

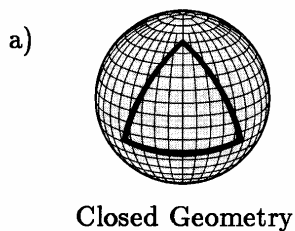
## 12. A view into modern cosmology

### 12.1. Robertson-Walker-metric and Friedmann equations.

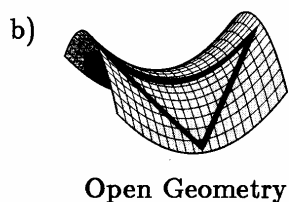
Modern Cosmology started with Einsteins „General Relativity“. About one year after Einstein published his GR in its final form he treated in a subsequent paper (1917) cosmology. He made two fundamental and far reaching assumptions: The universe should be **homogeneous and isotropic**. You may find these assumptions premature. Since when you view the sky at a clear night it does look neither homogeneous nor isotropic. Nevertheless, if the distribution of matter would be averaged over very large volumes then these assumptions seem to be reasonable. And if we eventually consider the early universe as observed in the cosmic microwave background (CMB) Einstein’s assumptions proved to be excellent (with deviations  $< 10^{-4}$ ). Einstein made a further supposition in this early paper. He assumed a static universe. Today we are already used to the idea of an expanding universe and therefore we find Einstein’s static universe a bit weird. In order to accomplish a static model Einstein had to add to his equations a **cosmological constant**  $\Lambda$ . A few years later a young Russian physicist and meteorologist, Alexander Friedmann, discovered that Einstein’s solution was unstable but that nevertheless stable solutions can be found without using  $\Lambda$  (1922, 1924).

The metric of a homogeneous and isotropic space is given in the following general form

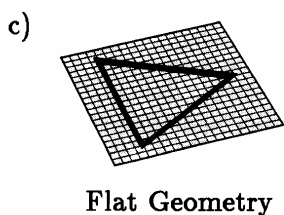
$$ds^2 = c^2 dt^2 - a(t)^2 \left[ \frac{dr^2}{1 - \kappa r^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] \quad \text{where} \quad \begin{array}{l} \kappa = +1 \\ \kappa = 0 \\ \kappa = -1 \end{array} \quad (12.1)$$



$\kappa = +1$  describes a closed space with constant positive curvature. a) at left shows a 2d closed surface with constant curvature.



$\kappa = -1$  describes an open space with constant negative curvature.. b) the surface at left is open and has negative curvature but is not isotropic.



$\kappa = 0$  describes a flat geometry i.e. the Euclidian space.  
c) The flat plane at left symbolizes a flat or Euclidian geometry.

Fig. 12.1. The geometry of a homogeneous and isotropic 3-space.

(12.1) is called **Roberson-Walker metric**. The elements of its metric tensor are the following

$$g_{00} = 1, \quad g_{11} = \frac{a^2(t)}{1 - \kappa r^2}, \quad g_{22} = a^2(t)r^2, \quad g_{33} = a^2(t)r^2 \sin^2 \theta \quad (12.2)$$

We define with (12.1) the *proper distance* with the null geodesic  $ds^2 = 0$  and  $d\Omega = 0$  which yields the light travel time in a universe where the Robertson-Walker metric holds

$$D(r,t) = ct = a(t) \cdot \int_0^t \frac{dr}{\sqrt{1 - \kappa r^2}} = a(t) \begin{cases} \cdot \sin^{-1} r & \kappa = +1 \\ \cdot \sinh^{-1} r & \kappa = -1 \\ \cdot r & \kappa = 0 \end{cases} \quad (12.3)$$

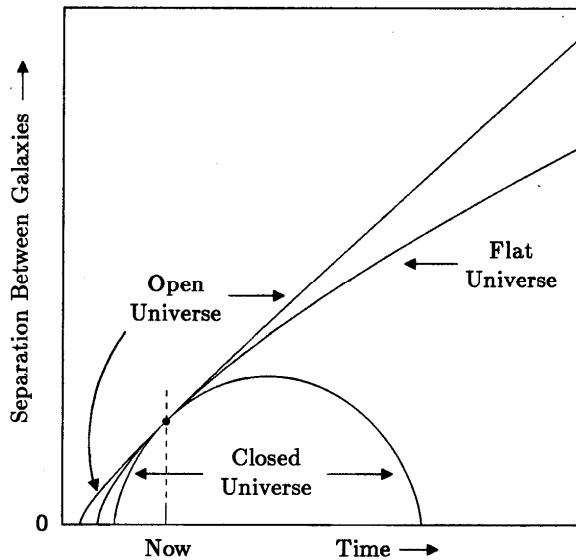
Inserting (12.2) in the Einstein equations  $R_{kl} - \frac{1}{2}R = T_{kl}$ , where  $T_{kl}$  contains energy/mass density  $\varepsilon$  and pressure  $P$  of the (friction-less) cosmic gas, gives two linear differential equations for  $\dot{a}$  and  $\ddot{a}$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P) \quad (12.4)$$

and

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{c^2}{a^2}\kappa \quad (12.5)$$

In honour to Alexander Friedmann 12.4 and 12.5 are called Friedmann Equations. They describe the dynamics of the (model-) universe. Again the parameter  $\kappa$  defines the respective geometry.



From top to bottom:

$\kappa = -1$  describes an **open universe** with negative curved space which expands for ever.

$\kappa = 0$  describes a **flat universe** with an **Euclidian space** with a long lasting expansion which will finally cease ( $t = \infty, \dot{a} = 0$ ).

$\kappa = +1$  describes a **closed universe** with positive curved space. It expands, reaches a maximum and collapses again.

Fig.12.2. Plotted solutions of (12.5) for  $\kappa = +1, 0, -1$

## 12.2. Cosmic expansion and Hubble law.

As can be seen from fig.12.2 and equ. (12.5) a precise knowledge of the Hubble constant

$$\left(\frac{\dot{a}}{a}\right)^2 \Big|_{t=0} = H_0^2 \quad (12.6)$$

is necessary to decide which geometry (and which dynamics) is realized. We would like to connect  $H_0$  with the cosmic redshift  $z$ . For this purpose we use the expression of the comoving distance  $r$  which we get from (12.3) after division by  $a(t)$

$$c \int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_0^r \frac{dr}{\sqrt{1 - \kappa r^2}} \quad (12.7)$$

The right hand side does not depend on time. We take the differential at present ( $t = t_0$ ) and at arbitrary earlier time ( $t = t_1$ ). Due to (12.7) they are equal

$$\frac{\delta t_0}{a(t_0)} = \frac{\delta t_1}{a(t_1)} \quad (12.8)$$

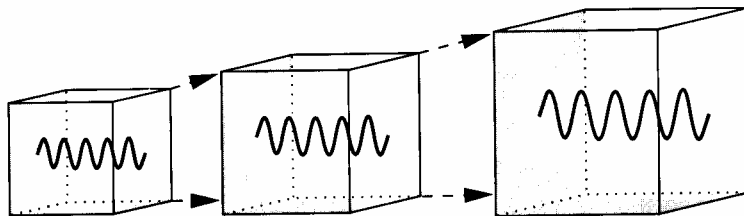
Now consider  $\delta t_1$  and  $\delta t_0$  which may stand for the inverse of frequencies  $1/\nu_1$  and  $1/\nu_0$

$$\frac{\nu_0}{\nu_1} = \frac{\lambda_1}{\lambda_0} = \frac{a(t_1)}{a(t_0)} \quad (12.9)$$

The *redshift* is then defined as

$$z = \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{\nu_1}{\nu_0} - 1 = \frac{a(t_0)}{a(t_1)} - 1 \quad (12.10)$$

$\lambda_0$  is the wavelength which the terrestrial observer sees and  $\lambda_1$  is the wavelength measured in the rest frame of the emitter



**Fig.12.3. The wavelength of light which reaches us is increased since the time of emission by the cosmic expansion.**

The scale parameter at present time is usually set  $a(t_0) = 1$ . Hubble's law arises from an expansion of (12.3) with respect to  $t$  (for  $\kappa = 0$ )

$$\frac{a(t)}{a(t_0)} = (1 + (t - t_0)H_0 + \dots) \quad (12.11)$$

and

$$\frac{a(t_0)}{a(t)} = (1 - (t - t_0)H_0 + \dots) \quad (12.11a)$$

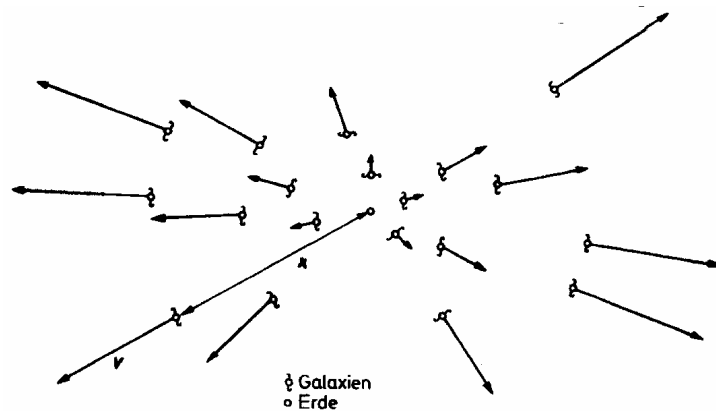
Comparing with (12.10) gives

$$z = H_0(t_0 - t) \quad (12.12)$$

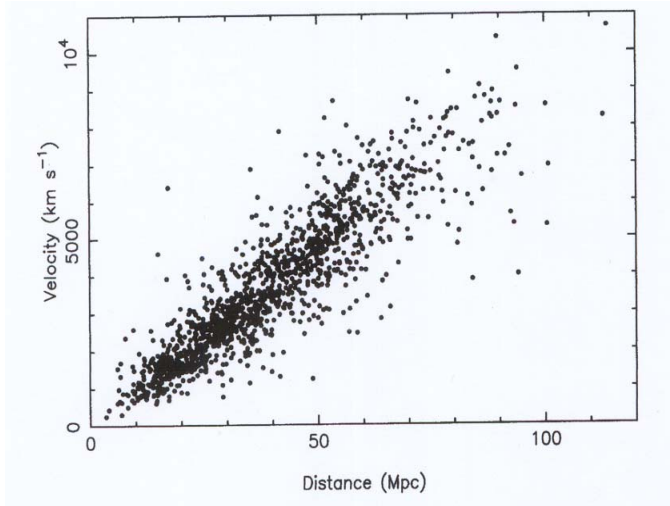
For close objects (and for all objects if  $\kappa = 0$ ) the expression  $D = c(t_0 - t)$  is the proper distance

$$cz = H_0(t_0 - t)c = H_0D \quad (12.12a)$$

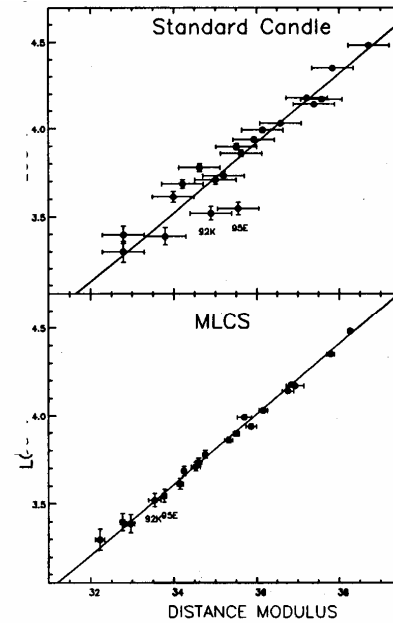
This is the Hubble law, correct for small redshifts only. At 1922 the redshift was measured already for 41 near spiral galaxies with  $z$  up to 0,006. The problem which Hubble faced in the following years was the determination of  $H_0$ . More than 50 year were necessary to eliminate all systematic errors in the common methods used. Hubble's value was about 8 times larger than the present one. This had to do with the properties  $\delta$  Cephei stars which he used as standard candles.



**Fig.12.4.** The impression that all galaxies are flying away from us. The farther they are away the faster they move. However, we are not living at a special cosmic location since an observer in any other galaxy would have the same impression. It is just the result of Hubble's law.



**Fig. 12.5.** A modern “Hubble plot”. A part from the Hubble flow the galaxies conduct peculiar motions depending on the masses in their environment.



**Fig.12.6.** Supernovae Ia are today used as reference light sources (standard candles) for distance determinations. Above: uncorrected data, below: corrections have been made

Their physics was badly known at Hubble’s time. Today it is believed that the error of the present value of  $H_0$  is only about 2%.

### 12.3. Solutions of the Friedmann equations.

The density of matter changes with volume as  $\varepsilon_m \propto a^{-3}$ . Plugging this into the Friedmann equations (for simplicity we again choose  $\kappa = 0$ ) we find the evolution of a matter dominated universe

$$a(t) = \left(\frac{3}{2}\right)^{2/3} \cdot (H_0 t)^{2/3} \quad (12.13)$$

The critical density of matter is  $\varepsilon_{c,0}$  is reached when it fulfils (12.5) for  $\kappa = 0$ .

$$\varepsilon_{c,0} / c^2 = 1,88 \cdot 10^{-26} h^2 \text{ kg/m}^3 = 0,95 \cdot 10^{-26} \text{ kg/m}^3 \quad (12.14)$$

Note that there are an infinite number of solutions for  $\kappa = \pm 1$  respectively, however there is only one solution for  $\kappa = 0$ . So to achieve  $\kappa = 0$  needs always some fine tuning. The left hand expression of (12.14) is correct for  $\kappa = 0$  and  $H_0 = 100 \text{ km}\cdot\text{s}^{-1}/\text{Mpc}$ . To allow for later corrections a factor  $h$  was introduced  $h = \frac{H_0}{100 \text{ km}\cdot\text{s}^{-1}/\text{Mpc}}$ . The present value is  $h = 0,71$  which is inserted at the right hand side of (12.14).

The density of radiation scales

$$\varepsilon_R \propto a^{-4} \quad (12.15)$$

You will recognize this easily when you consider the energy density to be  $\sigma T^4 \propto \frac{N h\nu}{V} \propto \frac{1/a}{a^3}$

With (12.15) a very simple law for a radiation dominated universe is derived

$$a(t) \propto t^{1/2} \quad (12.16)$$

or more detailed

$$a^2(t) = 2 \cdot \Omega_{R0}^{1/2} \cdot H_0 \cdot t \quad (12.17)$$

The value of  $\varepsilon_{R0}$  is the present radiation density obtained from the CMB to be

$$\varepsilon_{R0} = 4,2 \cdot 10^{-14} \text{ Joule} / m^3 \quad (12.18)$$

or

$$\frac{\varepsilon_{R0}}{c^2} = 4,7 \cdot 10^{-31} \text{ kg} / m^3$$

Equ. (12.15) implies also

$$T \propto a^{-1} = z + 1 \quad (12.15a)$$

The temperature increases with redshift.

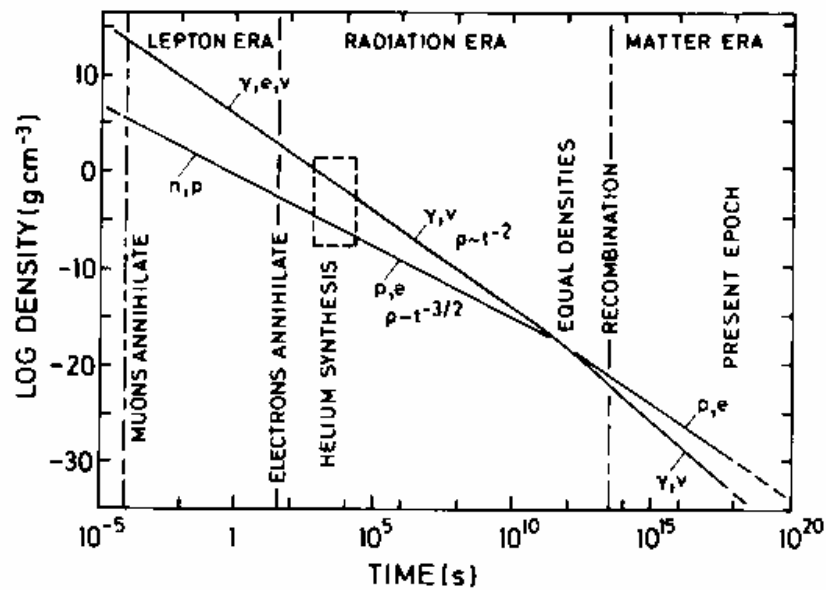
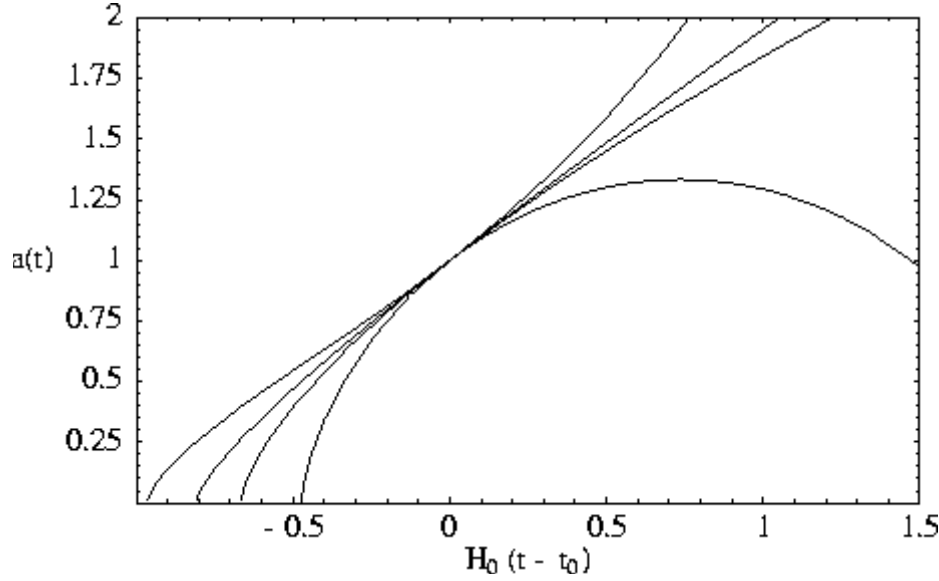


Fig. 12.7. During the cosmic evolution the density of radiation decays faster than the matter density

## 12.4. Dark energy or quintessence and dark matter.

Do we live in a matter dominated universe? Unfortunately the mean cosmic density is very difficult to measure. Let us first discuss the consequences of a fully matter dominated universe with  $\kappa = 0$ .



**12.8. Lowest curve  $\varepsilon_M > \varepsilon_C$ , second curve from below  $\varepsilon_M = \varepsilon_C$ , third curve  $\varepsilon_M < \varepsilon_C$ , upper curve  $\varepsilon_M = 0,3 \cdot \varepsilon_C$  and  $\varepsilon_\Lambda = 0,7 \cdot \varepsilon_C$ .**

The Hubble constant has the dimension of reciprocal time which is called **Hubble time**

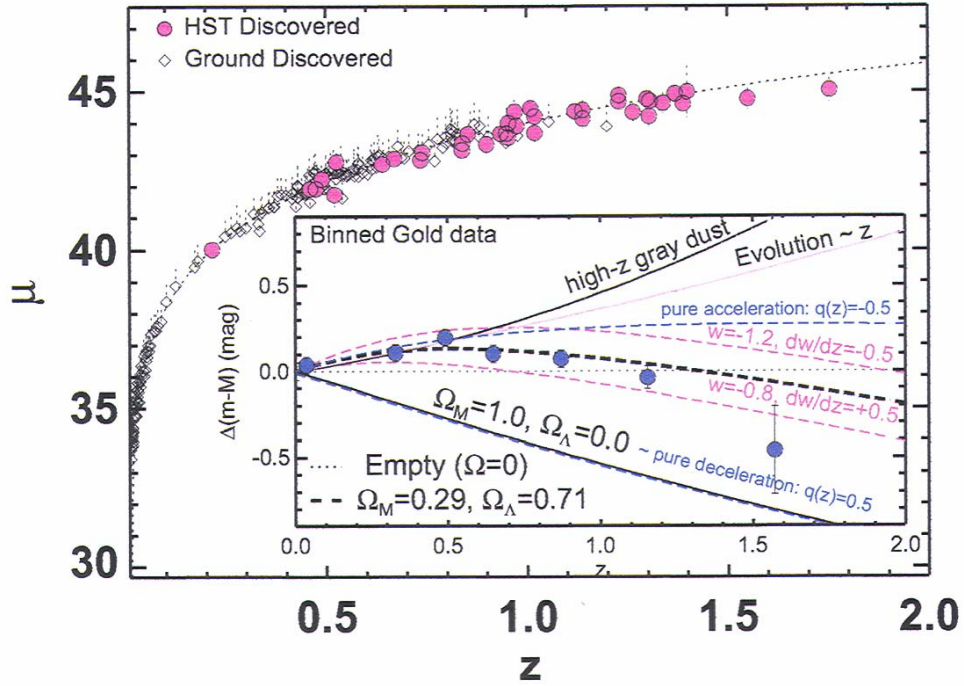
$$H_0^{-1} = t_H \cong 13,8 \cdot 10^9 \text{ years} \quad (12.19)$$

The Hubble time extends from  $(t - t_0) = 0$  to  $a(t) = 0$  where the tangent crosses the horizontal time axis at  $H_0(t - t_0) \cong -1$ . However, in a purely matter dominated universe ( s.12.13) we find that the scale parameter is zero,  $a(t) \rightarrow 0$ , for  $(2/3)t_0 = 9,2 \cdot 10^9 \text{ yrs}$ . This is in contradiction to observations since many stars are already older than  $10^{10} \text{ yrs}$ . Either we have  $\kappa \neq 0$  or the matter/energy content  $\Omega = \varepsilon / \varepsilon_c$  has a different and inhomogeneous composition. There are good reasons for the second option.

SN Ia are not only reliable standard candles but they are also very bright sources and allow astronomers to look deep into space and far back into the past. In these observations it was found that the intensity of remote SNe is weaker as expected in a matter dominated evolution (s. 12.13). These observations together with CMB data suggest that there must be an additional acceleration which changes the sign on the right hand side of the first Friedmann equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P) > 0 \quad (12.20)$$

The unknown medium must be a kind of energy since any additional mass density would make  $\ddot{a}/a$  again negative and would decelerate the expansion. We now could follow Einstein and add a cosmologic constant  $\Lambda$  to the energy momentum tensor.



**Fig.12.9.** Indication of accelerated expansion in a plot of magnitude minus absolute magnitude  $\mu = m - M$  versus cosmic redshift  $z$ . The absolute magnitude  $M$  is constant and the same for all SNe Ia. The magnitude allows to derive the luminosity distance  $D_{lum} = r(1+z)$ .

This would give an additional term to the energy-momentum tensor  $T'_{kl} = T_{kl} + g_{kl}\varepsilon'$  with

$$g_{kl}\varepsilon' = \begin{pmatrix} \varepsilon_{\Lambda} & 0 & 0 & 0 \\ 0 & -p_{\Lambda} & 0 & 0 \\ 0 & 0 & -p_{\Lambda} & 0 \\ 0 & 0 & 0 & -p_{\Lambda} \end{pmatrix} \quad (12.21)$$

and an equation of state

$$P_{\Lambda} = -\varepsilon_{\Lambda} \quad (12.22)$$

Or more generally

$$P_{\Lambda} = w\varepsilon_{\Lambda}$$

which yields

$$\frac{\ddot{a}}{a} = +\frac{8\pi G}{3c^2}\varepsilon_{\Lambda} \quad (12.23)$$

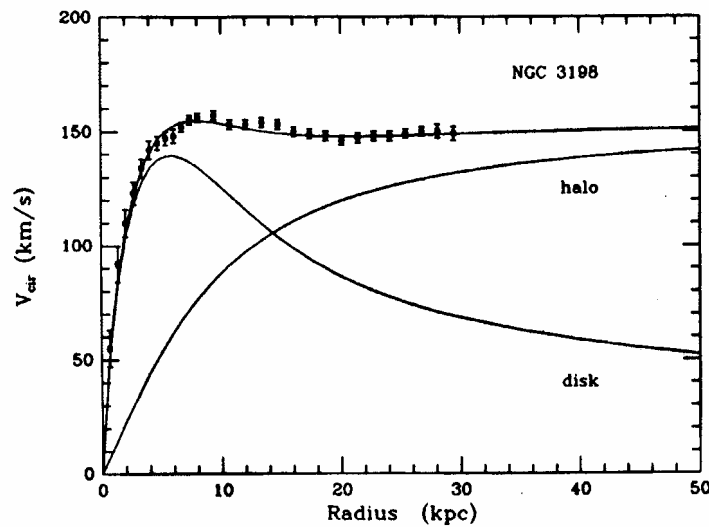
A negative pressure would have the accelerating and anti-gravitational effect we are looking for. Einstein's gravitational constant, however, offers no interpretation and is simply a mathematical trick.

An alternative assumption introduces a primordial field which dominated the very early universe and decayed during the evolution of structure to the present low value. This



mysterious field got the name “*quintessence*”. It should be time dependent. However the observations failed to show any time dependence at larger z-values..

What about matter? Let us call the known matter on earth, in stars, in the interstellar and intergalactic space *baryonic matter*. The abundances of light elements give us an upper limit for the fraction of all baryonic matter. It is about 4% of the critical density or  $\Omega_{BM} = 0,04\Omega$ . The nuclei of light elements were formed in the big bang, more precisely at temperatures of about  $10^9$  Kelvin in the first 3 minutes after the begin of the cosmic evolution. 4% is surprisingly low. And it is even more surprising how low the contribution of luminous baryonic matter is in stars: only a tiny fraction of about  $0,002\Omega$ . Obviously much of baryonic matter is invisible and exists either as fully ionized plasma of very low density in clusters of galaxies verified by X-radiation. Other baryonic matter forms large bumpy clouds of molecular hydrogen detected by its Ly $\alpha$ -absorption lines (Ly $\alpha$ - forest) in QSO-spectra.



**Fig.12.10.** a plot of stellar velocities obtained from the Doppler shift of its spectral lines. Since for circular orbits  $v^2 = GM(r)/r$  the mass within the circular orbits is immediately obtained. From  $v^2 \approx \text{constant}$  it follows  $M(r) \propto r$  (s. also ch. 11.5).

But curious enough most matter is non-baryonic, *dark matter*, of completely unknown nature. Indeed, about 26% of the critical density is dark matter, which interacts with baryonic matter only by its gravity. Dark matter forms extended halos around all larger galaxies. This has been concluded from a study of the Doppler-velocities of stars in the outer parts of galaxies (s. fig. 12.10).

Either contributions, the dark energy and the dark matter, appear as parameter in the fit of CMB fluctuations. Regarding the high quality of CMB-data these parameter are very precise and reliable. Summing up all relevant contributions the normalized matter/energy-density for  $\kappa = 0$  becomes

$$\frac{\varepsilon}{\varepsilon_C} = \Omega = \Omega_\Lambda + \Omega_M = 1 \text{ with } \Omega_\Lambda = 0,70 \text{ and } \Omega_{M0} = 0,30 \quad (12.24)$$

The radiation density contributes not before very high z-values. How well is the condition  $\kappa = 0$  fulfilled? From the CMB-data one finds  $\Omega = 1,00 \pm 0,02$ .

## 12.5. Distances in cosmology.

We can now rewrite (12.5) to obtain a relation between the light travelling distance  $c(t - t_0)$  and redshift  $z$

$$\frac{\dot{a}}{a} = H(t) = H_0 \sqrt{\Omega_{0M} a^{-3} + \Omega_\Lambda} = H_0 E(t) \quad (12.25)$$

If there is a non vanishing curvature ( $-\kappa c^2/a^2$  has to be taken into account) a term  $\Omega_\kappa a^{-2}$  has to be added under the square root. Note that  $\epsilon_\Lambda$  and therefore  $\Omega_\Lambda$  does not depend on  $a(t)$ . The radiation density is small and its contribution at  $z < 100$  is negligible.

Instead of  $t$  which cannot directly be obtained we will use  $z$  the redshift factor

$$c \int_0^t dt = H_0^{-1} \int_0^a \frac{c da}{aE(z)} = t_H \int_z^\infty \frac{c dz}{(z+1)E(z)} \quad (12.26)$$

The resulting time  $t$  is the cosmic age corresponding to  $z$  and  $c(t_0 - t)$  is the photon time of flight required to travel from the far object to us. The evaluation of the integral yields

$$t/t_H = \frac{2 \sinh^{-1} \left( \sqrt{(z+1)^{-3} |\Omega_{0M} - 1| / \Omega_{0M}} \right)}{3 \sqrt{|\Omega_{0M} - 1|}} \quad (12.27)$$

where  $t$  is the time corresponding to  $z$  counted from the beginning of the universe. The look-back time is then  $t_0 - t = t_H - t$  (s. fig. 12.8) and the **light travelling distance** to an object with redshift  $z$  for  $\kappa = 0$  is  $D = c(t_0 - t)$ .

The **co-moving distance** between two objects which are only moved by the Hubble flow is

$$r = D_c = H_0^{-1} \int_{a1}^1 \frac{c da}{a^2 E(a)} = H_0^{-1} \int_0^z \frac{c dz}{E(z)} \quad (12.29)$$

The **angular diameter distance**  $D_A$  has to be applied when a source (or lens) covers a certain solid angle as in typical lensing observations

$$D_A = \frac{r}{z+1} = \frac{D_c}{z+1} \quad (12.30)$$

Think of a source at co-moving radial coordinate  $r$  that emits light at  $t$  and is observed at present to subtend a small angle  $\theta$ . It will extend over a proper distance  $s$  normal to the line of sight.

$$s = a(t)r\theta \quad (12.31)$$

The angular distance is now defined as

$$s = \theta D_A \quad (12.32)$$

Solving (12.29) and (12.30) for  $D_A$  and replacing  $a(t)$  by  $1/(z+1)$  we regain (12.28). Note that  $D_A(z)$  is a non-monotonous function and has for  $\Omega_M = 0,3$ ,  $\theta_\Lambda = 0,7$  and  $\Omega = 1$  a maximum at  $z = 2,4$ .

When the magnitude is used for a distance scale as in observations of SNe Ia (12.9) the relevant distance measure is the so called luminescence distance  $D_L$  which is related to  $D_A$  by

$$D_L = (z+1)^2 D_A \quad (12.33)$$

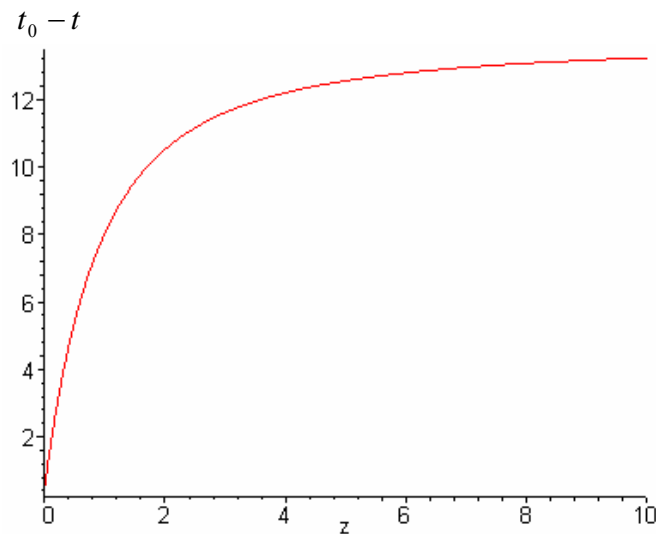
$z$	$D_C$ [Mpc]	$D_A$ [Mpc]	$D_L$ [Mpc]	$1/a$
0.5	1862.0	1241.3	2793.0	1.5
1.0	3257.3	1628.6	6514.6	2.0
1.5	4302.4	1720.9	10 755.9	2.5
2.0	5106.9	1702.3	15 320.7	3.0
2.5	5745.5	1641.5	20 109.1	3.5
3.0	6266.1	1566.5	25 064.7	4.0

**Tab.12.1.Redshift and scale parameter together with three distance measures.**

Now you may ask: “What is the correct distance?” The answer is: ”They are all correct. But the observation you perform defines the distance measure you have to use”.

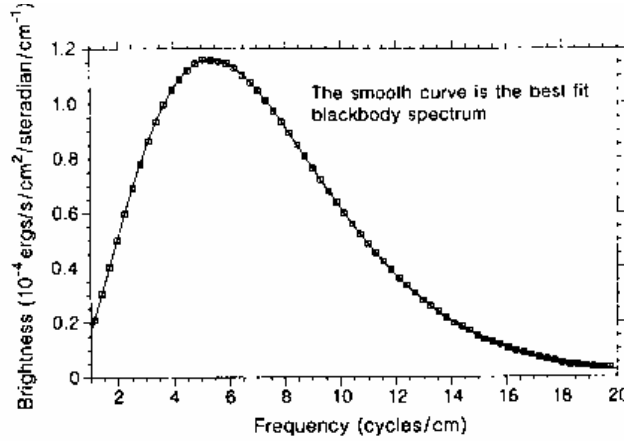
## 12.6. The cosmic microwave background (CMB).

In 1965 two physicists, Robert Wilson and Arno Penzias, worked at the Bell Telephone Laboratory at Holmdal (N.J.) on satellite communication. They were plagued by a background noise in their antenna which they were unable to eliminate.



**Fig. 12.11. A plot of (12.27, the look back time versus redshift**

Accidentally they heard of a group working under Robert Dicke at Princeton University on cosmic microwave radiation and asked for their advice. Few months later they published their results followed by a paper of Dicke's group with a description of the basic theoretical ideas. Wilson and Penzias received the Nobel price in 1978 for their accidental discovery of the CMB. It needed some time till NASA was ready to launch the COBE-satellite 1989 especially equipped to measure intensity and frequency dependence of the CMB. The results show, after subtraction of galactic radiation, a nearly homogeneous and isotropic radiation at least down to  $\Delta T/T \approx 10^{-4}$ . As a surprise the spectral distribution appeared as an ideal Planck curve of thermal radiation at  $T_0 = 2,726$  Kelvin.

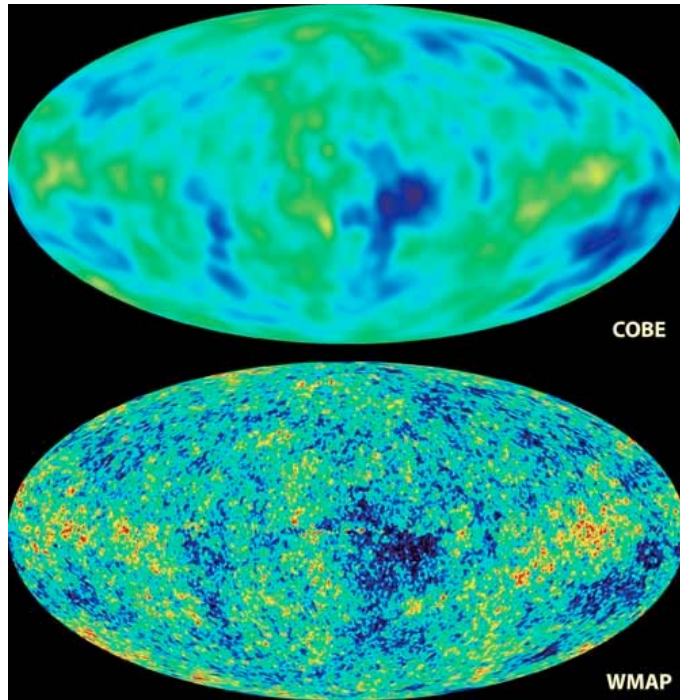


**Fig. 12.12. The black body spectrum of  $T = 2,726$  K of the cosmic microwave radiation.**

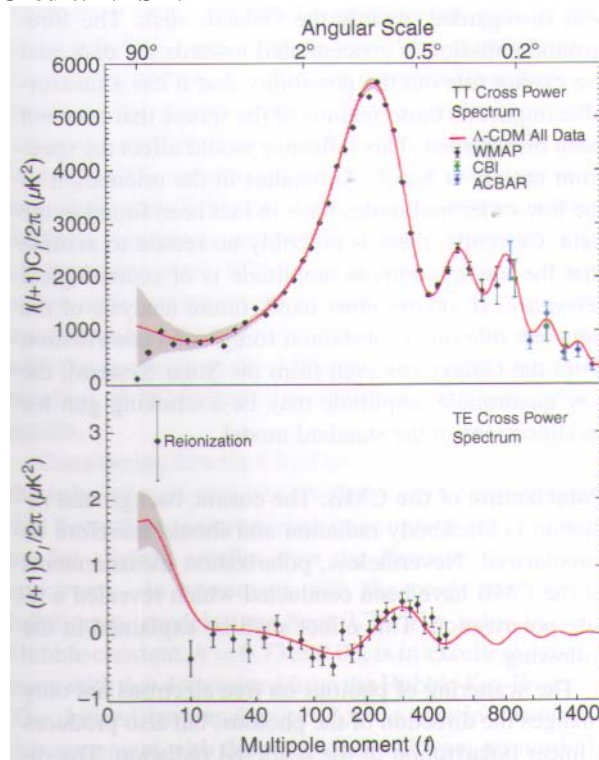
Deviations from homogeneity and isotropy appeared as angular fluctuations. They are of the order of some  $10^{-5}$ . CMB and represents the radiation cosmos released from matter (s. fig. 12.7). This happened when the temperature was low enough for photons to travel without scattering at electrons or absorption by hydrogen atoms. It follows from the famous Saha equation that hydrogen is sufficiently neutral at temperatures of  $T \leq 3000K$ . When the radiation cosmos has cooled down to 3000 K we find from (12.15a) a redshift of  $z \cong 1100$  K which corresponds to about 400 000 years after the big bang when radiation separates from matter.

The fluctuations were the next interesting aspect of CMB to consider. If the early universe would be completely homogeneous and isotropic there would no structure. Inhomogeneities of density or temperature form the seeds where structure evolving may start. In 2001 NASA launched a second satellite for the CMB investigation WMAP (Wilkinson Microwave Anisotropy Probe). Its instrumentation was designed to measure temperature correlations in 5 frequency channels over the full solid angle with an accuracy of  $\Delta T/T \approx 10^{-6}$ . In each measurement temperatures from two different positions  $\bar{n}_1 \equiv \mathcal{G}_1\varphi_1$  and  $\bar{n}_2 \equiv \mathcal{G}_2\varphi_2$  were correlated. The expansion in spherical harmonics represents the power spectrum plotted versus  $l$  the index of respective spherical harmonic

$$\frac{\langle (T(\bar{n}_1), T(\bar{n}_2))^2 \rangle}{\bar{T}^2} = \frac{l(l+1)}{2\pi} C_l \quad (12.34)$$



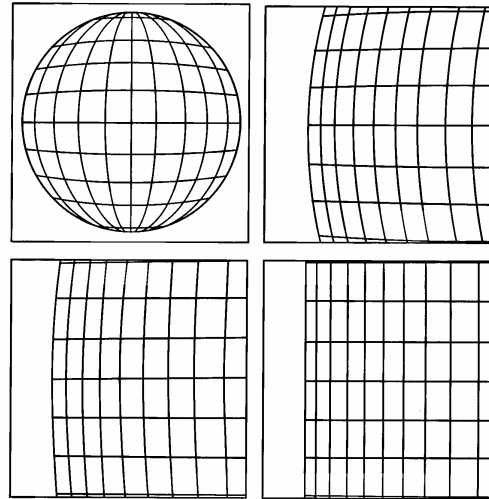
**Fig.12.13.** The fluctuations (highly intensified) as seen by COBE and WMAP shows the progress achieved in resolution Credit: NASA.



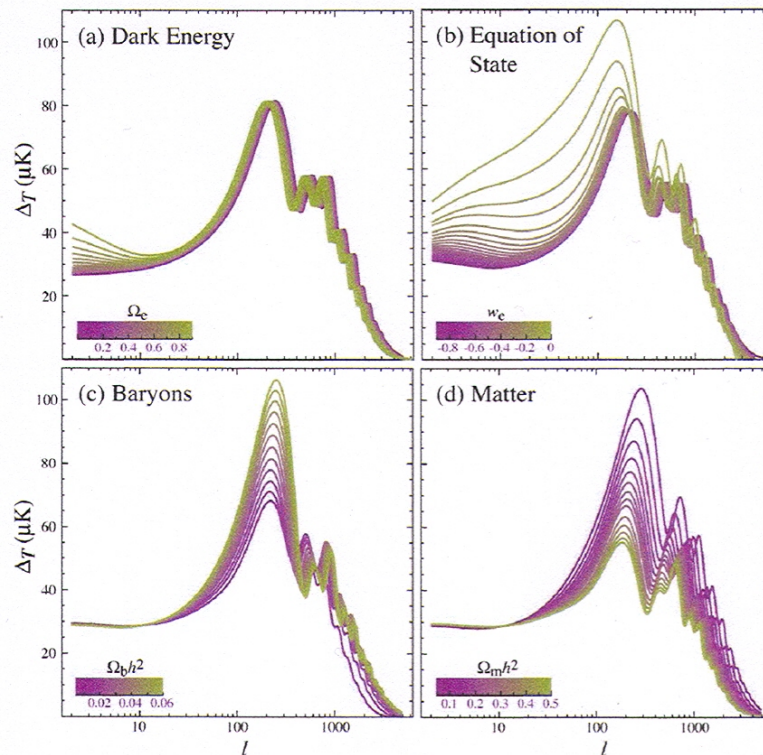
**Fig.12.14.** Power spectrum of fluctuations as measured by WMAP. The continuous black line is calculated from the concordance model  $\Omega = 1$ ,  $\Omega_\Lambda = 0,70$  and  $\Omega_{M_0} = 0,30$ . Credit: Bennett et al. 2003, APJS 148, 1.

Unfortunately we have not the time in this lecture to go in the details of this analysis. Although complicated the physical processes are well enough understood to put them in a program (CMBFAST) and investigate the influence of various cosmic parameters (s. fig. 12.15). Today one can say that CMB fluctuations have proved to be the richest source of

observational cosmology. The data have been collected in 3 series of measurement during 5 years of observation. For this purpose the satellite is kept at the L<sub>2</sub>-position 1,2 Million km off the earth. The accuracy of the measurements and the statistics obtained from the successive runs have improved our reliable knowledge so much that scientists nowadays say that an area of precision cosmology has been reached. Where do these fluctuations come from? They are much too large for mere statistical fluctuations known from statistical thermodynamics. They only possible alternative are quantum fluctuations in the ery early universe.



**Fig. 12.16.** An illustration how an inflationary expansion leads to an Euclidian space.



**12.15.** The influences of 4 cosmic parameters: a) the fraction of dark energy  $\Omega_\Lambda$ , the equation of state for dark energy or quintessence  $P_\Lambda = w_\Lambda \varepsilon_\Lambda$ , c) the fraction of baryons  $\Omega_{bm}$  and d) the fraction of all forms of matter  $\Omega_M$ . Credit: Hu and Dodelson2002, ARA & A 40, 171.

Think of these fluctuations as partial waves  $k_\mu$ . When in an early fast expansion of space the diameter of the horizon exceeds  $\pi/k_\mu$  then this partial wave is frozen and the respective density fluctuation becomes static. This happens at a very early time probably at  $t \approx 10^{-33}$  s during an area of exponential growth, called inflationary area. In the course of later expansion these fluctuations grow to macroscopic scale, still observable in the CMB. They may therefore be considered as a fingerprint of processes in the big bang which are otherwise completely unobservable. The assumption of an *inflation* is an important *hypothesis* which complements the Einstein-Friedmann standard cosmology. It answers open question, e.g. why do we live in a Euclidian universe ( $\Omega = 1$ )? This question is called the *flatness problem*. Fig. 12.16 illustrates how an initial curvature is flattened by an extreme inflationary expansion. The standard cosmology let space grow faster than the horizon leading to the *horizon problem*. A horizon involves all space-time points which are causally connected (or which are located within the light cone). In the very early area of standard cosmology there arise many such spots including causal connected events, but these spots remain unconnected among each other. If these very early events contributed to our known world we should see different laws of nature in different directions at the sky. That is not the case. The laws of nature are obviously the same in all observable parts of the universe. Now assume again that an inflationary expansion is so strong that it would blow up one of the spots with causal connection then the problem is solved since the blown up spot became our universe. The inflation hypothesis leaves details open. However it is the hope of future high resolution measurements (i.e. the European Planck satellite) to help to exclude at least some of the existing models of inflation.

## 12.7. Conclusion.

Time from Big Bang	Redshift $z$	Process
$10^{-33}$ s – $10^{-32}$ s		Inflation
200 s	$10^9$	Nuclei of light elements
400 000	1100	Decoupling of radiation
$50 \cdot 10^6$ yrs	20	First stars, first galaxies
$150 \cdot 10^6$ yrs	15	Reionisation of neutral H
$3 - 4 \cdot 10^9$ yrs	2 – 3	Star burst, QSO-activity
$9,2 \cdot 10^9$ yrs	1,3	Birth of solar system

**Table 12.1. A succession of important steps of cosmic evolution**

It is worthwhile to think critically about the subject of this last lecture 12. You may ask: “What of all this stuff is reliable knowledge, what is hypothesis and what mere speculation”? Well, the Friedmann equations are a consequence of Einstein’s GR and are certainly a reliable basis of modern cosmology. The value of the Hubble constant  $H_0$ , the critical nature of expansion  $\Omega = 1$ , i.e. the universe with Euclidean space are correct within the accuracy of measurements. The same is true for the fractions of mass  $\Omega_M = 0,3$  and energy  $\Omega_\Lambda = 0,7$ . All the interpretations of  $\Omega_M$  and  $\Omega_\Lambda$ , however, are highly speculative. The history of the universe from the first minutes to the separation of matter and radiation at 400 000 years later till the

formation of first stars and clusters in the first  $10^9$  years is roughly understood. The assumption of an inflationary era is at least plausible and has a strong explanatory power. The time of its beginning and its ending is completely speculative as well as all field theoretical models of inflation. They present possible scenarios which simply wait to be refuted by new CMB data taken with higher resolution.

### **References (further reading).**

Ned Wright's Cosmology Tutorial see

[http://www.astro.ucla.edu/~wright/Cosmo\\_01.html](http://www.astro.ucla.edu/~wright/Cosmo_01.html)

A. R. Liddle: An introduction to modern cosmology. 2<sup>nd</sup> edition. Wiley 2009

G. Börner: The early universe. Facts and fiction. 4<sup>th</sup> Edition. Springer 2003

Steven Weinberg: Cosmology. Oxford University Press 2008

V. Mukhanov: Physical foundations of Cosmology. Cambridge University Press 2005

Calculation of cosmic distances see <http://www.astro.ucla.edu/~wright/CosmoCalc.html>