Drude) absorption of both polarized and unpolarized terahertz radiation in quantum well (QW) structures causes an electric photocurrent in the presence of an in-plane magnetic field. Experimental and theoretical analysis evidences that the observed photocurrents are spin-dependent and related to the gyrotropy of the QWs. Microscopic models for the photogalvanic effects in QWs based on asymmetry of photoexcitation and relaxation processes are proposed. In most of the investigated structures the observed magneto-induced photocurrents are caused by spin-dependent relaxation of non-equilibrium carriers.
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1. INTRODUCTION

Much current interest in condensed matter physics is directed towards understanding of spin dependent phenomena. In particular, the spin of electrons and holes in solid state systems is the decisive ingredient for spintronic devices [1]. Recently spin photocurrents generated in QWs and bulk materials have attracted considerable attention [2, 3]. Among them are currents caused by a gradient of a spin-polarized electron density [4–6], the spin-galvanic effect [7], the circular photogalvanic effect in QWs [8], pure spin currents under simultaneous one- and two-photon coherent excitation [9, 10] and spin-polarized currents due to the photo-voltaic effect in p-n junctions [11]. Experimentally, a natural way to generate spin photocurrents is the optical excitation with circularly polarized radiation. The absorption of circularly polarized light results in optical spin orientation of free carriers due to a transfer of photon angular momenta to the carriers [12]. Because of the spin-orbit coupling such excitation may result in an electric current. A characteristic feature of this electric current is that it reverses its direction upon changing the radiation helicity from left-handed to right-handed and vice versa.

However, in an external magnetic field spin photocurrents may be generated even by unpolarized radiation as it has been proposed for bulk gyrotropic crystals [13, 14]. Here we report on an observation of these spin photocurrents in QW structures caused by the Drude absorption of terahertz radiation. We show that, microscopically, the effects under study are related to the gyrotropic properties of the structures. The gyrotropic point group symmetry makes no difference between components of axial and polar vectors, and hence allows an electric current \( j \propto IB \), where \( I \) is the light intensity and \( B \) is the applied magnetic field. Photocurrents which require simultaneously gyrotropy and the presence of a magnetic field may be gathered in a class of magneto-optical phenomena denoted as magneto-gyrotropic photogalvanic effects. So far such currents were intensively studied in low-dimensional structures at direct inter-band and inter-subband transitions [15–22]. In these investigations the magneto-induced photocurrents were related to spin independent mechanisms, except for Refs. [15, 20] where direct optical transitions between branches of the spin-split electron subband were considered. This mechanism requires, however, the spin splitting and the photon energy to be comparable whereas, in the conditions under study here, the spin splitting is much smaller than the photon energy and the light absorption
occurs due to indirect (Drude-like) optical transitions. It is clear that magneto-gyrotropic effects due to the Drude absorption may also be observed at excitation in the microwave range where the basic mechanism is free carrier absorption as well. This could link electronics to spin-optics. In most of the investigated structures, the photogalvanic measurements reveal a magneto-induced current which is independent of the direction of light in-plane linear polarization and related to spin-dependent relaxation of non-equilibrium carriers. In addition, our results show that, without a magnetic field, non-equilibrium free carrier heating can be accompanied by spin flow similar to spin currents induced in experiments with simultaneous one- and two-photon coherent excitation [10] or in the spin Hall effect [23, 24].

2. PHENOMENOLOGICAL THEORY

Illumination of gyrotropic nanostructures in the presence of a magnetic field may result in a photocurrent. There is a number of contributions to the magnetic field induced photogalvanic effect whose microscopic origins will be considered in Section 5. The contributions are characterized by different dependencies of the photocurrent magnitude and direction on the radiation polarization state and the orientation of the magnetic field with respect to the crystallographic axes. As a consequence, a proper choice of experimental geometry allows to investigate each contribution separately. Generally, the dependence of the photocurrent on the light polarization and orientation of the magnetic field may be obtained from phenomenological theory which does not require knowledge of the microscopic origin of the current. Within the linear approximation in the magnetic field strength $B$, the magneto-photogalvanic effect (MPGE) is given by

$$ j_{\alpha} = \sum_{\beta\gamma\delta} \phi_{\alpha\beta\gamma\delta} B_{\beta} \{ E_{\gamma} E_{\delta}^* \} + \sum_{\beta\gamma} \mu_{\alpha\beta\gamma} B_{\beta} \hat{e}_{\gamma} |E_0|^2 P_{\text{circ}} . $$

Here the fourth rank pseudo-tensor $\phi$ is symmetric in the last two indices, $E_{\gamma}$ are components of the complex amplitude of the radiation electric field $E$. In the following the field is presented as $E = E_0 e$ with $E_0$ being the modulus $|E|$ and $e$ indicating the (complex) polarization unit vector, $|e| = 1$. The symbol $\{ E_{\gamma} E_{\delta}^* \}$ means the symmetrized product of the electric field with its complex conjugate,

$$ \{ E_{\gamma} E_{\delta}^* \} = \frac{1}{2} \left( E_{\gamma} E_{\delta}^* + E_{\delta} E_{\gamma}^* \right) . $$
φ components of the tensors belong to one of the point groups. Depending on the equivalence or non-equivalence of the QW interfaces their symmetry may be non-zero even for unpolarized radiation.

While the second term requires circularly polarized radiation the first term is the helicity of the radiation and \( \hat{e} \) is the unit vector pointing in the direction of light propagation. While the second term requires circularly polarized radiation the first term may be non-zero even for unpolarized radiation.

We consider (001)-oriented QWs based on zinc-blende-lattice III-V or II-VI compounds. Depending on the equivalence or non-equivalence of the QW interfaces their symmetry may belong to one of the point groups \( D_{2d} \) or \( C_{2v} \), respectively. The present experiments have been carried out on the \( C_{2v} \) symmetry structures and, therefore, here we will focus on them only.

For the \( C_{2v} \) point group, it is convenient to write the components of the magneto-photocurrent in the coordinate system with \( x' \parallel [\overline{1}10] \) and \( y' \parallel [110] \) or in the system \( x \parallel [100] \) and \( y \parallel [010] \). The advantage of the former system is that the in-plane axes \( x', y' \) lie in the crystallographic planes (110) and (1\overline{1}0) which are the mirror reflection planes containing the two-fold axis \( C_2 \). In the system \( x', y', z \) for normal incidence of the light and the in-plane magnetic field, Eq. (1) is reduced to

\[
\begin{align*}
S_1 &= \frac{1}{2}(\phi_{x'x'x'} + \phi_{y'y'y'}) \\
S_1' &= \frac{1}{2}(\phi_{y'y'y'} + \phi_{x'x'x'}) \\
S_2 &= \frac{1}{2}(\phi_{x'x'x'} - \phi_{y'y'y'}) \\
S_2' &= \frac{1}{2}(\phi_{y'y'y'} - \phi_{x'x'x'}) \\
S_3 &= \phi_{x'x'y'} = \phi_{x'x'y'} \\
S_3' &= \phi_{y'y'x'} = \phi_{y'y'x'} \\
S_4 &= \mu_{x'x'z} \\
S_4' &= \mu_{y'y'z}
\end{align*}
\]

| TABLE I: Definition of the parameters \( S_i \) and \( S'_i \) \( (i = 1 \ldots 4) \) in Eqs. (3) in terms of non-zero components of the tensors \( \phi \) and \( \mu \) for the coordinates \( x' || [\overline{1}10] \), \( y' || [110] \) and \( z || [001] \). The \( C_{2v} \) symmetry and normal incidence of radiation along \( z \) are assumed.

In the second term on the right hand side of Eq. (1), \( \mu \) is a regular third rank tensor, \( P_{circ} \) is the helicity of the radiation and \( \hat{e} \) is the unit vector pointing in the direction of light propagation. While the second term requires circularly polarized radiation the first term may be non-zero even for unpolarized radiation.
This current is induced even by unpolarized radiation. Each following contribution has a special polarization dependence which permits to separate it experimentally from the others.

**Linearly polarized radiation.** For linearly polarized light, the terms described by parameters $S_2, S'_2$ and $S_3, S'_3$ are proportional to $|e_{x'}|^2 - |e_{y'}|^2 = \cos 2\alpha$ and $e_{x'}e_{y'}^* + e_{y'}e_{x'}^* = \sin 2\alpha$, respectively, where $\alpha$ is the angle between the plane of linear polarization and the $x'$ axis. Hence the current reaches maximum values for light polarized either along $x'$ or $y'$ for the second terms (parameters $S_2, S'_2$), or along the bisector of $x', y'$ for the third terms, proportional to $S_3, S'_3$. The last terms (parameters $S_4, S'_4$), being proportional to $P_{\text{circ}}$, vanish for linearly polarized excitation.

**Elliptically polarized radiation.** For elliptically polarized light all contributions are allowed. In the experiments discussed below, elliptically and, in particular, circularly polarized radiation was achieved by passing laser radiation, initially linearly polarized along $x'$ axis, through a $\lambda/4$-plate. Rotation of the plate results in a variation of both linear polarization and helicity as follows

\[ P_{\text{lin}} \equiv \frac{1}{2}(e_{x'}e_{y'}^* + e_{y'}e_{x'}^*) = \frac{1}{4}\sin 4\varphi, \]
\[ P'_{\text{lin}} \equiv \frac{1}{2}(|e_{x'}|^2 - |e_{y'}|^2) = \frac{1 + \cos 4\varphi}{4}, \]
\[ P_{\text{circ}} = \sin 2\varphi. \]

Two Stokes parameters $P_{\text{lin}}, P'_{\text{lin}}$ describe the degrees of linear polarization and $\varphi$ is the angle between the optical axis of $\lambda/4$ plate and the direction of the initial polarization $x'$.

As described above, the first terms on the right hand side of Eqs. (3) are independent of the radiation polarization. The polarization dependencies of magneto-induced photocurrents caused by second and third terms in Eqs. (3) are proportional to $P'_{\text{lin}}$ and $P_{\text{lin}}$, respectively. These terms vanish if the radiation is circularly polarized. In contrast, the last terms in Eqs. (3) describe a photocurrent proportional to the helicity of radiation. It is zero for linearly polarized radiation and reaches its maximum for left- or right-handed circular polarization. Switching helicity $P_{\text{circ}}$ from $+1$ to $-1$ reverses the current direction.
TABLE II: Definition of the parameters $S^+_i$ and $S^-_i$ ($i = 1 \ldots 4$) in Eqs. (7) in terms of non-zero components of the tensors $\phi$ and $\mu$ for the coordinates $x \parallel [100]$, $y \parallel [010]$ and $z \parallel [001]$. The $C_{2v}$ symmetry and normal incidence of radiation along $z$ are assumed.

As we will see below the photocurrent analysis for $x \parallel [100]$ and $y \parallel [010]$ directions helps to conclude on the microscopic nature of the different contributions to the MPGE. In these axes Eqs. (3) read

$$j_x = S^+_1 B_x I + S^-_1 B_y I - (S^+_2 B_x + S^-_2 B_y) \left( e_x e^*_y + e_y e^*_x \right) I$$

$$+ \left( S^+_3 B_x - S^-_3 B_y \right) \left( |e_x|^2 - |e_y|^2 \right) I + (S^+_4 B_x - S^-_4 B_y) IP_{\text{circ}} ,$$

$$j_y = -S^-_1 B_x I - S^+_1 B_y I + (S^-_2 B_x + S^+_2 B_y) \left( e_x e^*_y + e_y e^*_x \right) I$$

$$+ \left( -S^-_3 B_x + S^+_3 B_y \right) \left( |e_x|^2 - |e_y|^2 \right) I + (-S^-_4 B_x + S^+_4 B_y) IP_{\text{circ}} , \quad (7)$$

where $S^\pm_l = (S_l \pm S^l_0)/2$ ($l = 1 \ldots 4$). The parameters $S^+_1$ to $S^+_4$ expressed via non-zero elements of the tensors $\phi$ and $\mu$ for the $C_{2v}$ symmetry are given in Table II. Equations (7) show that, for a magnetic field oriented along a cubic axis, all eight parameters $S^\pm_1$ contribute to the photocurrent components, either normal or parallel to the magnetic field. However, as well as for the magnetic field oriented along $x'$ or $y'$ the partial contributions can be separated analyzing polarization dependencies.
For the sake of completeness, in Appendices A and B we present the phenomenological equations for the magneto-photocurrents in the systems of the $T_d$ and $C_{\infty v}$ symmetries, respectively, representing the bulk zinc-blende-lattice semiconductors and axially-symmetric QWs with nonequivalent interfaces.

Summarizing the macroscopic picture we note that, for normal incidence of the radiation on a (001)-grown QW, a magnetic field applied in the interface plane is required to obtain a photocurrent. In Table III we present the relations between the photocurrent direction, the state of light polarization and the magnetic field orientation which follow from Eqs. (3) and Eqs. (7) and determine the appropriate experimental geometries (Section 4). In order to ease data analysis we give in Table IV polarization dependencies for geometries relevant to experiment. Specific polarization behavior of each term allows to discriminate between different terms in Eqs. (3).

| $B||x'$ | $1^{st}$ term | $2^{nd}$ term | $3^{rd}$ term | $4^{th}$ term |
|---------|-------------|-------------|-------------|-------------|
| $j'x'/I$ | 0           | 0           | $S_3 B_{x'} (e_x' e_{y'}^* + e_{y'} e_{x'}^*)$ | $S_4 B_{x'} P_{circ}$ |
| $j'y'/I$ | $S_1^t B_{x'}$ | $S_2^t B_{x'} (|e_x'|^2 - |e_{y'}|^2)$ | 0           | 0           |

| $B||y'$ | $1^{st}$ term | $2^{nd}$ term | $3^{rd}$ term | $4^{th}$ term |
|---------|-------------|-------------|-------------|-------------|
| $j'x'/I$ | $S_1 B_{y'}$ | $S_2 B_{y'} (|e_x'|^2 - |e_{y'}|^2)$ | 0           | 0           |
| $j'y'/I$ | 0           | 0           | $S_3^t B_{y'} (e_x' e_{y'}^* + e_{y'} e_{x'}^*)$ | $S_4^t B_{y'} P_{circ}$ |

| $B||x$ | $1^{st}$ term | $2^{nd}$ term | $3^{rd}$ term | $4^{th}$ term |
|--------|-------------|-------------|-------------|-------------|
| $j_x/I$ | $S_1^t B_x$ | $-S_2^t B_x (e_x e_y^* + e_y e_x^*)$ | $S_3^t B_x (|e_x|^2 - |e_y|^2)$ | $S_4^t B_x P_{circ}$ |
| $j_y/I$ | $-S_1^t B_x$ | $S_2^t B_x (e_x e_y^* + e_y e_x^*)$ | $-S_3^t B_x (|e_x|^2 - |e_y|^2)$ | $-S_4^t B_x P_{circ}$ |

| $B||y$ | $1^{st}$ term | $2^{nd}$ term | $3^{rd}$ term | $4^{th}$ term |
|--------|-------------|-------------|-------------|-------------|
| $j_x/I$ | $S_1^t B_y$ | $-S_2^t B_y (e_x e_y^* + e_y e_x^*)$ | $-S_3^t B_y (|e_x|^2 - |e_y|^2)$ | $-S_4^t B_y P_{circ}$ |
| $j_y/I$ | $-S_1^t B_y$ | $S_2^t B_y (e_x e_y^* + e_y e_x^*)$ | $S_3^t B_y (|e_x|^2 - |e_y|^2)$ | $S_4^t B_y P_{circ}$ |

TABLE III: Contribution of the different terms in Eqs. (3) and Eqs. (7) to the current at different magnetic field orientations. The two left columns indicate the magnetic field orientation and the photocurrent component, respectively.
TABLE IV: Polarization dependencies of different terms in Eqs. (3) at $\mathbf{B} \parallel y'$.

<table>
<thead>
<tr>
<th></th>
<th>1&lt;sup&gt;st&lt;/sup&gt; term</th>
<th>2&lt;sup&gt;nd&lt;/sup&gt; term</th>
<th>3&lt;sup&gt;rd&lt;/sup&gt; term</th>
<th>4&lt;sup&gt;th&lt;/sup&gt; term</th>
</tr>
</thead>
<tbody>
<tr>
<td>$j \parallel x'$</td>
<td>$j_x\phi$</td>
<td>$S_1IB_y(1 + \cos 4\varphi)/2$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$j \parallel y'$</td>
<td>$S_1IB_y\cos 2\alpha$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

3. METHODS

The experiments were carried out on MBE-grown (001)-oriented $n$-type GaAs/Al$_{0.3}$Ga$_{0.7}$As and InAs/AlGaSb QW structures. The characteristics of the investigated samples are given in Table V. The InAs/AlGaSb heterostructure were grown on a semi-insulating GaAs substrate. The quantum well is nominally undoped, but contains a two dimensional electron gas with the carrier density of $8 \cdot 10^{11}$ cm$^{-2}$ at 4.2 K located in the InAs channel. Details of the growth procedure are given in [25]. All GaAs samples are modulation-doped. For samples A2–A4 Si-$\delta$-doping, either one-sided with spacer layer thicknesses of 70 nm (A3) and 80 nm (A4), or double-sided with 70 nm spacer layer thickness (A2), has been used. In contrast, for sample A5 the AlGaAs barrier layer separating the QWs has been homogeneously Si-doped on a length of 30 nm. In the sample with a QW separation of 40 nm, this results in a spacer thickness of only 5 nm. Therefore, in addition to the different impurity distribution compared to the samples A2–A4, the sample A5 has much lower mobility.

All samples have two pairs of ohmic contacts at the corners corresponding to the $x \parallel [100]$ and $y \parallel [010]$ directions, and two additional pairs of contacts centered at opposite sample edges with the connecting lines along $x' \parallel [\overline{1}10]$ and $y' \parallel [110]$ (see inset in Fig. 1). The external magnetic field $\mathbf{B}$ up to 1 T was applied parallel to the interface plane.

A pulsed optically pumped terahertz laser was used for optical excitation [26]. With NH$_3$ as active gas 100 ns pulses of linearly polarized radiation with $\sim$10 kW power have been obtained at wavelengths 148 $\mu$m and 90 $\mu$m. The terahertz radiation induces free carrier absorption in the lowest conduction subband $e_1$ because the photon energy is smaller than the subband separation and much larger than the $\mathbf{k}$-linear spin splitting. The samples were
TABLE V: Parameters for non-illuminated samples at $T = 4.2$ K.

<table>
<thead>
<tr>
<th>Structure</th>
<th>Mobility cm²/V·s</th>
<th>Electron density cm⁻²</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1 (001)-InAs single QW of 15 nm width</td>
<td>$\approx 3 \cdot 10^5$</td>
<td>$8 \cdot 10^{11}$</td>
</tr>
<tr>
<td>A2 (001)-GaAs double QW of 9.0 and 10.8 nm width</td>
<td>$1.4 \cdot 10^5$</td>
<td>$1.12 \cdot 10^{11}$</td>
</tr>
<tr>
<td>A3 (001)-GaAs heterojunction</td>
<td>$3.53 \cdot 10^6$</td>
<td>$1.08 \cdot 10^{11}$</td>
</tr>
<tr>
<td>A4 (001)-GaAs heterojunction</td>
<td>$3.5 \cdot 10^6$</td>
<td>$1.1 \cdot 10^{11}$</td>
</tr>
<tr>
<td>A5 (001)-GaAs multiple QW (30 QWs of 8.2 nm width)</td>
<td>$2.57 \cdot 10^4$</td>
<td>$9.3 \cdot 10^{11}$</td>
</tr>
</tbody>
</table>

irradiated along the growth direction.

In order to vary the angle between the polarization vector of the linearly polarized light and the magnetic field we placed a metal mesh polarizer behind a crystalline quartz $\lambda/4$-plate. After passing through the $\lambda/4$-plate initially linearly polarized laser light became circularly polarized. Rotation of the metal grid enabled us to obtain linearly polarized radiation with angle $\alpha = 0^\circ \div 360^\circ$ between the $x'$ axis and the plane of linear polarization of the light incident upon the sample.

To obtain elliptically and, in particular, circularly polarized radiation the mesh polarizer behind the quartz $\lambda/4$-plate was removed. The helicity $P_{\text{circ}}$ of the incident light was varied by rotating the $\lambda/4$-plate according to $P_{\text{circ}} = \sin 2\varphi$ as given by Eq. (6). For $\varphi = n \cdot \pi/2$ with integer $n$ the radiation was linearly polarized. Circular polarization was achieved with $\varphi = (2n + 1) \cdot (\pi/4)$, where even values of $n$ including $n = 0$ yield the right-handed circular polarization $\sigma_+$ and odd $n$ give the left-handed circular polarization $\sigma_-$. The photocurrent $j$ was measured at room temperature in unbiased structures via the voltage drop across a $50 \Omega$ load resistor in closed circuit configuration. The voltage was measured with a storage oscilloscope. The measured current pulses of 100 ns duration reflected the corresponding laser pulses.

4. EXPERIMENTAL RESULTS

As follows from Eqs. (3), the most suitable experimental arrangement for independent investigation of different contributions to the magneto-induced photogalvanic effect is achieved by applying magnetic field along one of the crystallographic axes $x' \parallel [1\bar{1}0], y' \parallel [110]$ and mea-
FIG. 1: Magnetic field dependence of the photocurrent measured in sample A1 at room temperature with the magnetic field $B$ parallel to the $y'$ direction. Normally incident optical excitation of $P \approx 4$ kW is performed at wavelength $\lambda = 148 \mu$m with linear ($E \parallel x'$), right-handed circular ($\sigma_+$), and left-handed circular ($\sigma_-$) polarization. The measured current component is parallel to $B$. The inset shows the experimental geometry.

suring the in-plane current along or normal to the magnetic field direction. Then, currents flowing perpendicular to the magnetic field, contain contributions proportional only to the parameters $S_1$ and $S_2$ if $B \parallel y'$ (or $S'_1$ and $S'_2$ if $B \parallel x'$), whereas, currents flowing parallel to the magnetic field arise only from terms proportional to $S_3$ and $S_4$ (or $S'_3$ and $S'_4$). Further separation of contributions may be obtained by making use of the difference in their polarization dependencies. The results obtained for $\lambda = 90 \mu$m and $\lambda = 148 \mu$m are qualitatively the same. Therefore we present only data obtained for $\lambda = 148 \mu$m.

4.1. Photocurrent parallel to the magnetic field ($j \parallel B \parallel y' \parallel [110]$)

According to Eqs. (3) and Table IV only two contributions proportional to $S'_3$ and $S'_4$ are allowed in this configuration. While the $S'_3$ contribution results in a current for linear or elliptical polarization, the $S'_4$ one vanishes for linear polarization and assumes its maximum at circular polarization.

Irradiation of the samples A1–A4 subjected to an in-plane magnetic field with normally incident linearly polarized radiation cause no photocurrent. However, elliptically polarized light yields a helicity dependent current. Typical magnetic field and helicity dependencies of this current are shown in Figs. 1 and 2. The polarity of the current changes upon
FIG. 2: Photocurrent as a function of the phase angle $\varphi$ defining the helicity. The photocurrent signal is measured in sample A1 at room temperature in the configuration $j \parallel B \parallel y'$ for two opposite directions of the magnetic field under normal incidence of the radiation with $\lambda = 148 \ \mu m$ ($P \approx 4 \ kW$). The broken and full lines are fitted after Eq. (6).

reversal of the applied magnetic field as well as upon changing the helicity from right- to left-handed. The polarization behavior of the current is well described by $j_y' \propto IB_y'P_{\text{circ}}$. This means that the current is dominated by the last term on the right side of the second equation (3) (parameter $S'_4$) while the third term is vanishingly small. Observation of a photocurrent proportional to $P_{\text{circ}}$ has already been reported previously. This is the spin-galvanic effect [7]. The effect is caused by the optical orientation of carriers, subsequent Larmor precession of the oriented electronic spins and asymmetric spin relaxation processes. Though, in general, the spin-galvanic current does not require an application of magnetic field, it may be considered as a magneto-photogalvanic effect under the above experimental conditions.

One of our QW structures, sample A5, showed a quite different behavior. In this sample the dependence of the magneto-induced photocurrent on $\varphi$ is well described by $j_y' \propto IB_y'\sin 4\varphi$ (see Fig. 3). In contrast to the samples A1–A4, in the sample A5 the spin-galvanic effect is overweighed by the contribution of the third term in Eqs. (3). The latter should also appear under excitation with linearly polarized radiation. Figure 4 shows the dependence of the photocurrent on the angle $\alpha$ for one direction of the magnetic field. The current $j_y'$ is proportional to $IB_y'\sin 2\alpha$ as expected for the third term in Eqs. (3).
FIG. 3: Photocurrent in the sample A5 as a function of the phase angle $\varphi$ defining the helicity for magnetic fields of two opposite directions. The photocurrent excited by normally incident radiation of $\lambda = 148 \mu m$ ($P \approx 17$ kW) is measured at room temperature, $j \parallel B \parallel y'$, The broken and full lines are fitted after Eq. (4).

FIG. 4: Photocurrent in the sample A5 as a function of the azimuth angle $\alpha$. The photocurrent $j \parallel B \parallel y'$ excited by normally incident linearly polarized radiation of $\lambda = 148 \mu m$ ($P \approx 17$ kW) and measured at room temperature. The broken and full lines are fitted according to Table IV, 3rd term.

4.2. Current perpendicular to the magnetic field ($j \perp B \parallel y' \parallel [110]$)

In the transverse geometry only contributions proportional to the parameters $S_1$ and $S_2$ are allowed. Here the samples A1 to A4 and A5 again show different behavior.

The data of a magnetic field induced photocurrent perpendicular to $B$ in samples A1–A4 are illustrated in Fig. 5. The magnetic field dependence for sample A1 is shown for three
FIG. 5: Magnetic field dependence of the photocurrent measured in sample A1 at room temperature with the magnetic field \( B \) parallel to the \( y' \) axis. Data are given for normally incident optical excitation of \( P \approx 4 \text{ kW} \) at the wavelength \( \lambda = 148 \mu \text{m} \) for linear \( (E \parallel x') \), right-handed circular \( (\sigma_+) \), and left-handed circular \( (\sigma_-) \) polarization. The current is measured in the direction perpendicular to \( B \).

different polarization states. Neither rotation of the polarization plane of the linearly polarized radiation nor variation of helicity changes the signal magnitude. Thus we conclude that the current strength and sign are independent of polarization. On the other hand, the current changes its direction upon the magnetic field reversal. This behavior is described by \( j_{x'} \propto IB_{y'} \) and corresponds to the first term on the right hand side of the first equation in Eqs. (3). The absence of a \( \phi \)-dependence indicates that the second term in Eqs. (3) is negligibly small. Note, that the dominant contribution to the polarization independent magneto-photogalvanic effect, described by the first term on the right side of Eqs. (3), is observed for the same set of samples (A1–A4) where the longitudinal photocurrent is caused by the spin-galvanic effect.

In sample A5 a clear polarization dependence, characteristic for the second terms in Eqs. (3), has been detected. The magnetic field and the polarization dependencies obtained from this sample are shown in Figs. 6, 7 and 8, respectively. For the sample A5 the \( \phi \)-dependence can be well fitted by \( S_1 + S_2(1 + \cos 4\phi)/2 \) while the \( \alpha \)-dependence is \( S_1 + S_2 \cos 2\alpha \), as expected for the first and second terms in Eqs. (3).
FIG. 6: Magnetic field dependence of the photocurrent measured in sample A5 at room temperature with the magnetic field \( B \) parallel to the \( y' \) axis. Data are presented for normally incident optical excitation \( P \approx 17 \text{ kW} \) at the wavelength \( \lambda = 148 \text{ \mu m} \) for the linear \((E \parallel x')\), right-handed circular \((\sigma_+)\), and left-handed circular \((\sigma_-)\) polarization. The current is measured in the direction perpendicular to \( B \).

FIG. 7: Photocurrent in sample A5 as a function of the phase angle \( \varphi \) defining the Stokes parameters, see Eq. (5). The photocurrent excited by normally incident radiation of \( \lambda = 148 \text{ \mu m} \) \( 9P \approx 17 \text{ kW} \) is measured at room temperature, \( j \perp B \parallel y' \). The full and broken lines are fitted according to Table IV, the 1\(^{st}\) and 2\(^{nd}\) terms.

### 4.3. Magnetic field applied along the \( x' \parallel [1\bar{1}0] \) direction

Rotation of \( B \) by 90\(^{\circ}\) with respect to the previous geometry interchanges the role of the axes \( x' \) and \( y' \). Now the magnetic field is applied along the \([1\bar{1}0]\) crystallographic direction. The magnetic field and polarization dependencies observed experimentally in both configu-
FIG. 8: Photocurrent in sample A5 for $j \perp B \parallel y'$ as a function of the azimuth angle $\alpha$. The photocurrent excited by normally incident radiation of $\lambda = 148 \mu m$ ($P \approx 17$ kW) is measured at room temperature for magnetic fields of two opposite directions. The broken and full lines are fitted according to Table IV, the 1st and 2nd terms.

The observed difference in photocurrents is expected for $C_{2v}$ point symmetry of the QW where the axes [110] and [110] are non-equivalent. This is taken into account in Eqs. (3) by introducing independent parameters $S_i$ and $S'_i$ ($i = 1 \ldots 4$).

4.4. Magnetic field applied along the crystallographic axis $x \parallel [100]$

Under application of $B$ along one of the in-plane cubic axes in a (001)-grown structure, all contributions to the photocurrent are allowed. This can be seen from Eqs. (7) and Table III. In all samples both longitudinal and transverse currents are observed for linearly (Fig. 9) as well as circularly (Fig. 10) polarized excitation. In the absence of the magnetic field the current signals vanish for all directions. For the samples A1–A4 a clear spin-galvanic current proportional to helicity $P_{\text{circ}}$ and superimposed on a helicity independent contribution is detected (see Fig. 10). The possibility of extracting the spin-galvanic effect is of particular importance in experiments aimed at the separation of Rashba- and Dresselhaus-like contributions to the spin-orbit interaction as has been recently reported [27].
FIG. 9: Magnetic field dependence of the photocurrent measured in sample A1 with the magnetic field \( B \) parallel to the [100] axis under photoexcitation with normally incident light of the wavelength \( \lambda = 148 \, \mu\text{m} \) (\( P \approx 4 \, \text{kW} \)) for linear polarization \( E \parallel y \). The current is measured in the directions parallel (\( j_x \)) and perpendicular (\( j_y \)) to \( B \).

FIG. 10: Magnetic field dependence of the photocurrent measured in sample A1 with the magnetic field \( B \) parallel to the [100] axis. Optical excitation of \( P \approx 4 \, \text{kW} \) at normal incidence was applied at wavelength \( \lambda = 148 \, \mu\text{m} \) for linear (\( E \parallel y \)), right-handed circular (\( \sigma_+ \)), and left-handed circular (\( \sigma_- \)) polarization. The current is measured in the direction parallel to \( B \).

5. MICROSCOPIC MODELS

The term magneto-photogalvanic effects (MPGE) stands for the generation of magnetic field induced photocurrent under polarized or unpolarized optical excitation. In this Section we give a survey of possible microscopic mechanisms leading to MPGE. Besides mechanisms discussed in literature we also present here novel mechanisms. We start by recalling
non-gyrotropic spin-independent mechanisms used to interpret MPGE observed in bulk non-
centrosymmetric semiconductors (Section 5.1). They are based on the cyclotron motion of
free carriers in both the real and the $k$-space. Since in a QW subjected to an in-plane mag-
netic field, the cyclotron motion is suppressed one needs to seek for alternative mechanisms.
As we will demonstrate below (Sections 5.3 to 5.5), the generation of magneto-induced pho-
tocurrent in quantum wells requires both gyrotropy and magnetic field and therefore the
effects belong to the magneto-gyrotropic class.

5.1. Bulk semiconductors of the $T_d$ point symmetry

In this Section we outline briefly microscopic mechanisms responsible for magneto-
photocurrents generated in bulk materials of the $T_d$ symmetry.

*Non-gyrotropic, spin-independent mechanisms.* The phenomenological description of the
MPGE in the $T_d$-class bulk crystals are described by Eqs. (29)–(31) in Appendix A. Micro-
scopically, the contribution proportional to $S_2$ in Eq. (29) can be easily interpreted [28, 29]
as the Hall rotation of the zero-magnetic field photocurrent. At zero magnetic field the
current $j^{(0)}$ in response to linear polarized radiation is given by

$$
j^{(0)}_x \propto e_y e_z^* + e_z e_y^*, \quad j^{(0)}_y \propto e_z e_x^* + e_x e_z^*, \quad j^{(0)}_z \propto e_x e_y^* + e_y e_x^*.
$$

Applying a magnetic field $B$ yields a current $j$ in the direction parallel to the vector $B \times j^{(0)}$.
The coefficient $S_1$, on the other hand, determines the contribution to the photocurrent arising
even if $j^{(0)} = 0$, e.g., for $e \parallel x$. This particular contribution can be described microscopically
as follows [30] (see also [31, 32]): (a) optical alignment of free-carrier momenta described
by an anisotropic correction to the free-carrier non-equilibrium distribution function, $\delta f(k)$,
proportional to $k_\alpha k_\beta / k^2$; (b) new terms $k_\gamma k_\delta / k^2$ appear due to cyclotron rotation of the free-
carrier distribution function; (c) momentum scattering of free carriers results in an electric
current $j_\eta \propto C_{\eta+1, \eta+2}$, where $\eta = (1, 2, 3) \equiv (x, y, z)$, $C_{\gamma, \delta}$ are the coefficients in the expan-
sion of $\delta f(k)$ over $k_\gamma k_\delta / k^2$. Here, the cyclic permutation of indices is assumed. The current
appears under one-phonon induced free carrier shifts in the real space (the so-called shift con-
tribution) or due to two-phonon asymmetric scattering (the ballistic contribution) [33, 34].
For the polarization $e \parallel x$, the anisotropic part of the free-carrier non-equilibrium distribu-
tion function is proportional to $k^2_x / k^2$. For $B \parallel y$, the cyclotron rotation of this anisotropic
distribution leads to the term $\delta f(k) \propto k_x k_z / k^2$. The further momentum relaxation yields an electric current in the $y$ direction. It should be mentioned that a similar mechanism contributes to $S_2$. It is clear that both this mechanism and the photo-Hall mechanism are spin-independent since the free-carrier spin is not involved here. Note that both mechanisms do exist in bulk crystals of the $T_d$ symmetry which are non-gyrotropic. Therefore they can be classified as non-gyrotropic and spin-independent.

An important point to stress is that the above mechanisms vanish in QWs for an in-plane magnetic field. Because the free-carrier motion is quantized in growth direction the anisotropic correction $\delta f(k) \propto k_\eta k_z / k^2$ ($\eta = x, y$) to the distribution function does not exist.

**Non-gyrotropic, spin-dependent mechanisms.** Two non-gyrotropic but spin-dependent mechanisms causing magnetic field induced photocurrents were proposed for bulk zinc-blende-lattice semiconductors in [19, 35]. In [35] the photocurrent is calculated for optical transitions between spin-split Landau-level subbands under electron spin resonance conditions in the limit of strong magnetic field. Taking into account both the spin-dependent Dresselhaus term, cubic in the wavevector $k$,

$$\mathcal{H}^{(3)}(k) = \gamma[\sigma_x k_x(k_y^2 - k_z^2) + \sigma_y k_y(k_z^2 - k_x^2) + \sigma_z k_z(k_x^2 - k_y^2)]$$  \hspace{1cm} (8)$$

and the quadratic in $k$ Zeeman term

$$\mathcal{H}^{(2)}(B) = G(\sigma \cdot k)(B \cdot k)$$  \hspace{1cm} (9)$$

in the bulk electron Hamiltonian, spin-flip optical transitions lead to asymmetric photoexcitation of electrons in the $k$-space and, hence, to a photocurrent. At a fixed radiation frequency the photocurrent has a resonant nonlinear dependence on the magnetic field and contains contributions both even and odd as a function of $B$. In Ref. [19] the photocurrent under impurity-to-band optical transitions in bulk InSb was described taking into account the quantum-interference of different transition channels one of which includes an intermediate intra-impurity spin-flip process. This photocurrent is proportional to photon momentum and depends on the light propagation direction. Therefore, it can be classified as the photon drag effect which occurs under impurity-to-band optical transitions and is substantially modified by the intra-impurity electron spin resonance. Since in the present work the experiments were performed under normal incidence of radiation of two dimensional structure we will not consider the photon drag effect in the following discussion.
5.2. Effects of gyrotropy in (001)-grown quantum wells

The (001)-grown quantum well structures are characterized by a reduced symmetry $D_{2d}$ (symmetric QWs) or $C_{2v}$ (asymmetric QWs). Generally, for symmetry operations of these point groups, the in-plane components of a polar vector $R$ and an axial vector $L$ transform according to the same representations. In the $C_{2v}$ group there are two invariants which can be constructed from the products $R_\alpha L_\beta$, namely, $I_1 = R_x L_x - R_y L_y = R_{x'} L_{y'} + R_{y'} L_{x'}$, \( (10) \)

$I_2 = R_x L_y - R_y L_x = R_{x'} L_{y'} - R_{y'} L_{x'} \equiv (R \times L)_z. \( (11) \)

The $D_{2d}$ symmetry allows only one invariant, $I_1$. In the following $I_1$- and $I_2$-like functions or operators are referred to as the gyrotropic invariants.

In order to verify that a given function, $I(k', k)$, linear in $B$ or $\sigma$ contains a gyrotropic invariant one can use a simple criterion, namely, multiply $I$ by $k_\eta$ and $k'_\eta (\eta = x, y)$, average the product over the directions of $k'$ and $k$ and check that the average is nonzero. Three examples of gyrotropic invariants relevant to the present work are given below.

The first is the spin-orbit part of the electron effective Hamiltonian,

\[
\mathcal{H}_{BIA}^{(1)} = \beta_{BIA} (\sigma_x k_x - \sigma_y k_y), \quad \mathcal{H}_{SIA}^{(1)} = \beta_{SIA} (\sigma_x k_y - \sigma_y k_x),
\]

\[
\mathcal{H}_{BIA}^{(3)} = \gamma_{BIA} (\sigma_x k_x k_y^2 - \sigma_y k_y k_x^2), \quad \mathcal{H}_{SIA}^{(3)} = \gamma_{SIA} (\sigma_x k_y - \sigma_y k_x)k^2.
\]

Here $\sigma_\alpha$ are the spin Pauli matrices, $k_x$ and $k_y$ are the components of the 2D electron wavevector, $\gamma_{BIA}$ coincides with the parameter $\gamma$ introduced by Eq. (8), $\mathcal{H}_{BIA}^{(1)}$ and $\mathcal{H}_{SIA}^{(1)}$ are the so-called Dresselhaus and Rashba terms being linear in $k$ or, respectively, bulk inversion asymmetry (BIA) and structure inversion asymmetry (SIA) terms. The terms $\mathcal{H}_{BIA}^{(1)}$ and $\mathcal{H}_{BIA}^{(3)}$, linear and cubic in $k$, stem from averaging the cubic-$k$ spin-dependent Hamiltonian Eq. (8).

The second example of a gyrotropic invariant is the well known diamagnetic band shift existing in asymmetric QWs [36–38], see also [39–41]. This spin-independent contribution to the electron effective Hamiltonian reads

\[
\mathcal{H}_{SIA}^{\text{dia}} = \tilde{\alpha}_{SIA} (B_x k_y - B_y k_x). \( 13 \)
The coefficient $\tilde{\alpha}_{SIA}^{(\nu)}$ in the $\nu$-th electron subband is given by $\tilde{\alpha}_{SIA}^{(\nu)} = (\hbar e/m^*)\bar{z}_\nu$, where $m^*$ is the effective electron mass, and $\bar{z}_\nu = \langle e\nu | z | e\nu \rangle$ is the center of mass of the electron envelope function in this subband.

The last example is an asymmetric part of electron-phonon interaction. In contrast to the previous two examples it does not modify the single-electron spectrum but can give rise to spin dependent effects. It leads, e.g., to spin photocurrents considered in Sections 5.3 and 5.4. The asymmetric part of electron-phonon interaction is given by

$$\hat{V}_{cl-phon}(k', k) = \Xi_c \sum_j \epsilon_{jj} + \Xi_{cv} \xi \sum_j \left[ (k' + k) \times \sigma \right]_j \epsilon_{j+1,j+2}. \quad (14)$$

Here $\epsilon_{jj'}$ is the phonon-induced strain tensor dependent on the phonon wavevector $q = k' - k$, $\Xi_c$ and $\Xi_{cv}$ are the intra- and inter-band constants of the deformation potential. For zinc-blende-lattice QWs the coefficient $\xi$ is given by [42]

$$\xi = \frac{i\hbar p_{cv}}{3m_0 \varepsilon_g (\varepsilon_g + \Delta_{so})}, \quad (15)$$

where $m_0$ is the free-electron mass, $\varepsilon_g$ and $\Delta_{so}$ are the band gap and the valence band spin-orbit splitting of the bulk semiconductor used in the QW layer, $p_{cv} = \langle S | \hat{p}_z | Z \rangle$ is the interband matrix element of the momentum operator between the Bloch functions of the conduction and valence bands, $S$ and $Z$.

Compared with the non-gyrotropic class $T_d$ the presence of gyrotropic invariants in the electron effective Hamiltonian in QWs of the $D_{2d}$- and $C_{2v}$-symmetry enable new mechanisms of the MPGE. At present we are unaware of any non-gyrotropic mechanism of the MPGE in QW structures in the presence of an in-plane magnetic field. Thus, it is natural to classify such contributions to the MPGE as magneto-gyrotropic photocurrents. Below we consider microscopic mechanisms of magneto-gyrotropic photocurrents, both spin-dependent and spin-independent. To illustrate them we present model pictures for three different mechanisms connected to acoustic phonon assisted optical transitions. Optical phonon- or defect-assisted transitions and those involving electron-electron collisions may be considered in the same way.

### 5.3. Photocurrent due to spin-dependent asymmetry of optical excitation

The first possible mechanism of current generation in QWs in the presence of a magnetic field is related to the asymmetry of optical excitation. The characteristic feature of this
FIG. 11: Microscopic origin of photocurrent caused by asymmetric photoexcitation in an in-plane magnetic field. The spin subband (+1/2) is preferably occupied due to the Zeeman splitting. The rates of optical transitions for opposite wavevectors $k$ are different, $W_1 < W_2$. The $k$-linear spin splitting is neglected in the band structure because it is unimportant for this mechanism.

mechanism is a sensitivity to the polarization of light. In our experiments we employ free-electron absorption. Indirect optical transitions require a momentum transfer from phonons to electrons. A photocurrent induced by these transitions appears due to an asymmetry of either electron-photon or electron-phonon interaction in the $k$-space. Below we take into account the gyrotropic invariants within the first order of the perturbation theory. Therefore while considering the spin-dependent magneto-gyrotropic effects, we can replace the contribution to the electron Hamiltonian linear in the Pauli spin matrices by only one of the terms proportional to the matrix $\sigma_j$ and perform the separate calculations for each index $j$. Then spin-conserving and spin-flip mechanisms can be treated independently.

5.3.1. Spin-dependent spin-conserving asymmetry of photoexcitation due to asymmetric electron-phonon interaction. In gyrotropic media the electron-phonon interaction $V_{el-phon}$ contains, in addition to the main contribution, an asymmetric spin-dependent term $\propto \sigma_\alpha(k_\beta + k'_\beta)$ given by Eq. (14), see also [14, 42–44]. Microscopically this contribution is caused by structural and bulk inversion asymmetry alike Rashba/Dresselhaus band spin splitting in the $k$-space. The asymmetry of electron-phonon interaction results in non-equal rates of indirect optical transitions for opposite wavevectors in each spin subband with $s_\alpha = \pm 1/2$. This causes an asymmetric distribution of photoexcited carriers within the subband $s_\alpha$ and yields therefore a flow, $i_\alpha$, of electrons in this subband. This situation is
sketched in Fig. 11 for the spin-up (\( s = 1/2 \)) subband. The single and double horizontal arrows in Fig. 11 indicate the difference in electron-phonon interaction strength for positive and negative wavevectors. The important point now is that single and double arrows are interchanged for the other spin direction (see Eq. (14)). Indeed the enhancement of the electron-phonon interaction rate for a specific \( k \)-vectors depends on the spin direction. Therefore for the other spin subband, the situation is reversed. This is analogous to the well known spin-orbit interaction where the shift of the \( \varepsilon(k) \) dispersion depends also on the spin direction. Thus without magnetic field two oppositely directed and equal currents in spin-up and spin-down subbands cancel each other exactly. This non-equilibrium electron distribution in the \( k \)-space is characterized by zero electric current but nonzero pure spin current \( i_{\text{spin}} = (1/2)(i_{1/2} - i_{-1/2}) \) [45]. The application of a magnetic field results, due to the Zeeman effect, in different equilibrium populations of the subbands. This is seen in Fig. 11, where the Zeeman splitting is largely exaggerated to simplify visualization. Currents flowing in opposite directions become non-equivalent resulting in a spin polarized net electric current. Since the current is caused by asymmetry of photoexcitation, it may depend on the polarization of radiation.

Generally, indirect optical transitions are treated in perturbation theory as second-order processes involving virtual intermediate states. The compound matrix element of phonon-mediated transition \( (s, k) \rightarrow (s', k') \) with the intermediate state in the same subband \( e1 \) can be written as

\[
M^{(\pm)}_{s'k',sk} = \sum_{s''} \left[ \frac{V^{(\pm)}_{s'k',s''k} R_{s'',s}(k)}{\varepsilon_s(k) - \varepsilon_{s''}(k) + \hbar \omega} + \frac{R_{s',s''}(k') V^{(\pm)}_{s''k',sk}}{\varepsilon_s(k) - \varepsilon_{s''}(k') \mp \hbar \Omega(q)} \right],
\]

where \( R_{s',s}(k) \) is the direct optical matrix element, \( V^{(\pm)}_{s'k',sk} \) is the matrix element of phonon-induced scattering, the upper (lower) sign in \( \pm \) and \( \mp \) means the indirect transition involving absorption (emission) of a phonon; \( s, s' \) and \( s'' \) are the spin indices.

While considering the spin-conserving electron transitions, we use the basis of electron states with the spin components \( s = \pm 1/2 \) parallel to the direction \( \eta \parallel B \), retain in the gyrotrropic invariants only the spin-independent terms containing \( \sigma_\eta \) and consider the processes \( (s, k) \rightarrow (s, k') \). Then, in Eq. (16) one can set \( s = s' = s'' \) and reduce the equation to

\[
M^{(\pm)}_{sk',sk} = V^{(\pm)}_{sk',sk} \left[ R_{s,s}(k) - R_{s,s}(k') \right] / \hbar \omega.
\]
The photocurrent density is given by

\[ j = \frac{2\pi}{\hbar} \sum_{k'k\epsilon\pm} [v_s(k')\tau'_p - v_s(k)\tau_p] |M_{ssk,k}^{(\pm)}|^2 \times \]

\[ \times \{ f^0_s(k)[1 - f^0_s(k')]N_q^{(\pm)} - f^0_s(k')[1 - f^0_s(k)]N_q^{(\mp)} \} \delta[\varepsilon_s(k') - \varepsilon_s(k) - \hbar\omega \pm \hbar\Omega(q)] , \]

where \( e \) is the electron charge, \( v_s(k) \) is the electron group velocity in the state \((s,k)\), \( \tau_p \) and \( \tau'_p \) are the electron momentum relaxation times in the initial and final states, \( f^0_s(k) \) is the electron equilibrium distribution function, \( q = k' - k \) is the phonon wavevector, \( N_q^{(\pm)} = N_q + (1 \pm 1)/2 \), and \( N_q \) is the phonon occupation number.

For the mechanism in question one retains in \( R_{s,s}(k) \) the main contribution \(-eA_0/cm^*)(\hbar k \cdot e)\) and uses the electron-phonon interaction in the form of Eq. (14) which can be rewritten as

\[ V_{sk',sk} = \Xi e_{ii} + \Xi e_{cv} \xi[(k' + k) \times \sigma_{ss}]z \xi_{xy} . \]  

Here \( A_0, e \) are the scalar amplitude and polarization unit vector of the light vector-potential, and \( e_{ii} \equiv \sum_i e_{ii} \).

Under indirect photoexcitation, the asymmetry of scattering described by Eq. (19) leads to electric currents of opposite directions in both spin subbands. The net electric current occurs due to the Zeeman splitting induced selective occupation of these branches in equilibrium. We remind that, in the first order in the magnetic field \( B \), the average equilibrium electron spin is given by

\[ S^{(0)} = -\frac{g\mu_B B}{4\bar{\varepsilon}} , \]

where \( g \) is the electron effective \( g \)-factor, \( \mu_B \) is the Bohr magneton, \( \bar{\varepsilon} \) is the characteristic electron energy defined for the 2D gas as \( \int d\varepsilon f(\varepsilon)/f(0) \), with \( f(\varepsilon) \) being the equilibrium distribution function at zero field, so that \( \bar{\varepsilon} \) equals the Fermi energy, \( \varepsilon_F \), and the thermal energy, \( k_B T \), for degenerate and non-degenerate electron gas, respectively. The current, induced by electron-phonon asymmetry under indirect photoexcitation, can be estimated as

\[ j \propto \frac{e\tau_p \Xi e_{cv} \xi}{\hbar} \eta_{ph} IS^{(0)} , \]

where \( \eta_{ph} \) is the phonon-assisted absorbance of the terahertz radiation.

For impurity-assisted photoexcitation, instead of Eq. (19), one can use the spin-dependent matrix element of scattering by an impurity,

\[ V_{sk',sk} = \{ V_0(q) + V_z(q) \xi[(k' + k) \times \sigma_{ss}]z \} e^{i(k-k')r_m} , \]
where $q = k' - k$, $V_0$ is the matrix element for intra-band electron scattering by the defect, $V_z$ is the matrix element of the defect potential taken between the conduction-band Bloch function $S$ and the valence-band function $Z$ (see [42] for details), $r_{im}$ is the in-plane position of the impurity.

5.3.2. Asymmetry of photoexcitation due to asymmetrical electron-phonon spin-flip scattering. Indirect optical transitions involving phonon-induced asymmetric spin-flip scattering also lead to an electric current if spin subbands get selectively occupied due to Zeeman splitting. The asymmetry can be due to a dependence of the spin-flip scattering rate on the transferred wavevector $k' - k$ in the system with the odd-$k$ spin splitting of the electron subbands, see [7]. Estimations show that this mechanism to the photocurrent is negligible compared to the previous mechanism 5.3.1.

5.3.3. Spin-dependent spin-conserving asymmetry of photoexcitation due to asymmetric electron-photon interaction. A magnetic field induced photocurrent under linearly polarized excitation can occur due to an asymmetry of electron-photon interaction. The asymmetry is described by the optical matrix element

$$R_{s,s}(k) = -\frac{eA_0}{c} \left[ \frac{\hbar(k \cdot e)}{m^*} + \frac{1}{\hbar} \sum_j e_j \frac{\partial}{\partial k_j} H_{ss}^{(3)}(k; \eta) \right],$$

(22)

where $H_{ss}^{(3)}(k; \eta)$ is the $\sigma_\eta$-dependent term in the cubic-$k$ contribution $H_{BIA}^{(3)}(k) + H_{SIA}^{(3)}(k)$ to the electron Hamiltonian. Here, for the electron-phonon matrix element, one can take the main spin-independent contribution including both the piezoelectric and deformation-potential mechanisms. Under indirect light absorption, the electron-photon asymmetry results in electric currents flowing in opposite directions in both spin branches. Similarly to the mechanism 5.3.1, the net electric current is nonzero due to the selective occupation of the Zeeman-split spin branches.

It should be stressed that the $H_{as}^{(3)}(k; \eta)$ term should also be taken into account in the $\delta$-function, the distribution function and the group velocity in the microscopical expression (18) for the photocurrent. Note that the linear-$k$ terms in the effective electron Hamiltonian, see Eq. (12), do not lead to a photocurrent in the first order in $\beta_{BIA}$ or $\beta_{SIA}$ because the linear-$k_i$ term in the function $\hbar^2 k_i^2 / 2m^* + \beta k_i$ disappears after the replacement $k_i \to \tilde{k}_i = k_i + \beta m^*/\hbar^2$.

5.3.4. Asymmetry of spin-flip photoexcitation due to asymmetric electron-photon interaction. To obtain the asymmetric photoexcitation for optical spin-flip processes we can take
into account, alongside with the term odd-\(k\), the quadratic-\(k\) Zeeman term similar to that introduced by Eq. (9). Then the spin-flip optical matrix element is given by

\[
R_{s,s}(k) = -\frac{eA_0}{\hbar c} \left\{ G\sigma_{s,s} \cdot \left[ e (B \cdot k) + k (B \cdot e) \right] + \sum_j e_j \frac{\partial}{\partial k_j} \mathcal{H}_{s,s}(k) \right\},
\]

(23)

where \(\bar{s} = -s\) and \(\mathcal{H}(k)\) is the odd-\(k\) contribution to the electron Hamiltonian, including both linear and cubic terms. Estimations show that the photocurrent due to the spin-conserving processes described by Eq. (22) is larger than that due to the spin-flip processes described by Eq. (23).

5.3.5. Spin-dependent asymmetry of indirect transitions via other bands or subbands. This contribution is described by Eq. (16) where the summation is performed over virtual states in subbands different from \(e_1\). The estimation shows that it is of the same order of magnitude as the contribution due to the mechanism 5.3.1.

Summarizing the above mechanisms we would like to stress that the characteristic feature of all of them is a sensitivity to the light linear polarization described in Eqs. (3) by the terms proportional to \(S_2, S'_2, S_3, S'_3\). Depending on the particular set of parameters, e.g., those in Eqs. (12, 14), the energy dependence of \(\tau_p\), the ratio between the photon energy, the electron average energy etc., one can obtain any value for the ratio between \(S_2\) and \(S_3\) as well as for the ratio between one of them and the coefficient \(S_1\).

5.4. Current due to spin-dependent asymmetry of electron relaxation

Energy and spin relaxation of a non-equilibrium electron gas in gyrotropic systems can also drive an electric current. The current is a result of relaxation of heated carriers, and hence its magnitude and direction are independent of the polarization of radiation. Several mechanisms related to the asymmetry of electron relaxation are considered below.

5.4.1. Asymmetry of electron energy relaxation. Another mechanism which stems from spin-dependent asymmetric terms in the electron-phonon interaction is the energy relaxation of hot carriers [14]. The light absorption by free electrons leads to an electron gas heating, i.e. to a non-equilibrium energy distribution of electrons. Here we assume, for simplicity, that the photoexcitation results in isotropic non-equilibrium distribution of carriers. Due to asymmetry of electron-phonon interaction discussed above, (see Eq. (14) and Section 5) hot electrons with opposite \(k\) have different relaxation rates. This situation is sketched in
**FIG. 12:** Microscopic origin of the electric current caused by asymmetry of the energy relaxation in the presence of an in-plane magnetic field. The spin subband (+1/2) is preferably occupied due to the Zeeman splitting. The \(k\)-linear spin splitting is neglected in the band structure because it is unimportant for this mechanism.

Fig. 12 for a spin-up subband \((s = 1/2)\), where two arrows of different thicknesses denote non-equal relaxation rates. As a result, an electric current is generated. Whether \(-k\) or \(+k\) states relax preferentially, depends on the spin direction. It is because the electron-phonon asymmetry is spin-dependent and has the opposite sign in the other spin subband. Similarly to the case described in the mechanism 5.3.1, the arrows in Fig. 12 need to be interchanged for the other spin subband. For \(B = 0\) the currents in the spin-up and spin-down subbands have opposite directions and cancel exactly. But as described in Section 5.3.1 a pure spin current flows which accumulates opposite spins at opposite edges of the sample. In the presence of a magnetic field the currents moving in the opposite directions do not cancel due to the non-equal population of the spin subbands (see Fig. 12) and a net electric current flows.

For the electron-phonon interaction given by Eq. (14) one has

\[
V_{s,s_k,k'} = \frac{1}{\varepsilon_{ii}} - \frac{1}{\varepsilon_{cv}} \xi(k'_y + k_y) \epsilon_{xy} \text{sign } s_x .
\]

Thus, the ratio of antisymmetric to symmetric parts of the scattering probability rate, \(W_{s_k,k'} = |V_{s_k,k'}|^2\), is given by \(W_{as}/W_s \sim (\varepsilon_{cv} \xi \epsilon_{xy} / \varepsilon_{ii})(k'_y + k_y)\). Since the antisymmetric component of the electron distribution function decays within the momentum
relaxation time $\tau_p$, one can write for the photocurrent

$$j_i \sim eN \frac{g\mu_B B_x}{\bar{\varepsilon}} W_s \left\langle \frac{\Xi_s \varepsilon_{xy}(k_y')}{\Xi} \left( \frac{\hbar k_y'}{m^*} - \frac{\hbar k_i}{m^*} \right) k_y \right\rangle,$$

where $N$ is the 2D electron density and the angle brackets mean averaging over the electron energy distribution. While the average for $j_y$ is zero, the $x$ component of the photocurrent can be estimated as

$$j_x \sim \frac{e\tau_p}{\hbar} \frac{\Xi_s \varepsilon_{xy}}{\Xi} \frac{g\mu_B B_x}{\bar{\varepsilon}} \eta I,$$

where $\eta$ is the fraction of the energy flux absorbed in the QW due to all possible indirect optical transitions. By deriving this equation we took into account the balance of energy

$$\sum_{k'k} [\varepsilon(k) - \varepsilon(k')] W_{k',k} = \eta I,$$

where $\varepsilon(k) = \hbar^2 k^2 / 2m^*$. An additional contribution to the relaxation photocurrent comes if we neglect the asymmetry of electron-phonon interaction by setting $\xi = 0$ but, instead, take cubic-$k$ terms into account in the electron effective Hamiltonian.

Compared to the mechanisms 5.3, the main characteristic feature of mechanism 5.4.1 is its independence of the in-plane linear-polarization orientation, i.e. $S_2 = S_2' = S_3 = S_3' = 0$. A particular choice of $V_s, k', s_x, k$ in the form of Eq. (24) leads to a photocurrent with $S_1' = S_1$ or, equivalently, $S_1^- = 0$. By adding a spin-dependent invariant of the type $I_2$ to the right-hand part of Eq. (24) one can also obtain a nonzero value of $S_1^-$.

### 5.4.2. Current due to spin-dependent asymmetry of spin relaxation (spin-galvanic effect).

This mechanism is based on the asymmetry of spin-flip relaxation processes and represents in fact the spin-galvanic effect [7] where the current is linked to spin polarization

$$j_i = Q_{ii'} (S_{i'} - S_{i'}^{(0)}) .$$

Here $S$ is the average electron spin and $S^{(0)}$ is its equilibrium value, see Eq. (20). In contrast to the majority of the mechanisms considered above which do not contain $k$-linear terms, these are crucial here.

In the previous considerations the spin-galvanic effect was described for a non-equilibrium spin polarization achieved by optical orientation where $S^{(0)}$ was negligible [3, 7]. Here we discuss a more general situation a non-zero $S^{(0)}$ caused by the Zeeman splitting in a magnetic field is explicitly taken into account. We show below that in addition to optical orientation
FIG. 13: Microscopic origin of the electric current caused by asymmetry of spin relaxation. Non-equilibrium spin is due to photoinduced depolarization of electron spins. Asymmetry of spin relaxation and, hence, an electric current is caused by $k$-linear spin splitting.

with circularly polarized light, it opens a new possibility to achieve a non-equilibrium spin polarization and, hence, an additional contribution to the photocurrent.

Fig. 13 illustrates this mechanism. In equilibrium the electrons preferably occupy the Zeeman split lower spin subband. By optical excitation with light of any polarization a non-equilibrium population as sketched in Fig. 13 can be achieved. This is a consequence of the fact that optical transitions from the highly occupied subband dominate. These optically excited electrons under energy relaxation return to both subbands. Thus, a non-equilibrium population of the spin subbands appears. To return to equilibrium spin-flip transitions are required. Since spin relaxation efficiently depends on initial and final $k$-vectors, the presence of $k$-linear terms leads to an asymmetry of spin relaxation (see bent arrows in Fig. 13), and hence to current flow. This mechanism was described in [7].

Following similar arguments as in Ref. [7, 46] one can estimate the spin-galvanic contribution to the polarization-independent magneto-induced photocurrent as

$$j \sim e \tau_p \beta g \mu_B B \eta I \bar{\varepsilon} / \bar{\hbar} \omega \zeta. \quad (27)$$

Here $\zeta$ is a factor describing the electron spin depolarization due to photoexcitation followed by the energy relaxation. It can be estimated as $\zeta \sim \tau_e / \tau_s$, where $\tau_e$ is electron energy relaxation time governed mainly by electron-electron collisions, and $\tau_s$ is the spin relaxation time. Assuming $\tau_e \sim 10^{-13} s$ and $\tau_s \sim 10^{-10} s$ at room temperature, the factor $\zeta$ is estimated as $10^{-3}$. 
5.5. Spin-independent mechanisms of magneto-induced photocurrent

The last group of mechanisms is based on a magnetic field induced shift of the energy dispersion in the \( k \)-space in gyrotropic materials. This mechanism was investigated theoretically and observed experimentally for direct inter-band transitions [21, 22] and proved to be efficient. To obtain such a current for indirect optical transitions one should take into account effects of the second order like non-parabolicity or transitions via virtual states in the other bands. Our estimations show that these processes are less efficient compared to mechanisms 5.3 and 5.4. However, to be complete, we consider below possible contributions of the diamagnetic shift to the current at the Drude absorption of radiation.

5.5.1. Spin-independent asymmetry of indirect transitions with intermediate states in the same subband. The experiments on the MPGE under direct optical transitions observed in asymmetric QW structures are interpreted in terms of the asymmetric spin-independent electron energy dispersion, \( \varepsilon(k, B) \neq \varepsilon(-k, B) \), analyzed by Gorbatsevich et al. [16], see also [17, 18]. The simplest contribution to the electron effective Hamiltonian representing such kind of asymmetric dispersion is the diamagnetic term \( \mathcal{H}_{\text{SIA}}^{(\text{dia})} \) in Eq. (13). In asymmetric QWs, \( \bar{\varepsilon}_\nu \) are nonzero and the subband dispersion is given by parabolas with their minima (or maxima in case of the valence band) shifted from the origin \( k_x = k_y = 0 \) by a value proportional to the in-plane magnetic field.

For indirect optical transitions these linear-\( k \) terms do not lead, in the first order, to a photocurrent. To obtain the current one needs to take into account the non-parabolic diamagnetic term

\[
\mathcal{H}_{\text{SIA}}^{(\text{dia},3)} = \mathcal{F}_{\text{SIA}}(B_x k_y - B_y k_x)k^2.
\]

The non-parabolicity parameter can be estimated by \( \mathcal{F}_{\text{SIA}} \sim (\hbar^2/m^*E_g)\tilde{\alpha}_{\text{SIA}} \). By analogy with the SIA diamagnetic term we can introduce the BIA diamagnetic term \( \mathcal{H}_{\text{BIA}}^{(\text{dia},3)} = \mathcal{F}_{\text{BIA}}(B_x k_x - B_y k_y)k^2 \). It is most likely that, in realistic QWs, the coefficient \( \mathcal{F}_{\text{BIA}} \) is small as compared to \( \mathcal{F}_{\text{SIA}} \).

5.5.2. Spin-independent asymmetry of indirect transitions via other bands and subbands. One can show, that even the linear-\( k \) diamagnetic terms can contribute to the photocurrent under indirect intra-subband optical transitions if the indirect transition involves intermediate states in other bands (or subbands) different from the conduction subband \( e1 \). Under normal incidence of the light, a reasonable choice could be a combination of direct intra-band
optical transitions with the piezoelectric electron-phonon interaction, for the first process, and inter-band virtual optical transitions as well as interband deformation-potential electron-phonon interaction, for the second process. An asymmetry of the indirect photoexcitation is obtained as a result of the interference between two indirect processes with the intermediate state in the same subband and elsewhere. Moreover, the diamagnetic dispersion asymmetry of the initial and intermediate bands should be taken into account in the energy denominator of the compound two-quantum matrix element for the transitions via other bands.

5.5.3. Spin-independent asymmetry of electron energy relaxation. Similarly to the spin-dependent mechanism 5.4.1, the diamagnetic cubic-\( \mathbf{k} \) term, see Eq. (28), can be responsible for the relaxational photocurrent. This relaxation mechanism is unlikely to give an essential contribution to the MPGE.

To summarize this group of mechanisms we note that, as in the case of spin-dependent mechanisms, the mechanisms 5.5.1 and 5.5.2 allow a pronounced dependence of the photocurrent on the orientation of the in-plane light polarization whereas the relaxation mechanism 5.5.3 is independent of the polarization state.

6. DISCUSSION

In all investigated QW structures, an illumination with terahertz radiation in the presence of an in-plane magnetic field results in a photocurrent in full agreement with the phenomenological theory described by Eqs. (3). The microscopic treatment presented in Section 5 shows that two classes of mechanisms dominate the magneto-gyrotropic effects. The current may be induced either by an asymmetry of optical excitation and/or by an asymmetry of relaxation. Though in all cases the absorption is mainly independent of the light polarization, the photocurrent depends on polarization for the first class of the mechanisms (see Section 5.3) but is independent of the direction of linear light polarization for the second class (see Section 5.4). Thus the polarization dependence of the magneto-gyrotropic photocurrent signals allows us to distinguish between the above two classes. The asymmetry of photoexcitation may contribute to all terms in Eqs. (3). Therefore, such photocurrent contributions should exhibit a characteristic polarization dependence given, for linearly polarized light, by the second and third terms in Eqs. (3) with the coefficients \( S_2, S'_2, S_3, S'_3 \). In contrast, the asymmetry of relaxation processes (see Section 5.4) contributes only to the
coefficients $S_1, S'_1, S_4, S'_4$.

The experimental data obtained on the samples A1 to A4 suggest that in these QW structures relaxation mechanisms, presented in Section 5.4, dominate. Indeed only current contributions described by the first and last terms in Eqs. (3) are detectable, whereas the second and third term contributions are vanishingly small. These samples are denoted as type I below. The results obtained for type I samples are valid in the wide temperature range from 4.2 K up to room temperature. The transverse photocurrent observed in the direction normal to the magnetic field $B$ applied along $\langle 110 \rangle$ is independent of the light polarization. It corresponds to the first term in Eqs. (3). Hence, this current is caused by the Drude absorption-induced electron gas heating followed by energy relaxation (mechanism 5.4.1) and/or spin relaxation (mechanism 5.4.2). The analysis (see Section 5.4) shows that in the absence of the magnetic field electron gas heating in gyrotropic QWs is accompanied by a pure spin flow. The longitudinal photocurrent component parallel to $B$, which appears under excitation with circularly polarized radiation only, arises due to spin relaxation of optically oriented carriers (spin-galvanic effect [3, 7]).

In contrast to the samples of type I, the experimental results obtained on the sample A5 (in the following denoted as type II) has characteristic polarization dependencies corresponding to the second ($S_2, S'_2$) and third ($S_3, S'_3$) terms in Eqs. (3). The photocurrent exhibits a pronounced dependence on the azimuthal angle $\alpha$ of the linear polarization, but it is equal for the right and left circular polarized light. This experimental finding proves that the main mechanism for current generation in type II sample is the asymmetry of photoexcitation considered in Section 5.3.

The question concerning the difference of type I and type II samples remains open. While experimentally the two classes of the mechanisms are clearly observed, it is not clear yet what determines large difference between the relevant $S$-coefficients. Not much difference is expected between the type I and II samples regarding the strength and asymmetry of electron-phonon interaction. The samples only differ in the type of doping and the electron mobility. The influence of impurity potentials (density, position, scattering mechanisms etc.) on microscopic level needs yet to be explored. In addition, the doping level of the type I samples is significantly lower and the mobility is higher than those in the type II samples. This can also affect the interplay between the excitation and relaxation mechanisms.

Finally we note, that under steady-state optical excitation, the contributions of the relax-
ation and photoexcitation mechanisms to magneto-induced photogalvanic effects are superimposed. However, they can be separated experimentally in time-resolved measurements. Indeed, under the ultra-short pulsed photoexcitation the current should decay, for the mechanisms considered above, within the energy ($\tau_\varepsilon$), spin ($\tau_s$) and momentum ($\tau_p$) relaxation times.

7. SUMMARY

We have studied photocurrents in $n$-doped zinc-blende based (001)-grown QWs generated by the Drude absorption of normally incident terahertz radiation in the presence of an in-plane magnetic field. The results agree with the phenomenological description based on the symmetry. Both experiment and theoretical analysis show that there are a variety of routes to generate spin polarized currents. As we used both magnetic fields and gyrotropic mechanisms we coined the notation “magneto-gyrotropic photogalvanic effects” for this class of phenomena.

Acknowledgements

The high quality InAs quantum wells were kindly provided by J. De Boeck and G. Borghs from IMEC Belgium. We thank L.E. Golub for helpful discussion. This work was supported by the DFG, the RFBR, the INTAS, programs of the RAS, and Foundation “Dynasty” - ICFPM.

8. APPENDICES

8.1. Appendix A. Point Groups $T_d$ and $D_{2d}$

In the $T_d$-class bulk crystals the MPGE linear in the magnetic field $B$ can be phenomenologically presented as [28, 47]

$$
j_x = 2S_1 \left( |e_y|^2 - |e_z|^2 \right) B_z I + S_2 \left[ (e_z e_x^* + e_x e_z^*) B_z - (e_y e_y^* + e_y e_y^*) B_y \right] I \quad (29)
$$

$$
-S_4 \left[ i(e \times e^*)_y B_z + i(e \times e^*)_z B_y \right] I ,
$$
and similar expressions for \( j_y \) and \( j_z \), where \( x \parallel [100], y \parallel [010], z \parallel [001] \). Note that here the notation of the coefficients is chosen as to be in accordance with the phenomenological equations (7). Under photoexcitation along the [001] axis, \( e_z = 0 \) and, in the presence of an external magnetic field \( \mathbf{B} \perp [001] \), one has

\[
\begin{align*}
j_x &= S_1[1 - (|e_x|^2 - |e_y|^2)]B_xI - B_yI \left[ S_2 \left( e_xe_y^* + e_ye_x^* \right) + S_4P_{\text{circ}} \right], \\
j_y &= -S_1[1 + (|e_x|^2 - |e_y|^2)]B_yI + B_xI \left[ S_2 \left( e_xe_y^* + e_ye_x^* \right) - S_4P_{\text{circ}} \right].
\end{align*}
\]

In the axes \( x' \parallel [1\bar{1}0], y' \parallel [110], z \parallel [001] \), Eqs. (30) assume the form

\[
\begin{align*}
j_{x'} &= S_1 \left[ B_{y'} - (e_{x'}e_{y'}^* + e_{y'}e_{x'}^*) \right] B_{x'} I + S_2 \left( |e_{x'}|^2 - |e_{y'}|^2 \right) B_{y'}I + S_4P_{\text{circ}}B_{x'}I, \\
j_{y'} &= S_1 \left[ B_{y'} - (e_{x'}e_{y'}^* + e_{y'}e_{x'}^*) \right] B_{y'} I - S_2 \left( |e_{x'}|^2 - |e_{y'}|^2 \right) B_{x'}I - S_4P_{\text{circ}}B_{y'}I.
\end{align*}
\]

Equations (30,31) are consistent with Eqs. (3,7) describing the magneto-induced photocurrents in the \( C_{2v} \)-symmetry systems and can be obtained from Eqs. (3,7) by setting \( S'_1 = S_1 = -S_3 = -S'_3, S'_2 = -S_2 \) or, equivalently, \( S_1^- = S_2^+ = S_3^- = S_4^+ = 0 \) and \( S_1^+ = -S_3^+ = S_1, S_2^- = S_2, S_3^- = S_4 \).

One can show that the phenomenological equations for the \( D_{2d} \) symmetry are obtained from Eqs. (3,7) if we set \( S'_1 = S_1, S_3 = S'_3, S'_2 = -S_2, S'_4 = -S_4 \). The only difference with Eqs. (30,31) is that \( S_1 \) and \( S_3 \) are now linearly independent.

8.2. Appendix B. Point Group \( C_{\infty v} \)

For a system of the \( C_{\infty v} \) symmetry, one has

\[
\begin{align*}
j_x &= S_1B_yI + S_2 \left[ (|e_x|^2 - |e_y|^2)B_y - (e_xe_y^* + e_ye_x^*)B_x \right] I + S_4B_xIP_{\text{circ}}, \\
j_y &= -S_1B_xI + S_2 \left[ (|e_x|^2 - |e_y|^2)B_x + (e_xe_y^* + e_ye_x^*)B_y \right] I + S_4B_yIP_{\text{circ}}.
\end{align*}
\]

where the form of the equation is independent of the orientation of Cartesian coordinates \((x,y)\) in a plane normal to the \( C_{\infty v} \)-axis. A comparison to Eqs. (3) for \( C_{2v} \) symmetry shows that the form of these equations is identical besides the coefficients \( S_i \). In this case we have \( S'_1 = -S_1, S_2 = S_2' = -S_3 = S_3', S'_3 = S_4 = S_4' \).


