8. Neutron Stars. Facts and Fiction.

8.1. Rotation-powered Pulsars

The existence of neutron stars had already been proposed (by Baade and Zwicky 1934, Landau 1932, 1938, Tolman 1939) long before their discovery as radio pulsars by Bell and Hewish 1967. For this discovery Anthony Hewish and Martin Ryle were awarded with the Nobel Price in 1974, but Hewish's capable thesis student Jocelin Bell missed out (a bleak chapter in the history of the price). Pulsars are observed by a very regular succession of radio pulses. Their repetiton period t_P ranges from 1,4 milliseconds to 8 seconds.

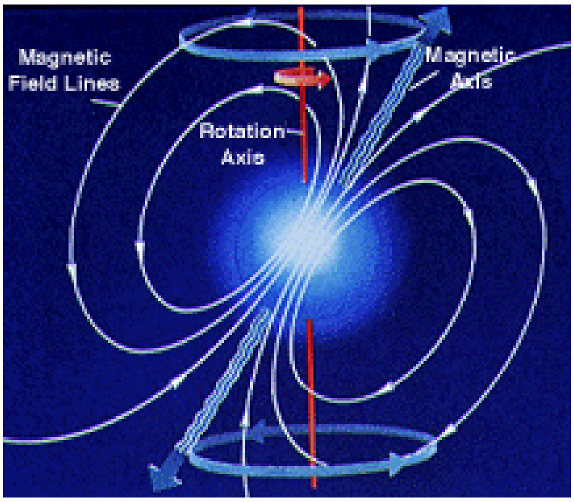


Fig. 8.1. Pulsar model. Axis of rotation and magnetic axis are not parallel to each other but form an oblique angle.

Fig.1. shows a very simple model which explains the main properties of a pulsar. There is an angle between the magnetic moment and the axis of rotation. The moving magnetic moment induces a strong electric field. Particles are generated and accelerated in the electric field. They move along helical paths around magnetic field lines and emit synchrotron radiation. Whenever the magnetic moment passes the line of sight a pulse of beamed radiation is observed. Thus the pulsar acts as a lighthouse.

The neutron star is not simply a giant nucleus but a star, i.e. bound mainly by gravitational forces. A NS may be considered as rigid sphere (or rotational ellipsoid). Then the gravitational acceleration GM/R^2 should always outweight the centrifugal acceleration $R\Omega^2$. In this way we find a lower limit of the rotation periods t_P of a NS

$$\frac{GM}{R^3} \geq \Omega^2 = \left[\frac{2\pi}{t_P}\right]^2 \quad \text{or} \quad t_P \geq 0{,}55\left(\frac{M_{sol}}{M}\right)^{1/2}\left(\frac{R}{10\,km}\right)^{3/2} ms \qquad (8.1)$$

At still shorter periods t_P the star becomes unstable. With $R = 11 km$ and $M = 1{,}4 M_{sol}$ we obtain $t_P \geq 0{,}61\ ms$ as lower limit. For White Dwarfs this lower limit would be 25 s and would not explain the much shorter periods of ms-pulsars. One of the shortest periods found is 1,5578 ms of a pulsar PSR B1937+21**.** As a rigid rotator the neutron star has an enormous amount of rotational energy available

$$E = \frac{1}{2} I \Omega^2 \tag{8.2}$$

A loss of rotational energy would be observed as a spin-down-effect $\dot{t}_P$

$$-\frac{dE}{dt} = -I \Omega \dot{\Omega} = \frac{4\pi^2 I \dot{t}_P}{t_P^3} \tag{8.3}$$

Independent from the specific processes which contribute to the observed radiation we may apply a well known expression for the radiated power of an oscillating magnetic dipole with moment p_m and angle β between moment and axis of rotation

$$-\frac{dE}{dt} = \frac{\mu_0}{4\pi} \frac{2 p_m^2 \omega^4}{3c^3} \sin^2 \beta \quad \text{Watt} \tag{8.4}$$

The moment p_m can be expressed in terms of the surface magnetic field B_0

$$p_m = \frac{1}{2\mu_0} B_0 R^3 \quad \text{Tesla} \cdot \text{m}^3 \tag{8.5}$$

so that we finally obtain

$$-\frac{dE}{dt} = \frac{2\pi^3 B_0{}^2 R^6}{3 \mu_0 c^3 t_P^4} \sin^2 \beta \quad \text{Watt} \tag{8.6}$$

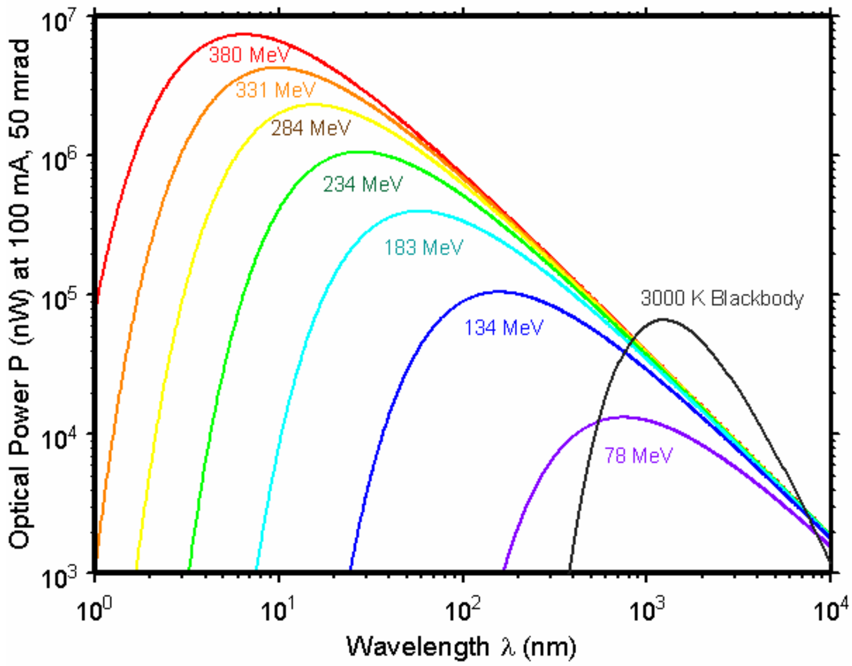


Fig. 8.2. Synchrotron spectral power versus observable wavelength with electron energy as parameter. The spectra cover a broad range of frequencies or wavelength as often observed in pulsar radiation. With highly relativistic electrons the beam becomes collimated with aperture $\left(1 - \upsilon^2 / c^2\right)^{-1}$.

The radiated power must be provided by the rotation of $1{,}4\,M_{sol}$ which acts as a huge energy reservoir

$$-\frac{dE}{dt} = -I\,\Omega\dot{\Omega} = \frac{4\pi^2\, I\, \dot{t}_P}{t_P^3} = -C\,\Omega^{n+1} \qquad (8.7)$$

Here *I* is the moment of inertia and $\dot{t}_P$ the decay of the pulse period which is found in the range $10^{-19} < \dot{t}_P < 10^{-13}$. From (8.2) and (8.3) we find a rough measure of the pulsar life time

$$\tau_{life} = -\frac{E}{\dot{E}} = -\frac{\Omega}{2\dot{\Omega}} \approx \frac{t_P}{2\dot{t}_P} \qquad (8.8)$$

Equ. (8.6) and (8.7) connect the quantities *B*, *I*, t_P and $\dot{t}_P$. If three have been measured an unknown fourth quantity can be determined (Note: 1 year is 31 536 000 seconds).

$$B\sin\beta = 3\cdot 10^{15}\left(t_P\ \dot{t}_P\left[\frac{I}{10^{38}\,kg\cdot m^2}\right]\right)^{1/2}\ Tesla \qquad (8.9)$$

This seems to be a nice simple model widely used by astronomers. However, we should keep in mind that the observed radiation is not magnetic dipole but ***synchrotron radiation***, i.e. a curvature radiation. Furthermore rotation of a star in GR is more complicated. The braking index is defined as

$$n = -\frac{\Omega\ddot{\Omega}}{\dot{\Omega}^2} \qquad (8.10)$$

and a power law can be formulated

$$\dot{\Omega} \propto -\Omega^n \qquad (8.11)$$

One would expect $n = 3$ for magnetic dipole radiation, but one finds $n < 3$. Although the repetition frequency is very stable the particular pulses change their shape quickly. The beam cone itself must be structured which suggests a complex and quickly variable pulsar plasma.

The Crab pulsar is typical and has the highest observable X-ray intensity so that it is used as a reference source. Here are some values:

Distance: 2 kpc
Source: SN from 1054 (observed by Chinese astronomers)
$L \approx 10^5 \cdot L_{sol}$
$t_P = 33\ ms$ or more precise 0.0334033474093999999 ± (2 x 10^{-13}) s
$\dot{t}_P = 30{,}8\,ns$ per day ; magnetic field ca. 10^8 Tesla

The radiation and the pulsar wind are both powered by the rotation of the NS and lead to a slowing down of the pulsar's rotation period. The wind forms a high energy plasma emitted from the open field lines near the pol regions of the NS and observable as a shock front in the surrounding nebula.

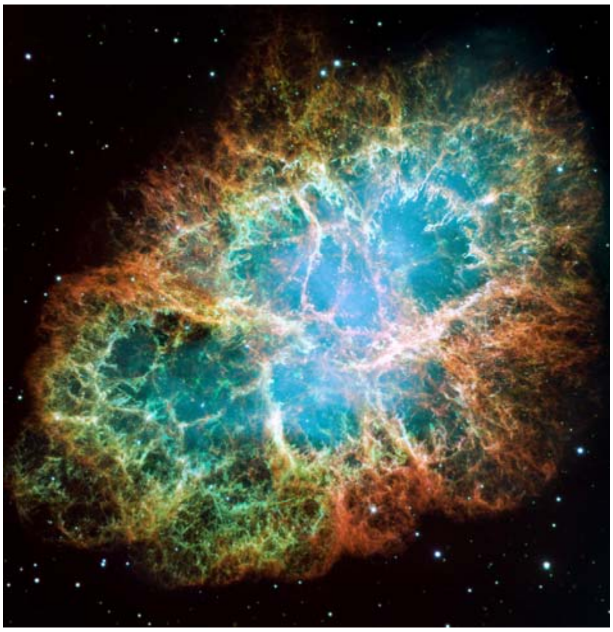
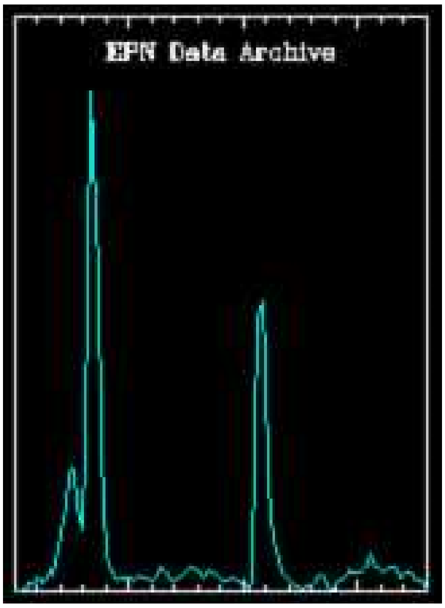
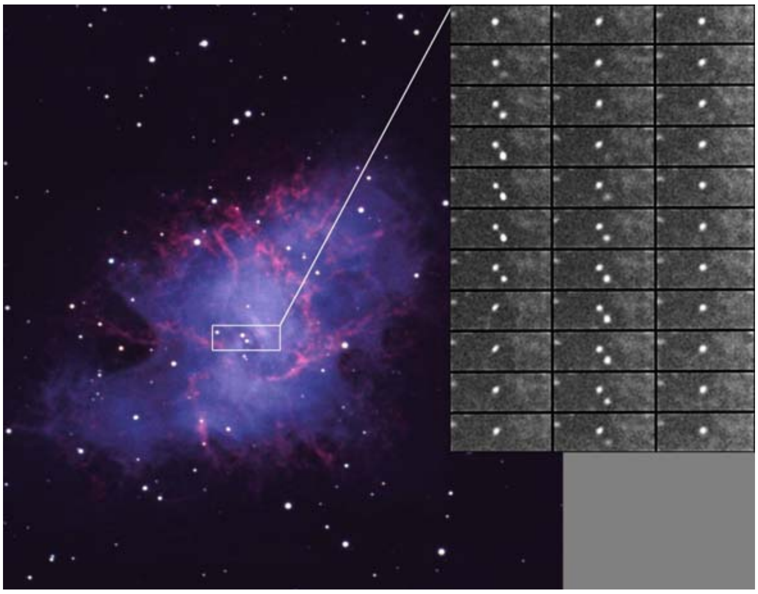


Fig. 8.3. a) Crab optical picture HST **b) Crab X-ray picture Chandra**





c) Radio pulses from Jodrell Bank **d) Optical pulsing (polarised light)**

Magnetic fields have mostly been determined by equ. (8.9). In some cases it was possible to observe the electron cyclotron resonance line (s Lect. 09). The motion of the high magnetic field induces extremely high voltages. We can perform a simply estimate

$$U = -\frac{d}{dt}\left(B_0 \cdot A\right) \qquad (8.12)$$

With the plane perpendicular to the rotation axis we have

$$A = \pi R^2 \sin^2 \beta \cos \omega t \qquad (8.13)$$

We obtain

$$U = \pi R^2 \omega B_0 \sin^2 \beta \sin \omega t \qquad (8.14)$$

We use for an estimate the following values $R = 12\ km$, $\sin^2 \beta = 0{,}5$, $\omega = \frac{2\pi}{t_P}$ and obtain

$$U = 1{,}4 \cdot 10^{17} \frac{1}{t_P}\ Volt$$

These extremely high voltages are found near the polar region. They are strong enough to free particle pairs from the surface of the NS.

Whereas nearly 2 000 pulsars with periods of 0,1 to 10 s have been found there are only a few hundred millisecond pulsars. The latter are either very young as the Crab or they got "reloaded" with angular momentum from accreted matter collected in an accretion disk from a normal companion.

8.2. Birth and cooling.

Due to our present knowledge a NS is born during the catastrophic event of a core collapse Supernova (SN) which ends the nuclear life of a star in the mass range of $8 M_{sol} < M < 20 M_{sol}$. After nuclear burning proceeded up to ^{56}Fe, the element with the highest nuclear binding energy, no more energy can be gained by fusion processes. Thus the supply of thermal energy ceases. The core is only preserved from gravitational collapse by the pressure of the degenerate electron gas. In this final stage of stellar evolution the core has reached about the size of the earth. It has attained a mass of 1,44 M_{sol}, the Chandrasekhar mass limit, at which the degenerate electron gas of the core becomes relativistic. Its support against gravity ceases, the core becomes unstable and collapses. Density and temperature increases further and a process of nuclear disintegration starts. Protons mutate into neutrons by an inverse β-process $p + e = n + v$ an endothermic process which consumes energy that is thermal pressure. The density of electrons vanishes and with it the pressure of the electron gas. During the quick implosion of the star the core converts into a hot neutron sphere. All the masses of the outer stellar parts move in free fall to the centre and are reflected there at the hard crust of the new born neutron star. A stream of neutrinos helps to boost the unbound matter off in a powerful explosion. Immediately after the SN event the temperature of the NS is about 10^{12} Kelvin.

Neutrinos are copiously produced acting as an efficient heat sink. Neutrino generation is performed by the so called URCA-processes $n \to p + e^- + \bar{v}$ and $p + e^- \to n + v$. Here v and $\bar{v}$ denote a neutrino and an antineutrino respectively. The process was first discussed by George Gamow and Mario Schoenberg while they were visiting a casino named *Casino-da-Urca* in Rio de Janeiro. Schoenberg is reported to have said to Gamow that "the energy disappears in the nucleus of the supernova as quickly as the money disappeared at that roulette table." In Gamow's South Russian dialect, *urca* can also mean a robber or gangster. Another URCA-process with momentum conservation is $n + n \to p + n + e^- + \bar{v}$ and $p + n + e^- \to n + n + v$. The cooling by URCA has a steep T^8-temperature dependence Somewhat less steep is the neutrino-pair production by photons $\gamma + A \to \gamma' + A + v + \bar{v}$. It needs the presence of a nucleus A and has a temperature dependence of T^6 But it is still not

clear how much this process contributes to the cooling. Finally thermal radiation $\propto T^4$ comes into play which controls the slower temperature decay.
Thermal radiation of NSs has been measured in the X-ray and in the optical regime also from solitary NSs. The results give temperatures of some 10^6 Kelvin. Nevertheless at these temperatures NS can be considered as “cooled down” since kT is far below the Fermi temperature of electrons or nucleons in the star. There exist some observational difficulties to determine the deviations of the NS’s thermal radiation from a black body. For this purpose it is necessary to construct a model atmosphere which is under the unusual conditions of the NS surface very difficult. The atmosphere of a neutron star is only a few centimetre thick. One of the troubles is in many NSs the presence of extremely high magnetic fields, e.g. a field of $B \cong 10^8$ $Tesla$ changes the binding energy of hydrogen atoms from 13,6 to 120 eV (s. lect. 09). More data and more realistic modelling will hopefully improve the present knowledge about temperature and age of NSs.

8.3. What’s in it? The internal structure of NSs.

How do we know what is inside a NS? As a first approach physicists have tried the apply the known laws of statistical thermodynamics to NSs. What has been left from a supernova event (besides shells of diluted high velocity plasma clouds) is a high density gas of electron protons and neutrons quickly cooling down. All quantum states are successively occupied up to the maximum, the Fermi momentum k_F (or Fermi energy). These quantum gases of fermions are often called Fermi gases. All models of the internal structure of NSs start with a plausible equation of state (EOS) of the neutral nuclear matter, i.e. a mixture of protons, neutrons and electrons. An overview is depicted in fig. 8.5. From densities of 10^6 g/cm^3 up to some 10^{11} g/cm^3 the pressure is dominated by relativistic electrons $P_e \propto \rho_e^{4/3}$ (full black line). Under the conditions of equilibrium and charge neutrality we have

$$\mu_e + \mu_p = \mu_n \qquad \text{and} \qquad k_p = k_e \tag{8.20}$$

where the expression

$$\mu_l = \sqrt{k_l^2 + m_l^2} \tag{8.21}$$

is the chemical potential (Fermi energy) of the respective particles “l” with momentum k_l.
Below the density

$$\rho \approx n_p\, m_p = 3{,}56 \cdot 10^{-8}\ fm^{-4} = 1{,}25 \cdot 10^7\ g \cdot cm^{-3}$$

we find (besides atomic nuclei) a neutral Fermi gas in equilibrium which is an equal mixture of protons and electrons without neutrons, above neutrons appear. The neutron drip point is reached at $\rho \approx 4 \cdot 10^{11}$ g/cm^3 when neutrons start to move freely between atomic nuclei.

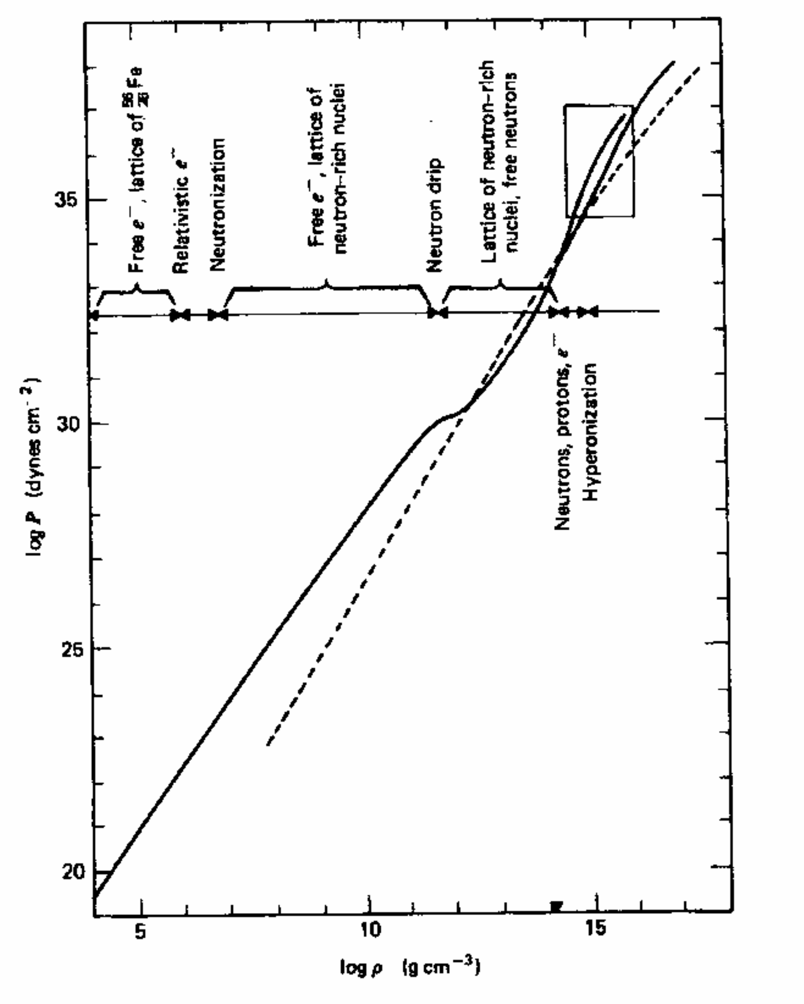


Fig. 8.4. Shown is a plot of the equation of state $P = f(\rho)$ for cold nuclear matter, i.e. the Fermi energy E_F is much larger than the thermal energy kT. See also text.

The electron pressure is overtaken above $\rho > 10^{11}$ by non-relativistic neutrons $P_n = K_{5/3}\,\rho_n^{5/3}$ (dashed line). The constant is given by $K_{5/3} = \frac{1}{20}\left[\frac{3}{\pi}\right]^{2/3} \frac{h^2}{m_n^{8/3}}$ $= 5{,}3802 \cdot 10^2$ Joule/kg. In this region $10^{11} < \rho < 10^{14}$ nuclear forces are attractive and the effective pressure is somewhat below the neutron pressure. The nuclear saturation density is at $2 \cdot 10^{14}$ g/cm^3. Beyond saturation the nuclear forces are repulsive (s. the inlet box). At these high densities there is a preponderance of neutrons. One finds for the densities in the ultra relativistic limit, i.e. $k^2 >> m^2$ (when other baryon types are ignored)

$$\rho_p \approx \frac{1}{8}\rho \quad \text{and} \quad \rho_n \approx \frac{7}{8}\rho \tag{8.22}$$

We find these conditions in the core and can therefore rightfully speak of a neutron star. Nuclear matter is considered as a saturated system. The short range repulsion and the Pauli principle are the reason for the rigid-sphere behaviour of nucleons. The nuclear radius scales as

$$R = r_n \cdot A^{1/3} \tag{8.23}$$

with $$r_n = 1{,}16\ fm = 1{,}16 \cdot 10^{-15}\ \text{m} \tag{8.24}$$

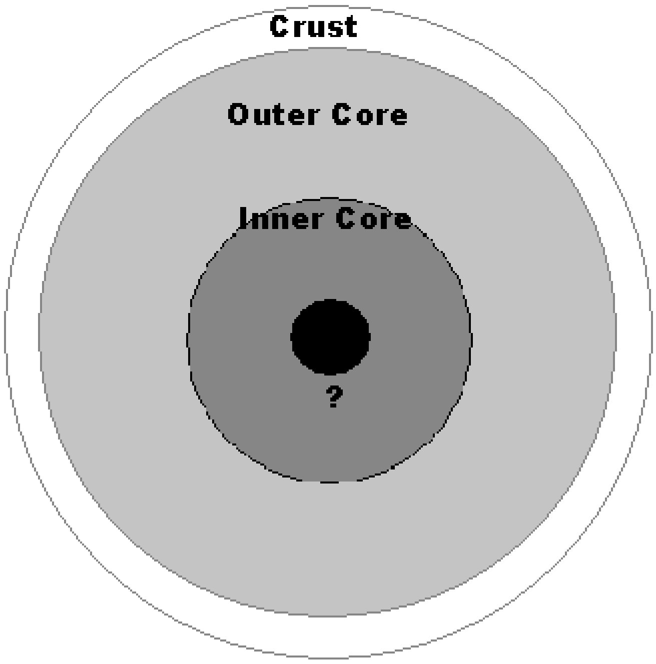


Fig. 8.5. Scheme of the internal structure of a NS: The crust is essentially composed of Fe, Ni, Si. It has a high density of some $10^6\ g \cdot cm^{-3}$. In the deeper crust the elements become neutron rich. Deeper in the NS when the outer core starts, the density reaches the neutron drip and undergoes a further steep increase. Neutrons can now leave the atoms as a nearly free particles. The matter is assumed to be solid up to densities of some 10^{12} when atomic nuclei are completely dissolved into protons, neutrons and electrons. The composition of the central part of the inner core is speculative. Above densities of $4{\cdot}10^{14}$ g/cm^3 π-mesons and hyperons might appear besides relativistic electrons and nucleons. Also a phase transition into a quark-gluon plasma is possible in the core.

There is an empirical expression in the liquid drop model for the nuclear mass with Z protons, N neutrons and $A = Z + N$ the number of nucleons

$$M(A,Z) = A\left[\frac{4\pi}{3} r_n^3 \varepsilon_n + a_{sym}\left(\frac{N-Z}{A}\right)^2\right] + 4\pi r_n^2 A^{2/3} \varepsilon_{surf} + \frac{3}{5}\frac{e^2 Z^2}{r_n A^{1/3}} \tag{8.25}$$

In this formula is $\varepsilon_n = 141\ \text{MeV}\,\text{fm}^{-3}$ at saturation. The binding energy per particle is defined as

$$\frac{B}{A} = M(A,Z) - m \tag{8.26}$$

where m is the averaged nucleonic mass m = 938,93 MeV. With increasing A the volume- and the symmetry-term dominate. In the limiting case of infinite symmetric nuclear matter one finds

$$\frac{B}{A} = -16{,}3\,\mathrm{MeV} \quad \text{and} \quad a_{sym} = 32{,}5\,MeV \tag{8.27}$$

for the binding energy per nucleon and the symmetry coefficient respectively. The binding energy is related to the EOS $\varepsilon(n)$ by

$$\frac{B}{A} = \left[\frac{\varepsilon}{n}\right]_n - m \tag{8.28}$$

.

Using $r_n = 1{,}16$ fm we may also obtain an equilibrium value of the number density

$$n_n = \frac{1}{\frac{4\pi}{3} r_n^3} = 0{,}153\ fm^{-3} \tag{8.29}$$

The final expression for the energy density contains besides the energy of nuclei the energy of the electrons $\varepsilon'(n_e)$, the number of free nucleons $\varepsilon_n(n_n)$ and the lattice energy of the solid phase $\varepsilon_{lattice}$

$$\varepsilon = n_N M(A,Z) + \varepsilon'(n_e) + \varepsilon_n(n) + \varepsilon_{lattice} \tag{8.30}$$

with

$$\varepsilon_{lattice} = -1{,}444 \cdot Z^2 e^2 n_e^{3/4} \tag{8.31}$$

Given the energy density the pressure can be derived

$$P(n) = n^2 \frac{\partial}{\partial n}\left(\frac{\varepsilon}{n}\right) \tag{8.32}$$

Assuming equilibrium the density at saturation is 0,15 fm^3. The energy per particle of nuclear matter reaches a minimum of about -16 MeV. Around this minimum the energy has a parabolic dependence. Its curvature is given in lowest order by the compression modulus

$$K = P^2 \frac{d}{dP^2}\left(\frac{\varepsilon(P)}{n(P)}\right) = 9n^2 \frac{d^2}{dn^2}\left(\frac{\varepsilon(n)}{n}\right) \tag{8.33}$$

which is likely between 230 and 240 MeV (M.Camenzind gives 234 MeV as most probable value).
The new born NS cools quickly below the Fermi energy of the particles (e, p, n) T << $T_{F,i}$. Under the conditions decribed above these are still “low temperatures” and we expect to find rather a quantum liquid than a quantum gas. There is all reason to assume that the neutrons in the core are supra-liquid and the protons superconducting. Since most NSs carry a very high magnetic field the superconducting vortices should be ordered.

8.4. How much mass is allowed? The limiting mass of NSs.

Masses of NSs are found all slightly below 1,4 solar mass. This lies near the Chandrasekhar limit of an iron core and suggests a common birth process during a core collapse of a massive star. One of the most accurate mass determinations has been possible in a rare binary pulsar PSRJ 0737-3039. The component A was found to have a mass of $M_A = 1{,}337 \pm 0{,}005$ where as component B has $M_B = 1{,}250 \pm 0{,}005$, which latter is the lightest NS observed up to now.

In a first step we make a very crude approximation and consider a NS with ***uniform density***. We again have to solve the Einstein equation but now with non vanishing energy momentum tensor

$$T_0^0 = \varepsilon \qquad \text{and} \qquad T_k^k = P \tag{8.49}$$

We will not derive the formalism but just write down the results relevant in our discussion:

$$M(r) = 4\pi \int_0^r \varepsilon(r')\, r'^2 dr' \qquad\qquad \frac{dP}{dr} = \frac{[P(r)+\varepsilon(r)][M(r)+4\pi r^3 P(r)]}{r[r-2M(r)]} \tag{8.50}$$

Equ. (8.50) are called Oppenheimer-Volkoff equations which we will now use for our simple model. The second equation replaces in GR the hydrostatic equation of Newtonian mechanics.

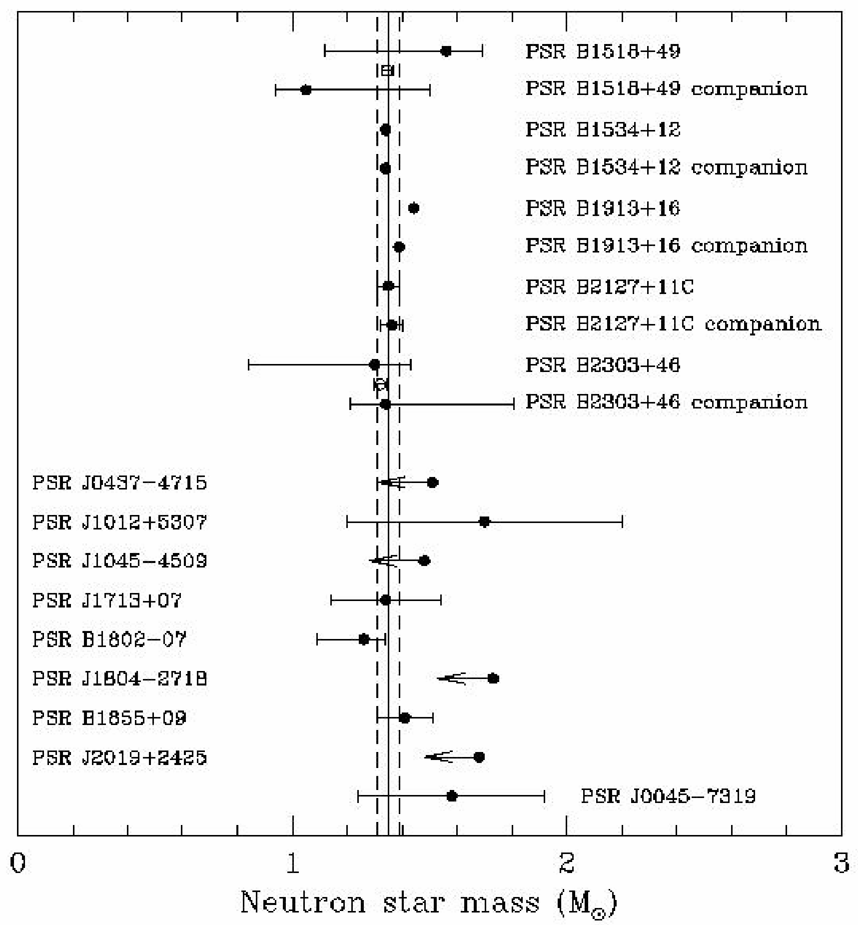


Fig. 8.6. Masses of neutron stars as measured in binary systems with pulsars.

It has a very remarkable feature: the right hand side contains also the pressure. In Newtonian mechanics a sufficient high pressure saves the star from gravitational collapse. In GR this is not always the case. Therefore gravitational collapse is inevitable.
Let the mass of the NS be *M* and the radius *R* and the constant density ε_C. Then we may write

$$M(r) = \frac{4\pi}{3} r^3 \varepsilon_C \quad \text{and} \quad M = \frac{4\pi}{3} R^3 \varepsilon_C \tag{8.34}$$

and

$$\frac{dP}{dr} = -\frac{4\pi r}{3} \frac{(P(r)+\varepsilon_C)(3P(r)+\varepsilon_C)}{1-8\pi \frac{\varepsilon_C r^2}{3}} \tag{8.35}$$

Rearranging variables we may write

$$-\int_r^R \frac{dP}{(P+\varepsilon_c)(3P+\varepsilon_c)} = \frac{4\pi}{3} \int_r^R \frac{r\,dr}{1-8\pi \frac{\varepsilon_C r^2}{3}} \tag{8.36}$$

Note that the edge of the star at $r = R$ is defined by $P(r) = 0$. Then we find after integration

$$\frac{P(r)+\varepsilon_C}{3P(r)+\varepsilon_C} = \sqrt{\frac{1-2M/R}{1-2M\,r^2/R^3}} \tag{8.37}$$

Solving for $P(r)$ we finally obtain

$$P(r) = \varepsilon_C \left[\frac{\sqrt{1-2M/R} - \sqrt{1-2Mr^2/R^3}}{\sqrt{1-2Mr^2/R^3} - 3\sqrt{1-2M/R}} \right] \tag{8.38}$$

The central pressure P_C is reached for $r = 0$. Then (8.37) gives

$$1 - \left[\frac{P_C + \varepsilon_C}{3P_C + \varepsilon_C} \right]^2 = \frac{2M}{R} \tag{8.39}$$

We take the limit at large $P_C >> \varepsilon_C$ and find the inequality

$$\frac{2M}{R} < 1 - \left(\frac{1}{3} \right)^2 = \frac{8}{9} \tag{8.40}$$

We can calculate the corresponding limiting radius with the help of (8.28)

$$\frac{4\pi}{3} \varepsilon_C = \frac{M}{R^3} = \frac{M}{R} \cdot \frac{1}{R^2} \tag{8.41}$$

We find

$$R_{\lim} = \frac{1}{\sqrt{3\pi\varepsilon_C}} \quad \text{and} \quad M_{\lim} = \frac{4}{9} R_{\lim} \tag{8.42}$$

a lower limit for the radius R. For a NS of mass $M = 1{,}4\,M_{sol}$ a model of uniform density gives the limiting value $R \approx 4{,}7\,km$. The calculated mass limit depends on the model used. But most authors put it below 2 solar masses. We could easily do a bit better and use the equation of an ideal degenerate neutron $P \propto \rho^{5/3}$ which we have mentioned below fig. 8.4. Unfortunately after insertion into the Oppenheimer-Volkoff equations we see that we cannot find a simple analytic solution. Instead we have to solve equ. 8.27 numerically. The results are as follows

$$R = 14{,}64\left(\frac{\rho_c}{10^{15}\,g\,cm^{-3}}\right)^{-1/6}\text{km} \qquad M = 1{,}102\left(\frac{\rho_c}{10^{15}\,g\,cm^{-3}}\right)^{1/2}\cdot \text{M}_{\text{sol}} = \left(\frac{15{,}12\,\text{km}}{R}\right)^{3}\cdot \text{M}_{\text{sol}} \qquad (8.43)$$

These values look much more promising than those obtained with constant density. Note that for M $\rightarrow 0$ and $\rho_c \rightarrow 0$ we find $R \rightarrow \infty$, an unrealistic result, since neutrons become unstable at sufficiently low density. More realistic equations of state (EOS) have been developed using the full available knowledge about nuclear matter. The various attempts differ in the compression modulus K (s. equ. (8.33)) which mostly lies between 200 and 300 MeV (M. Camenznd gives K= 234 MeV as most probable value). A stiff EOS with higher K-values leads to higher central energy densities, larger radii and tolerates higher masses.

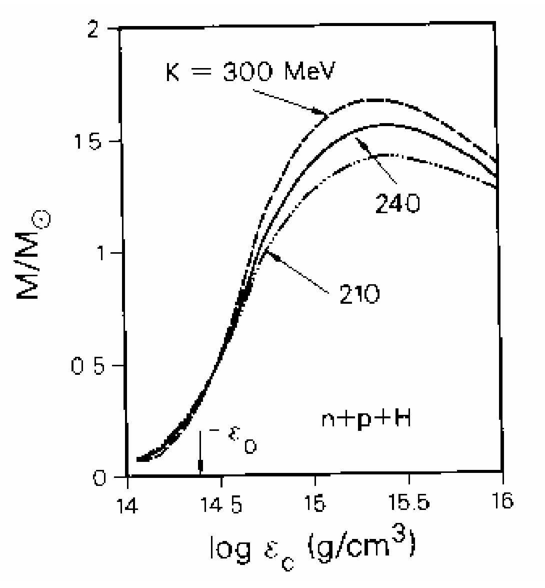


Fig. 8.7. NS sequences for 3 values of compression. Normal nuclear matter density is labeled as ε_0.

After N.K. Glendenning s. references below

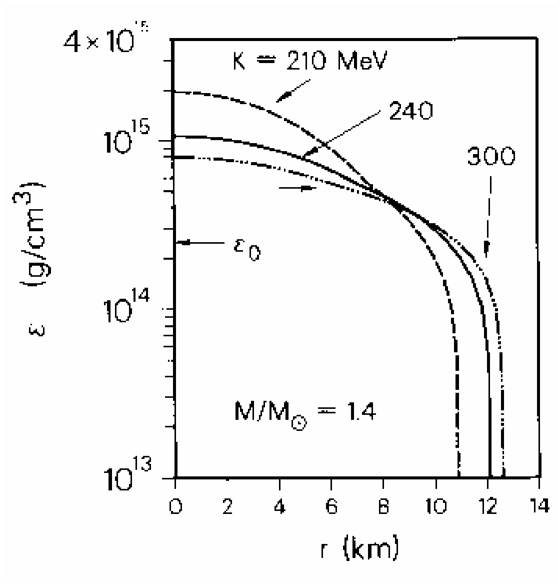


Fig. 8.8. Energy density distribution of three 1,4 solar mass NSs corresponding to three different compressions as indicated .

After N.K. Glendenning s. references below

We may use the droplet model to estimate the number of nucleons in a neutron star. If the nucleons are packed to their repulsive cores we may relate the radius R and the mass of the NS to the number A of baryons

$$R \approx r_0 \, A^{1/3} \;\; \text{and} \;\; M \approx A \cdot m \qquad (8.44)$$

where *m* is the baryon mass. We estimate

$$A \approx 2{,}6 \cdot 10^{57} \qquad (8.45)$$

A relativistic calculation has to solve the integral

$$A = 4\pi \int_0^R \left(1 - \frac{2M(r)}{r}\right)^{-1/2} r^2 n(r)\, dr \;) \qquad (8.46)$$

See fig. 8.9 for the results of model calculations where $A \approx 1{,}8 \cdot 10^{57}$.

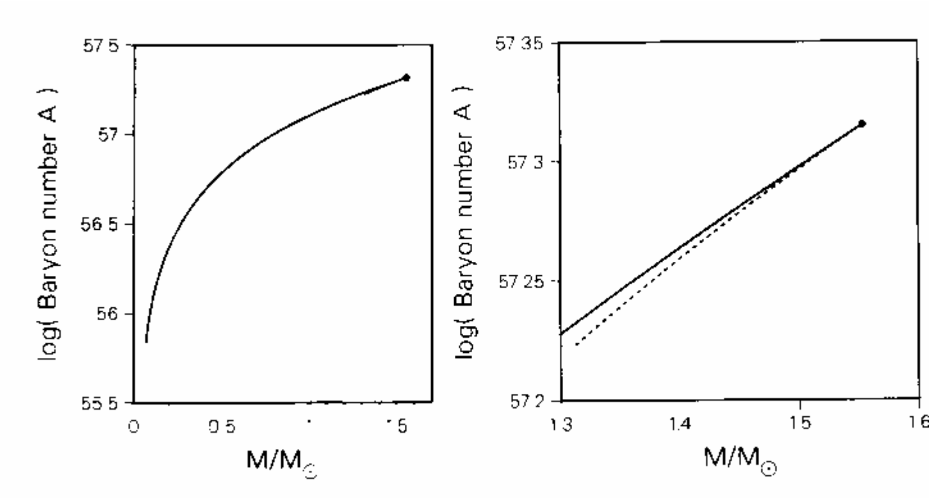


Fig. 8.9. Calculated number of nucleons after N.K. Glendenning s. below

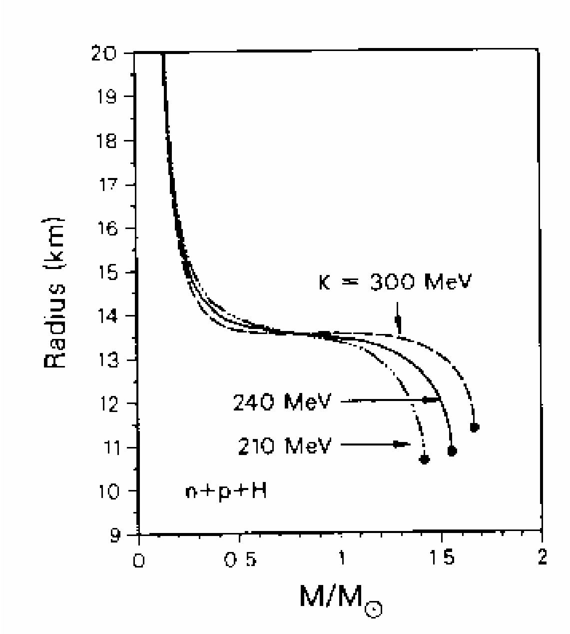


Fig 8.10. Radius-mass relation for three different EOSs.

With the known gravitational mass a Newtonian calculation of the gravitational energy yields

$$E_g \cong \int \frac{M(r)dM(r)}{r} = \frac{3}{5}\frac{M^2}{R} = \frac{3}{10}\frac{2M}{R}M \qquad (8.47)$$

If we take $r_S/R = 0{,}35$ then we find the gravitation to be 10,5% of Mc^2.

From the inequality (8.1) we obtain a lower limit of the energy density $\bar{\varepsilon} > 1.41 \cdot 10^{14}$ g/cm^3 for pulsars with periods $t_P > 1$ ms. Calculated values of the central denstity ε_c (s. fig. 8.7) reach $6{,}3 \cdot 10^{14}$ g cm^{-3} which matches nicely with our estimate in the inequality..

Surface redshifts $z = \left[1 - \frac{r_S}{R}\right]^{-1/2} - 1$ from binary pulsars have been obtained in the range $0{,}22 < z < 0{,}26$, which fits reasonably well in the diagram of calculated.

Although the spin down of a pulsar happens very smoothly there are sometimes sudden spin-ups, called pulsar glitches. About 170 of such glitches have been observed which had magnitudes between $10^{-6} > \Delta\Omega / \Omega > 10^{-9}$. The amount of rotational energy gained by a glitch is huge and amounts to as much as 10^{36} *Joule*. However, the structure of pulses is not affected. Therefore electromagnetic effects of the emission processes can safely been excluded. The responsible process must be in the NS's core. There was much hope that in the glitches the NSs may reveal something of their internal structure. Unfortunately after more than 40 years of observation and considerable theoretical effort there is still no consistent explanation.

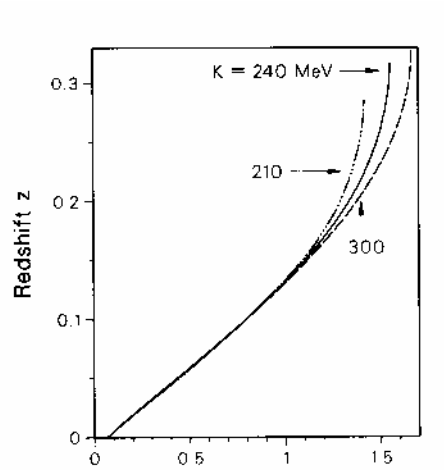


Fig. 8.11. Calculated surface reshift factors for models with three different compression moduli

One of the models which is still in discussion considers the superfluid phase as angular momentum reservoir. The spin down rate of superfluid is slower than that of the crust. Whenever there is a transfer of momentum from the superfluid to the crust a spin-up, a glitch, is observed.

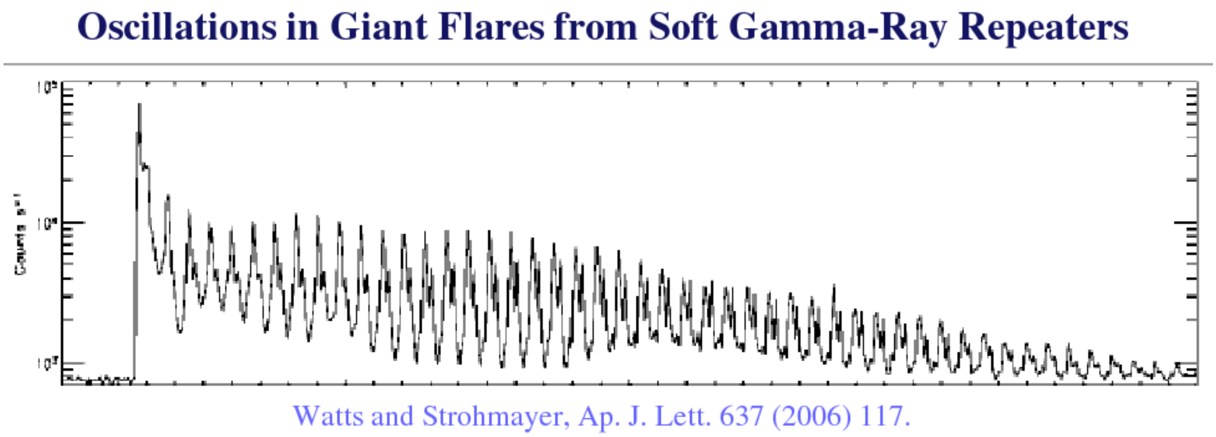


Fig. 8.12. Oscillations in the counting rate of a soft gamma repeater 1806-20. The frequencies of the eigenmodes can be extracted and compared with those from model calculations.

A more promising approach to test the internal structure seems to be astroseismology. Vibrations of normal stars have widely been measured by using "high resolution Doppler spectroscopy". The vibration times are in the range of minutes (or mHz). Starquakes on a NS are very strange events. Such event has recently been observed in the neutron star SGR 1806-20 about 50 000 Ly from the earth. The notation SGR assigns this object to the "soft gamma ray repeater", a class of NSs with extremely high magnetic fields of $10^{10} - 10^{11}\, Tesla\ or\ 10^{14} - 10^{15}\, Gauss$. A sudden reconnection of the field configuration can suddenly release more energy than our sun has radiated in 100 000 years. The burst event which lasts only 0,1 s triggers a shock wave in the crust (s. fig. 8.12). Following the burst quasi periodic oscillations have been observed with frequencies from 18 to 1800 Hz. Those with 18 and 26 Hz are assigned with torsion modes of the NS whereas the higher frequencies likely belong to shear modes of the solid crust. Its velocity is given by

$$\upsilon_S = \sqrt{\frac{\mu_S}{\rho}} \qquad (8.48)$$

Here $\mu_S \propto nZ^2/a$ is the shear modulus. Refering back to fig. 8.10 the most probable radius is R = 12 km, then we note that the innermost stable circular orbit lies at r = $3r_S$ = 12,6 km only slightly outside of the stellar surface. Dynamical instabilities inside r < $3r_S$ may thus facilitate
vibrations of the crust.

8.5. Problems

8.5.1. Consider a NS of 1,4 solar masses with radius of 12 km as a rigid sphere. Calculate the moment of inertia I. Use t_P and $\dot{t}_P$ from the Crab pulsar (script p.71) to determine the radiative power $\frac{dE}{dt}$ and the pulsar life time.

8.5.2. Use the moment of inertia I of 8.5.1. and the Crab parameter for an estimate of the magnetic field B.

8.5.3. In the inequality (8.1) we derived a lower limit of pulsar periods. Use $t_p = 1{,}0$ ms und derive with (8.1) a lower limit of the mean mass density of a NS. Note that for normal nuclear matter $\bar{\varepsilon} = 2{,}51 \cdot 10^{17}$ kg/m^3

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