5. Observing BH environments. Accretion disks. Jets.

5.1. The Eddington Luminosity

Accretion discs (AD) are very general phenomena in astrophysics. They are observed when a new star is born or when in a binary system one component is more evolved than the other so that its extended atmosphere has already filled the Roche lobe of the system. If the evolved component has reached its final state and is turned into a compact object, a neutron star or a black hole, then the inner part of the AD is a source of high energy radiation which contains considerable information about the compact object. In the case of active galactic nuclei (AGN) the central SMBH is surrounded by an extended ring of interstellar dust and molecular gases. Within this ring we find an AD the innermost region of it, its corona, is a source of high energy radiation.

In some cases large jets of ultra relativistic particles are observed which beam through the whole visible extension of the galaxy. The luminosity L (the radiation power) is often
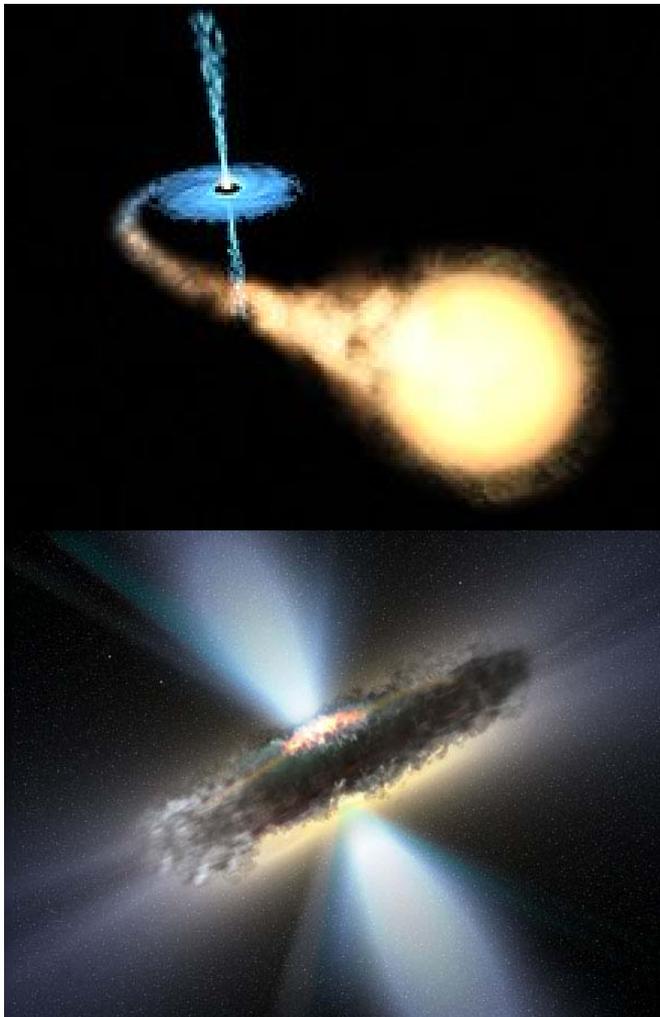

Fig. 5.1. X-ray binary System: An evolved star matter of which streams into a AD surrounding the compact companion, a NS or a BH.

Fig. 5.2 A SMBH and its jets. The SMBH and its AD is hidden behind a torus of gas and dust.

given in units of solar luminosity $L_{Sol} = 3,85 \cdot 10^{26} Watt$. Another important reference for accreting systems is the Eddington luminosity L_E . This is the limiting luminosity which is reached when the radiation pressure becomes as high as the gravitational force and prevents a

further increase of the accretion rate. To see this we consider the radiation flux F ($Joule/\sec \cdot m^2$) passing in a spherical system through an optical medium

$$F_{rad} = \frac{L}{4\pi \, r^2} \tag{4.1}$$

where R is the distance from the radiation source. The energy density of the radiation field at the distance R from the source is

$$\frac{F_{rad}}{c} = \frac{L}{c4\pi \, r^2} \left[\frac{Joule}{m^3} \right] \tag{4.2}$$

Radiation is scattered from a hot fully ionized plasma of accreting matter mainly by Thomson scattering at free electrons. The Thomson scattering cross section is a classical quantity

$$\sigma_0 = \frac{8\pi}{3} \left(\frac{e^2}{4\pi\varepsilon_0 m_e c^2} \right)^2 = 6{,}65 \cdot 10^{-29} \, m^2 = 0{,}1 \, barn = 10^{-3} \, fm^2 \tag{4.3}$$

The expression in the brackets is called **classical electron radius**. We proceed and ask for the radiation force acting on one electron in the plasma which should be equal to the gravitational force of a compact object of mass M acting on one proton (assuming the plasma is neutral)

$$\sigma_0 \frac{L}{4\pi \, c \, r^2} = G\frac{m_P M}{r^2} \tag{4.4}$$

The Luminosity which fulfils (4.4) is called Eddington luminosity

$$L_E = \frac{4\pi \, G \, m_P M \, c}{\sigma_0} = 1{,}3 \cdot 10^{31} \cdot \frac{M}{M_{sol}} \; Watt \quad or \quad \frac{L_E}{L_{sol}} = 3{,}2 \cdot 10^4 \cdot \frac{M}{M_{sol}} \tag{4.5}$$

In strong gravitational fields L_E of equ. (4.5) can be larger and has then to be multiplied by the Schwarzschild factor $\sqrt{g_{11}} = \left(1 - r_S/r\right)^{-\frac{1}{2}}$. There is a corresponding accretion rate, the Eddington accretion, which we obtain from the potential energy

$$L_E = \frac{GM \, \dot{M}}{r} = 3{,}16 \cdot 10^{-3} \, \frac{R_{sol}}{r} \frac{M}{M_{sol}} \frac{\dot{M}}{(10^{-10} M_{sol}/yr)} \; in \; units \; of \; L_{sol} \tag{4.6}$$

$L_{sol} = 3{,}85 \cdot 10^{26}$ *Watt*. This expression gives the luminosity in solar units when the accreting object has solar mass, solar radius and an accretion rate of 10^{-10} solar masses per year. A neutron star with 1,4 solar masses and 11,0 km radius would give $280 \cdot L_{sol}$ with this accretion rate . Finally the Eddington accretionrate for this NS.is1,14 $^{-8}$ M$_{sol}$/year.

Note that L_E depends on the inverse of R , meaning the smaller the accreting system the higher is the luminosity L_E. Equ. (4.6) leads for one solar mass to an accretion rate

$$\dot{M}_E = \frac{L_E R}{GM} = 1{,}5 \cdot 10^{-8} R_{10} \; [M_{sol} \; yr^{-1}] \tag{4.7}$$

This tells us that we find at Eddington luminosity an accretion rate of $1{,}5 \cdot 10^{-8}$ solar masses per year at a distance of 10 km when the compact object has a mass 1 M_{sol}. Note also in (4.6) the inverse dependence on R, the radius of the compact object. When we compare the radii of white dwarfs and neutron stars we find a factor 10^3. It follows that the Eddington luminosity L_E of a NS is 10^3 times higher than L_E of a WD. The maximum amount of energy liberated by free fall (of hydrogen) on the surface of a WD or NS is

$$\frac{3}{2} kT = G \frac{m_P M}{R} \tag{4.8}$$

where mp is the mass of a proton. This gives for a WD the maximum energy of 100 keV or 10^9 Kelvin and for a NS 100 MeV or 10^{12} Kelvin.

5.2. Angular momentum transport. Disc temperatures.

Matter which is transported into the center loses potential energy per each step Δr from $-G \frac{M \Delta M}{r}$ to $-G \frac{M \Delta M}{r - \Delta r}$..But it also looses angular momentum which should be transported outward for matter to accrete. The accreted material considered as an inviscid fluid forms a laminar flow as long as the Rayleigh criterion is obeyed

$$\partial \frac{r^2 \Omega}{\partial r} > 0 \tag{4.9}$$

This prevents the existence of hydrodynamic angular momentum transport. However, usually a subsonic turbulence is assumed as the source of viscosity v and is taken into account as a free parameter $\alpha = v/c_S \cdot H$ with c_S the velocity of sound. At present magneto-hydrodynamic (MHD) processes are assumed to cause the viscosity, however the relevant processes are still not well understood.

Usually the Kepler velocity $v_\phi(r) = \sqrt{GM/r}$ is much larger than the drift velocity $v_r(r)$. The potential energy U of material spiralling inwards can be completely or partly radiated away by black body radiation. We are going to make an estimate of the temperature profile of the disc assuming that the conversion of potential energy in radiation is complete. The mass ΔM spirals from radius r to $r - \Delta r$ and releases the potential energy

$$\Delta U = -GM \, \Delta M \left(\frac{1}{r} - \frac{1}{r - \Delta r} \right) \tag{4.10}$$

which is converted in thermal radiation (emitted from both sides of the disk)

$$\Delta U = 4\pi \, r \Delta r \cdot \sigma T^4 \Delta t \tag{4.11}$$

Here $\sigma = 5{,}67 \cdot 10^{-8} \, Watt \cdot m^{-2} K^{-4}$ is the Stefan-Boltzmann constant. Equating (4.10 and (4.11) gives

$$GM \dot{M} = 4\pi r^3 \sigma T^4 \qquad (4.12)$$

Solving for T we finally obtain

$$T = \left[G \frac{M\dot{M}}{4\pi\sigma r^3} \right]^{\frac{1}{4}} = \left[\frac{G}{4\pi\sigma} \frac{M^3}{r^3} \right]^{1/4} \left[\dot{M} \right]^{1/4} \left[\frac{1}{M} \right]^{1/2} \qquad (4.13)$$

or

$$T = \left[\frac{G}{4\pi\sigma} \frac{M^3}{r^3} \right]^{1/4} \left[\dot{M} \right]^{1/4} \left[\frac{1}{M} \right]^{1/2}$$

Replacing M/r by r_S / r we find

$$T = 1{,}82 \cdot 10^{10} \cdot \left[\frac{\dot{M}}{1 M_{sol} \cdot yr^{-1}} \right]^{\frac{1}{4}} [r_S / r]^{\frac{3}{4}} \left[\frac{M_{sol}}{M} \right]^{\frac{1}{2}} \qquad (4.23a)$$

If we set r equal to the inner disk edge, T would be the maximum temperature which surprisingly decreases with increasing BH mass M. This is the reason why T_{max} of an active galactic nucleus (AGN) is lower than that of a galactic BH.

Direct observations are possible for some special systems, e.g. Z Cha an eclipsing narrow binary system where the accreting component is a white dwarf. Fortunately the inclination angle "i" is nearly 90° so that an eclipse of parts of the bright disk by the primary star and the white dwarf companion results in an interesting light curve. An analysis confirms fairly well the dependence $T \propto r^{\frac{3}{4}}$ found above.
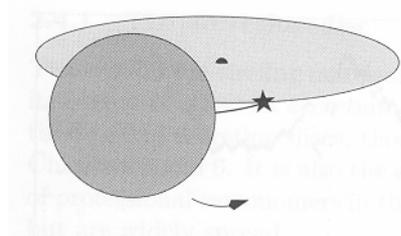

Fig.5.3.a) Model of the eclipsing system Z Cha in which a White Dwarf and a normal red dwarf star are orbiting around each other eclipsing always different parts of the accretion disc. Credit: Coel Hellier: Cataclysmic variable stars.
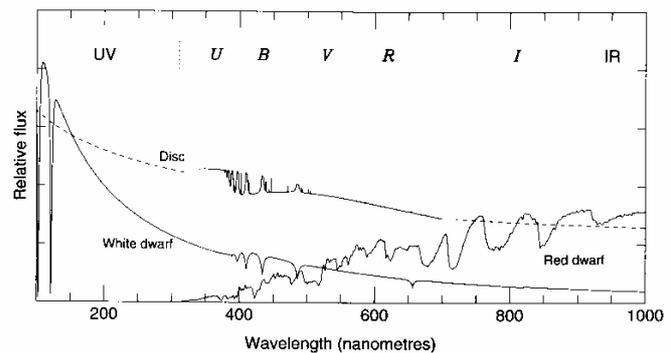
Fig.5.3.b) Schematic spectra from a system as Z Cha, called cataclysmic variable because of its huge brightness variations. Credit: Coel Hellier: Cataclysmic variable stars.

The assumption of a thin disk usually made refers to the geometry and simply means that the thickness H is much smaller than the radius r or $H \ll r$. The meaning of optical thick or thin is less obvious. We define the optical thickness by Beer's law. When a light beam of intensity

I and wavelength λ passes a sheet of matter with thickness *dz* then the intensity is weakened by extinction *k*

$$dI_\lambda = -k(\lambda, z) I_\lambda \, dz \qquad\qquad (4.14)$$

The optical thickness is defined as

$$\tau_\lambda = \int_0^z k(\lambda, z') dz' \qquad\qquad (4.15)$$

We speak of an ***opticalyl thick disc*** when $\tau > 1$ and an ***optically thin disc*** when $\tau < 1$. With the known thickness *H* of the disc the ratio

$$\bar{l}_\lambda \cong \frac{H}{\tau_\lambda} \qquad\qquad (4.16)$$

gives us a measure of the photon's mean free path $\bar{l}_\lambda$. A distinction between the limiting cases can be made from the spectra of the disc. In an optically thick disc the photons from spectral lines come from nearer the surface than do the continuum photons. We see the atomic lines in absorption. In an optically thin disc the photons come from anywhere and are not absorbed. Therefore we see the atomic lines in emission together with the continuum. When the temperature drops the optical thickness may also decrease and an optically thick disk can go over to a thin one. Such detailed studies of accretion disks are only possible in cases where the compact object is a WD and especially in such rare cases where eclipses occur. In cases of binariy systems with NSs their X-ray luminosity is so strong that irradiation effects prevail.

5.3. Various models and a unification.

In this section we will very briefly mention two different models of accretion. In the standard model of accretion disks (SAD) developed by Shakura and Sunyaev (1972 and 1973) the disk is in hydrodynamic equilibrium. The viscous shear generates heat $Q_v^+ = v\rho H (r \, d\Omega / dr)^2$. This occurs mainly by magneto-hydrodynamic processes in the plasma which are not well understood but fixed by a phenomenological constant α. This model is therefore often called α-disk model. The flow is cooled by radiation Q_r^-. In a thin plasma "bremsstrahlung" dominates whereas in a thick plasma the cooling is provided by thermal radiationt (s. equ. 4.11 - 4.13). The radiative cooling is efficient if

$$Q_v^+ - Q_r^- = 0. \qquad\qquad (4.16)$$

The other limiting case is called advection dominated accretion flow (ADAF) which was 1992 proposed by Narayan. In this model (4.16) is not fulfilled. A large part of Q_v^+ is kept in the plasma, raises its heat and thickens the disk. The hydrodynamic equilibrium is disturbed and leads to sub-Keplerian rotation (Andreas Müller, Diss. Heidelberg 2004)

$$\Omega = \frac{\upsilon_\phi}{r} \approx 0,4 \cdot \Omega_{Kepler} \tag{4.17}$$

ADAF solutions have low accretion rates $\dot{M} << \dot{M}_E$. Most of the frictional heat Q_v^+ disappears in the BH. An example is the BH in the center of our galaxy. The two models represent two limiting cases in which

$$f = \frac{Q_v^+ - Q_r^-}{Q_v^+} \text{ is either 1 or 0.} \tag{4.18}$$
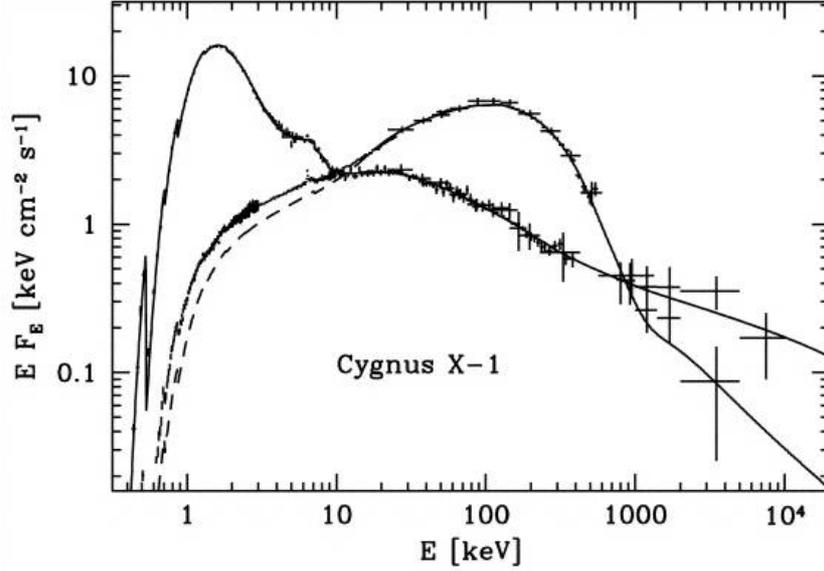

Fig. 5.4. Two spectra of Cyg X1 representing to different accretion states The upper curve was taken in the high/soft state, the lower one represent the low/hard state.

In the last 20 years several modification have been made and proposed under different names. Instead of going into further details we will follow A. Müller (Diss. 2004) and describe qualitatively a unification scheme of accretion on BHs. In many respects the scheme is independent of the mass of the BH. We will use several parameters to characterise the accretion flow ($\dot{m} = \dot{M}/\dot{M}_E$ accretion rate), the luminosity (L_E Eddinton luminosity, L_X X-ray luminosity) and the photon index α_P which describes the high energy behaviour of the spectra (intensity $\propto E^{-\alpha}$).

Table 5.1. gives some parameters and properties corresponding to the geometric models of Fig. 5.5. Although the various states of accretion apply to BHs of all masses the properties of active galactic nuclei are known to be dominated by large masses of molecular gas (H_2, H_2O, CO, CH, CN etc) .and dust (mainly C and oxides of Si, Fe and Ni). This supplied the SMBHs in the early times of the universe (z = 2 – 2,5) with a rich reservoir of matter leading to strong activity as observed in Quasi Stellar Objects (QSOs), BL Lac objects, and Radio-galaxies. In later phases of galactic evolution these reservoirs in the vicinity of the SMBH are mostly exhausted. Still prominent jets are observed which extend over 50 kpc. Finally the accretion settles into a quiescent state as is the case in the centre of our own galaxy. The central location of our galaxy is called for historical reasons Sgr A* , it hosts a SMBH of mass $4,3 \cdot 10^6 M_{Sol}$.

The radiation is shielded by a thick layer of interstellar dust. The estimated Luminosity is $L \approx 10^{-9} L_E$, the accretion flow mainly ADAF.
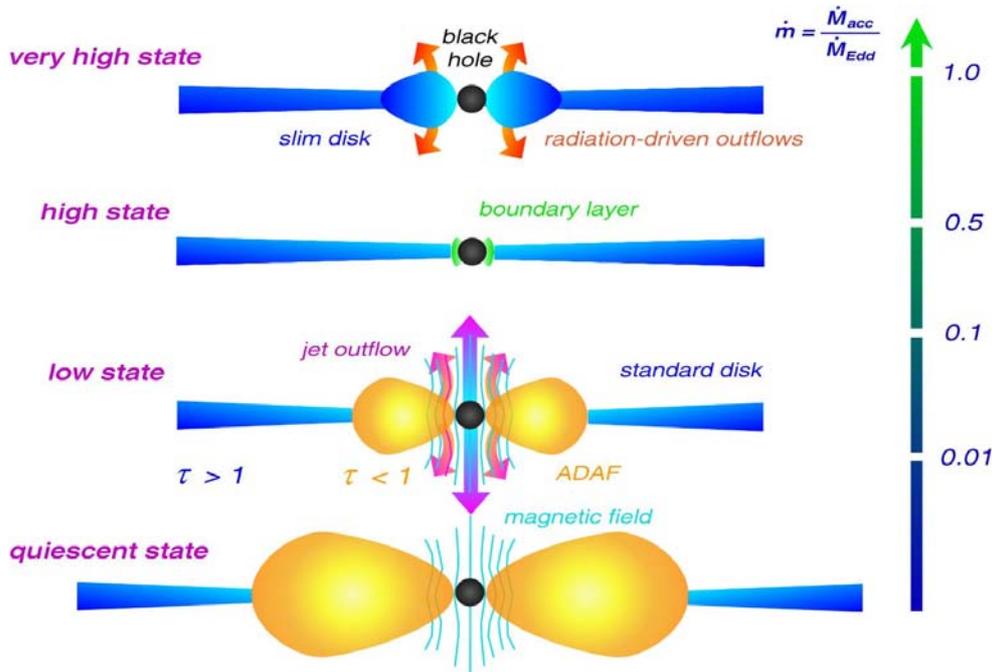

Fig. 5.5. Illustration of a unified schemeof BH accretion depending on the accretion rate given as vertical coordinate at the left side. The application of this scheme should be roughly independent of the BH's mass.

Accretion state	Luminosity	Accretion	Spectra	Peculiarities
Very high state	$L \approx L_E$ Very intense x-ray	$\dot{m} \approx 1$ Flow optically thick, $f < 1$	Non thermal plus blackbody radiation	Mostly ADAF
High/soft state	A modified black body ca. 1 keV	$f \approx 1$, $\dot{m} \geq 0,09$	Black body with subluminous high energy power tail $\alpha_P \cong 2,5$	Boundary layer acts as corona No ADAF
Low/hard state	$L_X \leq 0,1 \cdot L_E$	$0,1 > \dot{m} > 0,08$ $f \ll 1$ A hot ADAF within a cool slim disk	$\alpha_P \approx 1,5 - 1,9$ Photons from the disk gain energy in the ADAF by inverse Compton scattering. High energy cut-off at 100 keV . Fluorescence lines from Fe Kα at 6 - 7 keV	Extended outflow and production of jets
Quiescent/off state	$L \ll 10^2 L_E$	ADAF bigger in size,	$\alpha_P < 1,5$	No jet

		$\dot m < 0{,}01$ $f \ll 1$		

Table 5.1. Parameters and properties of the accretion states of BHs. Here is $\dot m = \dot M / \dot M_E$ and α$_P$ the exponent of the high energy slope.

Jets are very mysterious phenomena and are intimately connected with an accretion disk. They are even observed in star formation. In the context of our discussion we are exclusively concerned with the jets near BHs. These jets are collimated beams of ultra relativistic particles probably mainly electrons and positrons confined in a magnetic field. The jet hits the interstellar gas but reaches far out in the respective galaxy and ends in a shock front where it emits synchrotron radiation. The effort to construct a convincing model goes on for at least two decades. It has become clear that a purely hydrodynamic explanation remains insufficient. Magnetic fields have to be considered and MHD proves to be the appropriate framework of the modelling. It is possible that the magnetic field is transported with the plasma flow to the ergosphere (see lecture 6) of the rotating BH where it is dragged by the rotating spacetime forming a breaded structure. Poynting fluxes are generated that drive magnetically an outflow of matter.

Before we close this lecture we come back again to galactic BHs. The systems where one has found candidates of BHs are the so calles low mass X-ray binaries. However it is not always easy to decide between a neutron star and a BH. The neutron star has a solid crust where the heat Q_v^+ is completely converted in radiation (photons and neutrinos). Some characteristics of BHs are the lack of strong periodic feature of the X-ray intensity and low luminosity, whereas NSs may collect matter over the time of years. But in a suddenly nuclear explosion the H- and He-layer burns down and a burst of x-rays and visible light can be observed.. Galactic BHs however, are mostly found in a quiescent state (Low Mass X-ray Transients). The decisive observation must allow to determine the mass which for a BH should be larger than 3 M_{Sol}, which is clearly above the stability range of NSs. Only one galactic system was found with prominent relativistic jets: SS 433 also called a "mini-quasar".

Object	X-ray luminosity (erg s^{-1})	Distance (kpc)	Spectral type	V	Orbital period (days)	K velocity km s^{-1}	Mass function $M_\odot$	Mass $M_\odot$
Cyg X-1	2×10^{37}	2.5	O9.7I	9	5.6	76	0.25	>3
LMC X-3	3×10^{38}	55	B3V	17	1.7	235	2.3	>6
A0620 00	1×10^{38}	1	K5V	12–18	0.32	457	2.91	>3
V404 Cyg	2×10^{34}	4	K0	13–19	6.5	210	6.3	>6

Table 5.2. Properties of BH candidates from P.A. Charles and F.D. Seward: Exploring the X-ray universe. Cambridge university press 1995.
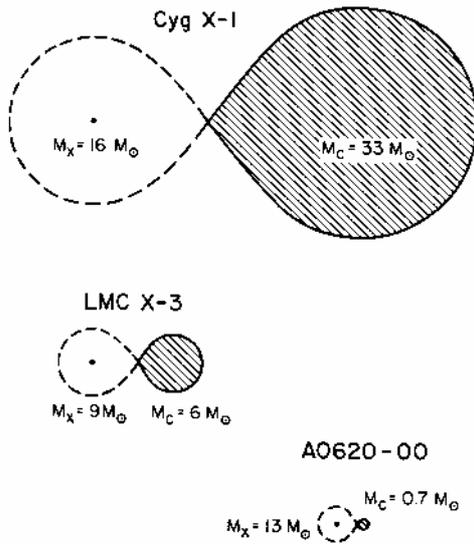
Fig. 5.6. Three galactic BH candidates.
The droplet shaped curves represent the Roche lobes and give approximately the correct geometry plotted in equal scale.
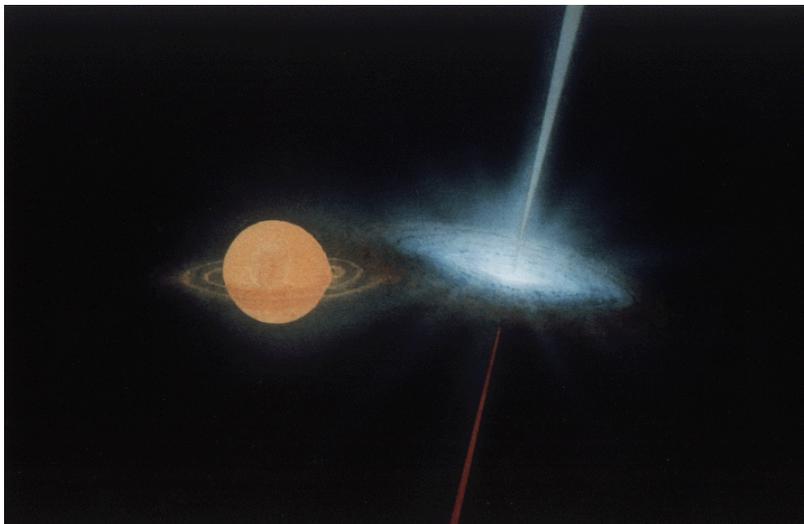

Fig.5.7. SS 433 an artists view. Credit :NASA

5.4. Problems.

5.4.1. A neutron star has the mass $M = 1,30\,M_{sol}$ and radius $R = 11\,km$. Calculate the Eddington luminosity and the corresponding Eddington accretion rate.

5.4.2. Estimate the fraction of rest mass which is converted into radiation.

5.4.3. Assume that the accretion disk of a BH with 10 solar masses is in the high soft state and that the inner edge coincides with the innermost marginal stable orbit. Assume further that the accretion rate is 1/10 Eddington. What is the disk temperature at the inner edge?

Solar Luminosity $L_{sol} = 3,85 \cdot 10^{26}$ Watt

Solar Radius $\quad R_{sol} = 6,96 \cdot 10^{8}$ m

Stefan-Boltzmann constant $\sigma = 5,67 \cdot 19^{-8}$ J m^2 s^{-1} K^{-4}

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