Relativistic nature of the Lorentz force. (1)

S - frame

$F = \frac{1}{4\pi\varepsilon_0 c^2} q \frac{2I}{r} v$; Current $I = \int j\, ds = j \underset{\uparrow}{A} = \rho_- v A$

cross section

[S] $F = \frac{1}{4\pi\varepsilon_0 c^2} \frac{2qv}{r} \cdot \rho_- v \cdot A = \frac{1}{4\pi\varepsilon_0} \frac{v^2}{c^2} \frac{2q\rho_- A}{r}$

S' - frame

Charge density depends on the volume which changes due to the relativistic contraction, i.e.

$$\underbrace{\rho_0 L_0 A}_{\text{(at rest) } Q} = \underbrace{\rho L_0 \sqrt{1-\frac{v^2}{c^2}}\, A}_{= (v \neq 0)\ Q \text{ (since charge is invariant)}} \Rightarrow \rho = \frac{\rho_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

ρ_+ is at rest in the frame $S \equiv \rho_0$

(1) $\rho_+' = \frac{\rho_+}{\sqrt{1-\frac{v^2}{c^2}}}$; ρ_- - is at rest in frame S', i.e. $\rho_-' \equiv \rho_0$

(2) $\rho_-' = \rho_- \sqrt{1-\frac{v^2}{c^2}}$

$\rho_- = \rho_+$

$$\rho' = \rho_+' + \rho_-' = \rho_+ \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - \sqrt{1-\frac{v^2}{c^2}} \right) = \rho_+ \frac{\frac{v^2}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$E' = \frac{\rho' A}{2\pi\varepsilon_0 r} \quad \text{(see Blatt 11)}$$

②

$$= \frac{A\rho_+}{2\pi\varepsilon_0 r} \frac{\frac{v^2}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$[S'] \quad F' = qE' = \frac{q}{2\pi\varepsilon_0} \frac{\rho_+ A}{r} \frac{\frac{v^2}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$F' = F$ at least for small velocities

if velocities are high then:

$$\begin{cases} \Delta p' = F'\Delta t' \\ \Delta p = F\Delta t \end{cases} \quad \text{and } \Delta p' = \Delta p \text{ independent of the frame}$$

$$F'\Delta t' = F\Delta t$$

However due to the time contraction

$$\Delta t' = \Delta t \sqrt{1-\frac{v^2}{c^2}}$$

Thus $F'\Delta t' = F\Delta t$ as expected

Lorentz - Transformation des elektromagnetischen Felds (3)

$$\beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\begin{cases} ct' = \gamma ct - \gamma\beta x \\ x' = -\gamma\beta ct + \gamma x \\ y' = y \\ z' = z \end{cases}$$

a) $\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} ? \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$

$$a^{\mu'}_{\ \mu} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} L & 0 \\ 0 & E \end{pmatrix}$$

b) $$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix} = \begin{pmatrix} A & B \\ -B^T & C \end{pmatrix}$$

$$F^{\mu'\nu'} = a^{\mu'}_{\ \mu} F^{\mu\nu} (a^T)_{\nu}^{\ \nu'} = \begin{pmatrix} L & 0 \\ 0 & E \end{pmatrix}\begin{pmatrix} A & B \\ -B^T & C \end{pmatrix}\begin{pmatrix} L^T & 0 \\ 0 & E \end{pmatrix} = \begin{pmatrix} L & 0 \\ 0 & E \end{pmatrix}\begin{pmatrix} AL^T & BE \\ -B^T L^T & CE \end{pmatrix} =$$

$$= \begin{pmatrix} LAL^T & LBE \\ -EB^TL^T & ECE \end{pmatrix}$$

$$LAL^T = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix}\begin{pmatrix} 0 & -E_x \\ E_x & 0 \end{pmatrix}\begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix}\begin{pmatrix} E_x\gamma\beta & -E_x\gamma \\ E_x\gamma & -\gamma\beta E_x \end{pmatrix} =$$

$$= \begin{pmatrix} E_x\gamma^2\beta - \gamma^2\beta E_x & -E_x\gamma^2 + \gamma^2\beta^2 E_x \\ -E_x\gamma^2\beta^2 + E_x\gamma^2 & E_x\gamma^2\beta - \gamma^2\beta E_x \end{pmatrix} = E_x\gamma^2\begin{pmatrix} 0 & \beta^2 - 1 \\ -\beta^2+1 & 0 \end{pmatrix} = E_x\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$LB = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} -E_y & -E_z \\ -B_z & B_y \end{pmatrix} = \gamma \begin{pmatrix} -E_y + \beta B_z & -E_z - \beta B_y \\ \beta E_y - B_z & \beta E_z + B_y \end{pmatrix}$$

$$F^{\mu'\nu'} = \begin{pmatrix} 0 & -E_x & -\gamma E_y + \gamma\beta B_z & -\gamma E_z - \gamma\beta B_y \\ E_x & 0 & \gamma\beta E_y - \gamma B_z & \gamma\beta E_z + \gamma B_y \\ \gamma E_y - \gamma\beta B_z & -\gamma\beta E_y + \gamma B_z & 0 & -B_x \\ \gamma E_z + \gamma\beta B_y & -\gamma\beta E_z - \gamma B_y & B_x & 0 \end{pmatrix}$$

Thus,

$E_x' = E_x$

$E_y' = \gamma E_y - \gamma\beta B_z$

$E_z' = \gamma E_z + \gamma\beta B_y$

$B_x' = B_x$

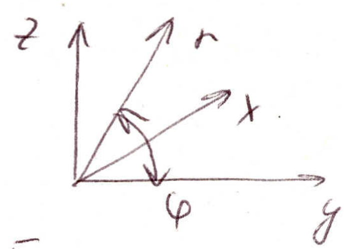
$B_y' = \gamma\beta E_z + \gamma B_y$

$B_z' = \gamma B_z - \gamma\beta E_y$

All the calculations are in CGS!

B Draht mit Ladungsdichte λ:

a) $E_r = \frac{2\lambda}{r}$; $E_r = \sqrt{E_y^2 + E_z^2}$



in the rest frame

$E_y = \frac{2\lambda}{r}\cos\varphi$, $B_{x,y,z} = 0$, $E_x = 0$

$E_z = \frac{2\lambda}{r}\sin\varphi$

Let the frame move along x. Then:

$E_x' = E_x$, $E_y' = \gamma E_y = \gamma \frac{2\lambda}{r}\cos\varphi$, $E_z' = \gamma E_z = \gamma \frac{2\lambda}{r}\sin\varphi$

$B_x' = 0$; $B_y' = \gamma\beta E_z = \gamma\beta \frac{2\lambda}{r}\sin\varphi$, $B_z' = -\gamma\beta E_y = -\gamma\beta \frac{2\lambda}{r}\cos\varphi$

Thus:

$\vec{E}' = \gamma \frac{2\lambda}{r}(\vec{e}_y\cos\varphi + \vec{e}_z\sin\varphi) = \gamma\frac{2\lambda}{r}\cdot\vec{e}_r = \gamma\vec{E}$

$\vec{B}' = \gamma\beta\frac{2\lambda}{r}(\vec{e}_y\sin\varphi - \vec{e}_z\cos\varphi) = -\gamma\beta\frac{2\lambda}{r}\vec{e}_\varphi$

i.e. $|\vec{B}'| = B_\varphi = -\gamma\beta\frac{2\lambda}{r}$

tangential

b) $v \ll c$

$\vec{E}' = \vec{E}$

$B_\varphi = -\frac{v}{c}\frac{2\lambda}{r} = -\frac{1}{c}\frac{2j}{r}$