Institut für Theoretische Physik
Prof. Dr. M. Grifoni WS06/07

Übungen zu Theoretische Physik III
Blatt 2

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Abzugeben am
Mo. 30.10.2006

Aufgabe 1: δ-function
Prove the following properties of the Dirac δ-function by using a test function $f(x)$:

a. $\delta(\alpha x) = \dfrac{\delta(x)}{|\alpha|}$ where α is a real number. (*2* points)

b. $\delta(g(x)) = \sum_i \dfrac{\delta(x - x_i)}{|g'(x_i)|}$, where x_i are the roots of $g(x)$ and $g'(x)$ is the derivative of g with respect to x. (*2* points)

c. $\Theta'(x) = \delta(x)$ where $\Theta(x)$ is the unit step function (*2* points)

$$\Theta(x) = \begin{cases} 1 & x \geq 0, \\ 0 & x < 0. \end{cases}$$

Aufgabe 2: Fourier transformation
The Fourier transformation of a function $f(x)$ is defined as

$$\hat{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) e^{-ikx} dx.$$

a. Calculate the Fourier transformation of the Dirac δ-function, $\delta(x)$. (*2* points)

b. Show that (*2* points)

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \hat{f}(k) e^{ikx} dk.$$

c. Calculate the Fourier transformation of the derivative $f'(x)$ in terms of $\hat{f}(x)$. (*2* points)

d. Calculate the Fourier transformation $h(x) = f(x)g(x)$ in terms of the Fourier transformations of $f(x)$ and $g(x)$. (*2* points)

Aufgabe 3: Nobel prize in physics 2006 (*6* points)
Read some materials about the Nobel prize in physics 2006 (Laureates: John C. Mather and George F. Smoot). Some materials may be found in the Nobel foundation's web site (`http://nobelprize.org/nobel_prizes/physics/laureates/2006/`). Give a brief description of the awarded work of the two laureates.

Aufgabe 4: Born-Sommerfeld quantization rule (☞ *to be discussed in class*)

a. Write down the Born-Sommerfeld quantization rules for momentum and angular momentum.

b. Use the quantization rule to calculate the energy of a free particle which moves in a cube with length L.

c. Use the quantization rule to calculate the energy of a one-dimensional harmonic oscillator with the potential $V(x) = \frac{1}{2}m\omega^2 x^2$.

d. Use the quantization rule to calculate the ground state energy (lowest energy) of a particle moving in a spherically symmetric potential $V(r) = kr$ with $r = |\boldsymbol{r}|$.