Institut für Theoretische Physik
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WS06/07

Übungen zu Theoretische Physik III
Blatt 10

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Aufgabe 1: Orthogonality of bound state solutions
Two states $\Psi_1(x)$ and $\Psi_2(x)$ are bound state solutions (real functions) of the Schrödinger equation with energy eigenvalues E_1 and $E_2 \neq E_1$, respectively. Show that $\Psi_1(x)$ and $\Psi_2(x)$ are orthogonal. (*4* points)

Aufgabe 2: Particle in expanding box
A particle of mass m is contained in a one-dimensional impenetrable box extending from $x = -L/2$ to $x = L/2$. The particle is in its ground state.

a. Find the eigenfunctions of the ground state and the first excited state. (*3* points)

b. The walls of the box are moved outward instantaneously to form a box extending from $-L \leq x \leq L$. Calculate the probability that the particle will stay in the ground state during this sudden expansion. (*3* points)

c. Calculate the probability that the particle jumps from the initial ground state to the first excited final state. (*3* points)

Aufgabe 3: Expectation values and time development of the harmonic oscillator
Calculate the expectation values of $\hat{x}$, $\hat{x}^2$, $\hat{p}$, $\hat{p}^2$, and $\langle(\Delta x)^2\rangle\langle(\Delta p)^2\rangle$ in a harmonic oscillator system. Choose the most convenient basis to perform these calculations. (*6* points)

Aufgabe 4: Particle in a central potential
Consider a particle of mass m in a central potential of the form

$$V(r) = \frac{c}{r^2} + \frac{1}{2}m\omega^2 r^2, \qquad c > 0.$$

a. Derive the asymptotic radial part of the stationary Schrödinger equation for $r \to 0$ and $r \to \infty$ and show that

$$u(r) = rR(r) = r^k e^{-\gamma r^2} g(r)$$

is a suitable ansatz for the radial solution. (*4* points)

b. Derive the differential equation for $g(r)$ and use the ansatz

$$g(r) = \sum_l a_l r^l$$

to solve it. (*4* points)

c. Why does the series in part (b) terminate at some finite $l = n$? Determine the allowed energy eigenvalues from this condition. (*2* points)

Aufgabe 5: Parity operator (☞ *to be discussed in class*)
Show that the operator

$$\Pi = \exp\left(i\pi\left(\frac{\hat{p}^2}{2\alpha} + \frac{\alpha}{2\hbar^2}\hat{x}^2 - \frac{1}{2}\right)\right)$$

acts like the parity operator, i.e., $\Pi\psi(x) = \psi(-x)$ for any wave function $\psi(x)$. Here α is a positive and real constant. (*6* points)
Hint: The operator $\frac{1}{2\alpha}\hat{p}^2 + \frac{\alpha}{2\hbar^2}\hat{x}^2$ is the Hamiltonian of a one dimensional harmonic oscillator. Any wave function can be expressed by the combination of the eigenstates of a one dimensional harmonic oscillator.

Aufgabe 6: Scattering off a combined δ/rigid potential (☞ *to be discussed in class*)
Consider an incoming wave $\phi_{\text{in}} = e^{ik_0x}$ with $k_0 = 2mE/\hbar^2$ from $-\infty$ scattering off a potential of the form

$$V(x) = \begin{cases} \frac{\hbar^2 V_0}{2m}\,\delta(x + x_0), & x \leq 0 \text{ and } x_0 > 0 \\ +\infty, & x > 0. \end{cases}$$

Calculate the amplitude of the reflected wave as a function of the energy E. What happens for $V_0 = 0$ and $V_0 = \infty$? (*6* points)

Aufgabe 7: Shallow square well (☞ *to be discussed in class*)
A particle of mass m is confined to move in one dimension by a potential $V(x)$,

$$V(x) = \begin{cases} \infty, & x < 0, \\ -V_0, & 0 < x < a, \\ 0, & a < x. \end{cases}$$

a. For the bound state derive the equation for determining the energy or corresponding wave vector. (*4* points)

b. Give the expression for the eigenfunction of a state with positive energy $E > 0$. (*4* points)

c. Show that the result of (c) define a phase shift for the potential, and derive an expression for the phase shift. (*2* points)

Frohe Weihnachten und ein gutes neues Jahr!