1. History and Concept of Black Holes

1.1. Compact objects in astronomy and the idea of Black Holes

Compact astronomical objects as white dwarfs (WDs), neutron stars (NSs) and Black Holes (BHs) are considered to be final states in the evolution of stars, when the fuel of nuclear fusion in the star's core is exhausted. Most stars have masses below 8 solar masses. They end as WDs, which are bodies with the dimensions of the earth and densities above 10^6 g/cm^3. They can exist with masses below about 1,4 solar masses. Above this so called Chandrasekhar limit they are unstable and would implode. While the death as a WD is observed as a relatively smooth end, stars with more than 8 solar masses (far less abandoned) undergo a catastrophic end as supernovae. The inner core is transformed during this fast developing catastrophe into a NS with a density well above that of nuclear matter (some 10^{14} g/cm^3). Only for those NSs which are members of a binary system the determination of the mass is possible. The masses found so far are all near 1,4 solar masses. The estimated diameter is about 25 km. In some rare cases one finds stars with masses equal or above 20 solar masses. They also end their radiating life in a catastrophic event even more energetic called a hypernova, there after a BH is left. A NS has a solid surface, a BH lacks such a surface. Instead one can define an event horizon. Its radius would be 3 km for a one solar mass BH. Both NS and BH are tiny objects and compared with astronomical dimensions and one can consider them as point like.

I take here the definition for a BH from Shapiro and Teukolsky: "A BH is simply a region of spacetime that cannot communicate with the external universe. The boundary of this region is called the surface of the BH or the ***event horizon***". The idea of a dark star or a black hole was already formulated by John Mitchell, a British natural philosopher and presented at the Royal Society at London in 1783. He had just combined Newton's theory of gravitation with his corpuscular theory of light and called them "dark stars". Pierre Simon Laplace took up the idea of "dark stars" in his famous work "***La Systeme du Monde***". However at 1808 by the time of the 3rd edition the wave theory of light has gained general acceptance after the discovery of interference. So Laplace deleted the mentioning of dark stars in the 3rd edition of his famous book on Newton's law of gravitation and its consequences. You will find a brief history of the concept of BHs at the end of this first lecture.

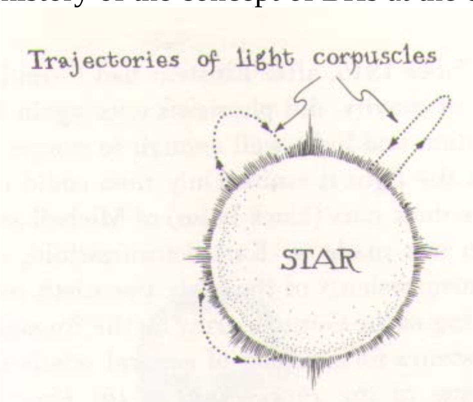

Fig.1. Light particles emitted from Mitchells "Dark Star" fall back to the surfacedue to Newton's law of gravity. Credit Kip S. Thorne: Black Holes and Time Warps. 1994

1.2. Escape velocity and Schwarzschild radius.

In order to investigate the concept of a BH with Newton's theory of gravity we consider the energy E of a test mass m in the gravitational field of a star of mass M, where M >> m. The energy is negative when the mass is bound (E<0) and positive when the mass is unbound (E > 0). For the limiting case (E=0) we have

$$\frac{m}{2}\upsilon^2 - G\frac{mM}{r} = 0 \qquad (1.1)$$

If we take r = R as the star's Radius then

$$\upsilon_e^2 = G\frac{2M}{R} \qquad (1.2)$$

This is the square of the escape velocity. To escape from the star's surface we should have $\upsilon > \upsilon_e$. Making M very large or R very small, $R \to r_S$ we would finally arrive at

$$c^2 = G\frac{2M}{r_S} \qquad (1.3)$$

In this case nothing can escape the star, neither matter nor light. The star is then a Black Hole, r_S is called "Schwarzschild radius". A Black Hole with mass of the sun $M = 1{,}989 \cdot 10^{30}\,kg$ has a Schwarzschild radius of

$$r_S = \frac{2GM_{Sun}}{c^2} = 2{,}94\ km \qquad (1.4)$$

1.3. Gravitational redshift and deflection of light.

Consider a spherical mass within a sphere of radius *r* then the ratio

$$\frac{r_S}{r}$$

gives us an estimate of the deviation from Newton's law i.e. the effects of General Relativity (GR) on the system considered. See table 1.1. As an example we will consider the "**gravitational redshift**". which is the frequency shift $\Delta\nu$ of light of frequency v emitted from the surface of a star or other massive object.

Object	Mass *M*	Radius *r*	r_S	r_S/r
Sun	$2 \cdot 10^{30}$ kg	$7 \cdot 10^{5}$ km	3 km	$4 \cdot 10^{-6}$
White Dwarf	0,7 solar masses = $2{,}8 \cdot 10^{30}$ kg	10^4 km	2 km	$2 \cdot 10^{-4}$
Neutron star	1,4 solar masses	12 km	4 km	1/3

Table 1.1

One finds for the gravitational redshift

$$\frac{\Delta \nu}{\nu} = \frac{r_S}{2r} \tag{1.5}$$

This relation is easily derived when we use the corpuscular or quantum theory of light. A photon hν travelling from the surface of a star (or planet) the distance Δr looses energy in the gravitational field. So we may write

$$h\nu' = h\nu - \frac{h\nu}{c^2} g\Delta r \tag{1.6}$$

Here we have assumed that the photon has a mass $\frac{h\nu}{c^2}$. We obtain a red shift (observed above the surface of the star)

$$\frac{\Delta \nu}{\nu} = \frac{\Delta r}{c^2} g \tag{1.7}$$

To make this result applicable to astrophysical purposes we replace the gravitational acceleration *g* and consider now a photon which escaped the star from *r* to infinity

$$g\,\Delta r \rightarrow \int_r^\infty \frac{GM}{r^2} dr = -\frac{GM}{r} \tag{1.8}$$

It suffers a redshift proportional to r_S/r

$$\frac{\Delta \nu}{\nu} = -\frac{GM}{c^2 r} = \frac{1}{2} \cdot \frac{r_S}{r} \tag{1.9}$$

This effect is called ***gravitational redshift***. If we compare (1.9) with the exact result from GR derived in the next lectures we find it to be a first order approximation.

In a later chapter we will explain and use gravitational lensing which is an important tool of modern astronomy. We will start with simple ***deflection of light*** by gravitational bodies. It will be left to the reader to show consistently that the deflection angle ϕ in Newtonian approximation is

$$\phi = \frac{r_S}{r}$$

To find this relation we write the force on the photon mass mph in x direction (see fig.)

$$F_x = -GMm_{ph} \frac{r_0}{\left(r_0^2 + y^2\right)^{3/2}} \tag{1.10}$$

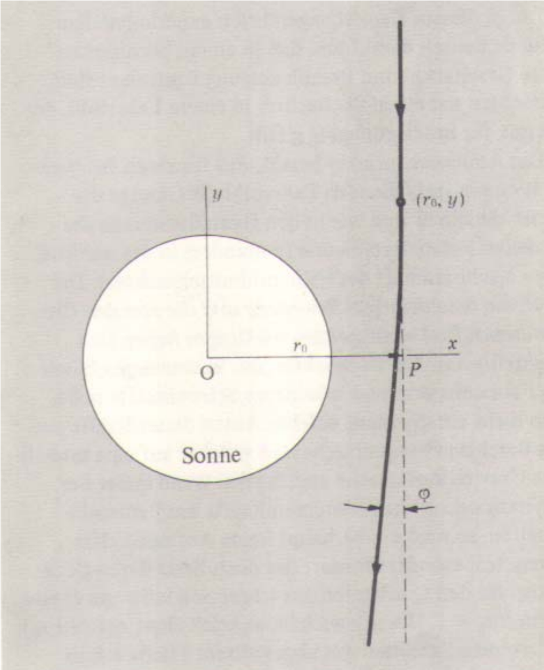

Fig.2 Deflection of photons by the sun. Before the photons reach the gravitational field of the sun they travel along the y – direction with the velocity c. Gravitation adds an x–component to the velocity vector. Measurement give the angle ϕ which is obtained from equ. (1.13)

$$\phi \cong \frac{\upsilon_x}{\upsilon_y} \cong \frac{\upsilon_x}{c}$$

(This fig. is from Berkeley Physics Course I. Ch. 14.)

With the transverse velocity component of the photon υ_x we find

$$m_{ph}\upsilon_x = \int F_x dt \qquad (1.11)$$

dt is the time when the photon travels along *dy*. So we can write $dt = {dy}/{c}$ and have to the integral

$$\upsilon_x \cong -\frac{2GM\, r_0}{c}\int_0^\infty \frac{dy}{(r_0^2 + y^2)^{3/2}} \cong -\frac{2GM}{cr_0} \qquad (1.12)$$

Then the deflection angle is

$$\phi \cong \frac{\upsilon_x}{\upsilon_y} = \frac{\upsilon_x}{c} \cong \frac{r_s}{r} \qquad (1.13)$$

However, the measured angle is in agreement with GR twice as large as (1.13) obtained from Newton's gravitation.

1.4. Problems: Achtung: Bitte Übungsblätter in den Grauen Kasten im Flur zum Treppenhaus stecken Aufschrift **Gebhardt: Schwarze Löcher Übungen**

1.4.1.Consider a BH of 10 solar masses. a) Determine the mean density when unrealistically the mass fills the space within the spherical event horizon. b) Do the same for a supermassive black hole (SMBH) of $10{\cdot}10^9$ solar masses. c) How does the mean density (within the event horizon) depend on the mass of the BH?

1.4.2. Work out the gravitational deflection of light along the equations (1.10) to (1.13) given in the lecture.

1.5. A. brief history of the concept of Black Holes.

1687	**Sir Isaac Newton**	**Described gravity in his publication, "Principia."**
1783	**John Michell**	**Conjectured that there might be an object massive enough to have an escape velocity greater than the speed of light.**
1796	**Simon Pierre LaPlace**	**Predicted the existence of black holes. "...[It] is therefore possible that the largest luminous bodies in the universe may, through this cause, be invisible." -- Le Système du Monde**
1915	**Albert Einstein**	**Published the Theory of General Relativity, which predicted spacetime curvature.**
1916	**Karl Schwarzchild**	**Used Einstein's Theory of General Relativity to define a black hole. Defined gravitational radius of black holes, later called the Schwarzchild radius.**
1926	**Sir Arthur Eddington**	**Relativity expert who, along with Einstein, opposed black hole theory.**
1935	**Subrahmanyan Chandrasekhar**	**Pioneer in theory of white dwarfs that led to an understanding of mass limits that decide whether a star will die as a white dwarf, neutron star or black hole.**
1964	**John Wheeler**	**Coined the term, "black hole."**
1964	**Jocelyn Bell-Burnell**	**Discovered neutron stars that, at the time, were the densest matter found through observations.**
1970	**Stephen Hawking**	**Defined modern theory of black holes, which describes the final fate of black holes.**
1970	**Cygnus X - 1**	**The first good black hole candidate that astronomers found. It emits x-rays and has a companion smaller than Earth but with a mass greater than that of a neutron star.**
1994	**Hubble Space Telescope**	**Provides best evidence to date of supermassive black holes that lurk in the center of some galaxies. The Space Telescope Imaging Spectrograph (STIS) revealed large orbiting velocities around the nucleus of these galaxies, suggesting a huge mass inside a very small region.**

Recommended books and articles.

Kip S. Thorn: Black holes and time warps. Einstein’s outrageous legacy. W.W. Norton & Comp. N.Y., Löndon 1994
A wonderful popular story of the history of GR and BHs from one of the founders of the new Sciende of BHs and strong gravity.

R. u. H. Sexl: Weisse Zwerge – schwarze Löcher. 2. erw. Aufl. Vieweg-Studium Nr.. 14
A very readable introduction to GR and gravitation, which does avoid the use of differential geometry but keeps to elementary calculus from high-school math.

Claus Kiefer: Gravitation. Fischer
Taschenbuch 2003 (Fischer – Kompakt)
The author (Professor of Theor. Physics of the Univ. Köln) tries to make a difficult subject easy readable and may be also understandable