Deeply Virtual Compton Scattering to the twist-four accuracy:
Impact of finite-$t$ and target mass corrections

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based on


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Hard exclusive processes involve off-forward matrix elements

**DVCS:** \( \gamma^* P \rightarrow \gamma P' \)

Form factors: \( \gamma^* \pi \rightarrow \gamma, B \rightarrow \rho \ell \bar{\nu}_\ell, \ldots \)

**Operator Product Expansion**

\[
J(x) J(0) \sim \sum_N C_N(x^2, \mu^2) O_N(\mu^2)
\]

involves

\[
\langle P' | O_N(\mu^2) | P \rangle \quad \langle \rho(p) | O_N(\mu^2) | 0 \rangle
\]

Kinematic variables: hadron mass \( m^2 \) momentum transfer \( t = (P - P')^2 \)

**How to calculate effects** \( \sim m^2/Q^2 \) and \( t/Q^2 \)?
Nucleon Tomography?

access to three-dimensional picture of the nucleon (M. Burkardt)

→ first two moments of transverse spin parton density


• paradigm shift: finite $t$ a “nuisance” → important tool
How to calculate effects $\sim m^2/Q^2$ and $t/Q^2$ in DVCS?

Early work:

- **DVCS:**
  - Extension of Nachtmann's approach to target mass corrections in DIS
  - Spin-rotation (Wandzura-Wilczek)
    - Belitsky, Müller: NPB589 (2000) 611
    - ...  
  - Results not gauge invariant
  - Results not translation invariant

- **B-decays:**
  - Ball, Braun: NPB543 (1999) 201
  - Problem localized but not solved
Contributions of different twist are intertwined by symmetries:

- Conservation of the electromagnetic current and translation invariance
  \[ \partial^\mu T\{j^{em}_\mu (x)j^{em}_\nu (0)\} = 0 \]
  \[ T\{j^{em}_\mu (2x)j^{em}_\nu (0)\} = e^{-iP \cdot x} T\{j^{em}_\mu (x)j^{em}_\nu (-x)\} e^{iP \cdot x} \]
  are valid in the sum of all twists but not for each twist separately

- Higher-twist contributions that restore gauge/translation invariance are due to descendants of leading-twist operators obtained by adding total derivatives
  \[ T\{j^{em}_\mu (x)j^{em}_\nu (0)\} = \sum_N a_N \mathcal{O}_N + \sum_N \left( b_N \partial^2 \mathcal{O}_N + c_N (\partial \mathcal{O})_N \right) + \text{other operators} \]
  \( \underbrace{\text{leading-twist}} \)

- “Kinematic” and “Dynamic” contributions must have autonomous scale-dependence

- Explicit diagonalization of the mixing matrix for twist-4 operators not feasible, but, conformal symmetry implies that this matrix is hermitian w.r.t. to a certain scalar product.

longitudinal plane \((q, q')\)

\[ n = q', \quad \tilde{n} = -q + \frac{Q^2}{Q^2 + t} q' \]

with this choice \(\Delta = q - q'\) is longitudinal and

\[ |P_\perp|^2 = -m^2 - \frac{t}{4} \frac{1 - \xi^2}{\xi^2} \sim t_{\min} - t \]

where

\[ P = \frac{1}{2} (p + p'), \quad \xi_{\text{BMP}} = \frac{(\Delta \cdot q')}{2(P \cdot q')} = \frac{x_B(1 + t/Q^2)}{2 - x_B(1 - t/Q^2)} \]

photon polarization vectors

\[ \varepsilon_{\mu}^0 = - \left( q_\mu - q'_\mu q^2/(qq') \right) / \sqrt{-q^2}, \]

\[ \varepsilon_{\mu}^\pm = (P_\perp^\perp \pm i \tilde{P}_\perp^\perp) / (\sqrt{2} |P_\perp|), \quad \tilde{P}_\perp^\perp = \varepsilon_{\mu \nu}^\perp P^\nu \]
BMP helicity amplitudes

\[
\mathcal{A}_{\mu\nu}(q, q', p) = i \int d^4x \ e^{-i(z_1 q - z_2 q')x} \langle p', s' \mid T\{J_\mu(z_1 x)J_\nu(z_2 x)\}\mid p, s \rangle
\]

\[
= \varepsilon_\mu^+ \varepsilon_\nu^- \mathcal{A}^{++} + \varepsilon_\mu^- \varepsilon_\nu^+ \mathcal{A}^{--} + \varepsilon_\mu^0 \varepsilon_\nu^- \mathcal{A}^{0+} + \varepsilon_\mu^0 \varepsilon_\nu^+ \mathcal{A}^{0-} + \varepsilon_\mu^- \varepsilon_\nu^- \mathcal{A}^{--} + \mathcal{A}_\mu^{(3)}
\]

for the calculation to the twist-4 accuracy one needs

- \(\mathcal{A}^{++}, \mathcal{A}^{--}\): \(1 + \frac{1}{Q^2}\)
- \(\mathcal{A}^{0+}, \mathcal{A}^{0-}\): \(\frac{1}{Q}\) ← agree with existing results
- \(\mathcal{A}^{--}, \mathcal{A}^{+-}\): \(\frac{1}{Q^2}\) ← straightforward
BMP Compton form factors (CFFs)

- Photon helicity amplitudes can be expanded in a given set of spinor bilinears

\[ A_{q}^{a \pm} = \tilde{H}_{q}^{a \pm} h + \tilde{E}_{q}^{a \pm} e \mp \tilde{H}_{q}^{a \mp} \tilde{h} \mp \tilde{E}_{q}^{a \mp} \tilde{e} \]

with, e.g., Belitsky, Müller, Ji: NPB 878 (2014) 214

\[ h = \frac{\bar{u}(p') (\gamma + \gamma') u(p)}{P \cdot (\gamma + \gamma')} \]

- The results read

Braun, Manashov, Pirnay: PRL109 (2012) 242001

\[
\begin{align*}
\mathbb{H}_{++} &= T_0 \otimes H + \frac{t}{Q^2} \left[ -\frac{1}{2} T_0 + T_1 + 2\xi D \xi T_2 \right] \otimes H + \frac{2t}{Q^2} \xi^2 \partial \xi \xi T_2 \otimes (H+E) \\
\mathbb{H}_{0+} &= -\frac{4|\xi P_{\perp}|}{\sqrt{2} Q} \left[ \xi \partial \xi T_1 \otimes H + \frac{t}{Q^2} \partial \xi \xi T_1 \otimes (H+E) \right] - \frac{t}{\sqrt{2} Q |\xi P_{\perp}|} \xi T_1 \otimes \left[ \xi (H+E) - \tilde{H} \right] \\
\mathbb{H}_{-+} &= \frac{4|\xi P_{\perp}|^2}{Q^2} \left[ \xi \partial^2 \xi \xi T_{1(+) \otimes H} + \frac{t}{Q^2} \partial^2 \xi \xi T_{1(+) \otimes (H+E)} \right] \\
&\quad + \frac{2t}{Q^2} \xi \left[ \xi \partial \xi \xi T_{1(+) \otimes (H+E)} + \partial \xi \xi T_1 \otimes \tilde{H} \right]
\end{align*}
\]
BMP Compton form factors (CFFs)

- Photon helicity amplitudes can be expanded in a given set of spinor bilinears

\[ A^a_{q \pm} = \mathbb{H}^q_{a \pm} h + \mathbb{E}^q_{a \pm} e \mp \mathbb{H}^q_{a \pm} \tilde{h} \mp \mathbb{E}^q_{a \pm} \tilde{e} \]

with, e.g.

\[ h = \frac{\bar{u}(p') (\bar{q} + q') u(p)}{P \cdot (\bar{q} + q')} \]

- The results read

Braun, Manashov, Pirnay: PRL109 (2012) 242001

\[
\begin{align*}
\mathbb{E}_{++} &= T_0 \otimes E + \frac{t}{Q^2} \left[ -\frac{1}{2} T_0 + T_1 + 2\xi D_\xi T_2 \right] \otimes E - \frac{8m^2}{Q^2} \xi^2 \partial_\xi \xi T_2 \otimes (H + E) \\
\mathbb{E}_{0+} &= -\frac{4|\xi P_\perp|}{\sqrt{2}Q} \left[ \xi \partial_\xi T_1 \otimes E \right] + \frac{4m^2}{\sqrt{2}Q|\xi P_\perp|} \xi T_1 \otimes \left[ \xi (H + E) - \tilde{H} \right] \\
\mathbb{E}_{--} &= \frac{4|\xi P_\perp|^2}{Q^2} \left[ \xi \partial_\xi^2 \xi T_1^{(+)} \otimes E \right] - \frac{8m^2}{Q^2} \xi \left[ \xi \partial_\xi \xi T_1^{(+)} \otimes (H + E) + \partial_\xi \xi T_1 \otimes \tilde{H} \right]
\end{align*}
\]

etc.
where $F = H, E, \tilde{H}, \tilde{E}$ are $C$-even GPDs

\[
T \otimes F = \sum_q e_q^2 \int_{-1}^{1} \frac{dx}{2 \xi} T \left( \frac{\xi + x - i\epsilon}{2(\xi - i\epsilon)} \right) F(x, \xi, t)
\]

the coefficient functions $T^\pm_k$ are given by the following expressions:

\[
T_0(u) = \frac{1}{1 - u}
\]
\[
T_1(u) \equiv T_1^{(-)}(u) = -\frac{\ln(1 - u)}{u}
\]
\[
T_1^{(+)}(u) = \frac{(1 - 2u) \ln(1 - u)}{u}
\]
\[
T_2(u) = \frac{\text{Li}_2(1) - \text{Li}_2(u)}{1 - u} + \frac{\ln(1 - u)}{2u}
\]

and

\[
D_\xi = \partial_\xi + 2 \frac{\xi P \perp |P|}{t} \partial^2_\xi = \partial_\xi - \frac{t - t_{\text{min}}}{2t} (1 - \xi^2) \partial^2_\xi
\]
Main features:

- Two expansion parameters

\[
\frac{t}{Q^2}, \quad \frac{t - t_{\text{min}}}{Q^2} \sim \frac{|\xi P_\perp|^2}{Q^2}
\]

- All mass corrections for scalar targets absorbed in \( t_{\text{min}} = -4m^2\xi^2/(1 - \xi^2) \); always overcompensated by finite-\( t \) corrections in the physical region

- Some extra \( m^2/Q^2 \) corrections for nucleon due to spinor algebra; disappear in certain CFF combinations

- Factorization checked to \( 1/Q^2 \) accuracy
- Gauge and translation invariance checked to \( 1/Q^2 \) accuracy
- Correct threshold behavior \( t \to t_{\text{min}}, \xi \to 1 \)
From CFFs to DVCS observables

- The only existing calculation to the required accuracy: BMJ

Belitsky, Müller, Ji: NPB 878 (2014) 214

!!! Subtlety: BMJ use a different reference frame to define photon helicity amplitudes; hence a different set of CFFs (calligraphic) related to BMP CFFs (blackboard bold) by a kinematic trafo

\[
F_{\pm +} = F_{\pm +} + \frac{\kappa}{2} \left[ F_{++} + F_{-+} \right] - \kappa_0 F_{0+},
\]

\[
F_{0+} = - (1 + \kappa) F_{0+} + \kappa_0 \left[ F_{++} + F_{-+} \right]
\]

where

\[
\kappa_0 \sim \sqrt{\left( t_{\text{min}} - t \right)/Q^2}, \quad \kappa \sim \left( t_{\text{min}} - t \right)/Q^2
\]

Adopted strategy is, thus,

\[
\text{BMP CFFs} \xrightarrow{\text{exact}} \text{BMJ CFFs} \xrightarrow{\text{exact}} \text{observables}
\]

\[
\mathcal{O}(1/Q^2)
\]
### Defining the Leading Twist approximation

<table>
<thead>
<tr>
<th>Kumerički-Müller convention (KM)</th>
<th>Braun-Manashov-Pirnay convention (BMP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{L}<em>\text{KM}$: $\begin{cases} \mathcal{F}</em>{++} = T_0 \otimes F, &amp; \mathcal{F}<em>{0+} = 0, \ \mathcal{F}</em>{--} = 0, &amp; \xi = \xi_{\text{KM}} \end{cases}$</td>
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<td></td>
</tr>
</tbody>
</table>

### Changing frame of reference results in

- **Different skewedness parameter**
  
  $\xi_{\text{KM}} = \frac{x_B}{2 - x_B}$ vs. $\xi_{\text{BMP}} = \frac{x_B(1 + t/Q^2)}{2 - x_B(1 - t/Q^2)}$

- **Numerically significant excitation of helicity-flip CFFs** $\mathcal{F}_{0+}, \mathcal{F}_{--}$
Unpolarized target

GPD model: GK12

Figure: Unpolarized cross section [upper panels] and electron helicity dependent cross section difference [lower panels] from HALL A
(new) Transversely polarized target


Figure: Transverse target spin asymmetries by HERMES collaboration

Summary and conclusions

- Target mass and finite-$t$ corrections to DVCS are known to twist-4 accuracy. They are relatively simple and can be implemented with moderate effort.

- **Premium:**
  
  Gauge and translation invariance of the Compton tensor is restored to $1/Q^2$ accuracy. Convention-dependence of the common leading-twist calculations is removed. Theoretically motivated limits $-t/Q^2 \lesssim 1/4$.

- For several key observables, the lion share of the twist-4 effects is captured by going over to the BMP frame.

- **Standardization badly needed for all steps, starting from the Compton tensor**

  \[
  A_{\mu \nu}(q, q', p) = \epsilon_\mu^+ \epsilon_\nu^- A^{++} + \epsilon_\mu^- \epsilon_\nu^+ A^{--} + \epsilon_\mu^0 \epsilon_\nu^- A^{0+} \\
  + \epsilon_\mu^0 \epsilon_\nu^+ A^{0-} + \epsilon_\mu^+ \epsilon_\nu^+ A^{+-} + \epsilon_\mu^- \epsilon_\nu^- A^{--} + q'_\nu B_\mu
  \]
Backup slides
Unpolarized target (2)

Figure: Single electron beam spin asymmetry by CLAS collaboration

Unpolarized target (3)

Figure: The single electron beam spin asymmetry [left panel] in the charge-odd sector and the unpolarized beam charge asymmetry [right panel] measured by the HERMES collaboration.

Longitudinally polarized targets

Figure: Longitudinal proton spin asymmetry from CLAS [left panel], measured with an electron beam, and HERMES [right panel], measured with a positron beam

Collider kinematics

Braun, Manashov, Müller, Pirnay: arXiv:1401.7621

Figure: The DVCS cross section from H1 (squares, diamonds, triangles) and ZEUS (circles)