Hard Exclusive Dijet Production on Hadrons and Nuclei

or

Color Explosion Imaging of Hadron Distribution Amplitudes

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Coulomb Explosion Imaging of Small Molecules

Color Explosion Imaging of Hadron DA

- In the diffraction dissociation of the $|q\bar{q}\rangle$ into dijets, $x$ can be measured by the momentum ratio of the jets:

$$x_{\text{measured}} \sim \frac{p_{\text{jet1}}}{p_{\text{jet1}} + p_{\text{jet2}}}$$
suggests

\[ M_{\pi \to 2 \text{ jets}} = \int_{0}^{1} dz' \int_{0}^{1} dx_1 \phi_\pi(z', \mu_F^2) T_H(z', x_1, \mu_F^2) F_\zeta^g(x_1, \mu_F^2) \]

where \( F_\zeta^g(x_1, \mu_F^2) \) is the skewed gluon distribution
\( \phi_\pi(z, \mu_F^2) \) only includes low transverse momenta \( k_\perp^2 < \mu_F^2 \)
\( T_H(z', x_1, \mu_F^2) \) is calculated with all partons on-shell

Kinematical constraint:

\[ \zeta = x_1 - x_2 = M^2/s = q_\perp^2/(z\bar{z}s) \]

Braun et al. PLB 509 (2001) 43; Chernyak, PLB 516 (2001) 116

calculated a hard gluon exchange contribution to \( T_H(z', x_1, \mu_F^2) \)
More precisely:

amplitude for pion dissociation into a quark-antiquark pair

\[
\langle 0 | \bar{d}(0) \gamma_\mu \gamma_5 u(x) | \pi^+(p) \rangle = i f_{\pi} p_\mu \int_0^1 du \ e^{-iupx} \ \Phi_\pi(u, x^2) + \ldots
\]

can be factorized into a product (convolution) of the coefficient function and the distribution amplitude (DA)

\[
\Phi_\pi(u, x^2) = c(u, x^2, \mu^2) \otimes \phi_\pi(u, \mu^2)
\]

with

\[
\langle 0 | \bar{d}(0) \gamma_\mu \gamma_5 u(z) | \pi^+(p) \rangle = i f_{\pi} p_\mu \int_0^1 du \ e^{-iupz} \ \phi_\pi(u, \mu)
\]

where in difference to the above \( z^2 = 0 \)

- Behavior of wave functions at small transverse separations \( x^2 \)
  is traded for the scale dependence of the DAs

\[
\phi_\pi(u, \mu) = 6u(1 - u) \left[ 1 + \phi_2(\mu) 6[1 - 5u(1 - u)] + \ldots \right]
\]

The scale dependence of \( \phi_n(\mu) \) is such that at sufficiently high scales only a few first terms are important

- Chernyak & Zhitnitsky '82 \( \phi_2(0.5 \text{ GeV}) \simeq 0.67 \pm \ldots \)
Photon Distribution Amplitudes

Quark and antiquark in the real photon can have either the same or the opposite helicities ⇒

**Chirality conserving:**

\[
\langle 0 | \bar{q}(0) \gamma^+ \frac{1 \pm \gamma_5}{2} q(x^-, r) | \gamma^{(\lambda)} (q) \rangle = \\
= \frac{i N_c e_q}{4 \pi^2 r^2} q_+ \int_0^1 du \ e^{-iuq_+x} - [ (e^{(\lambda)} \cdot r) (2u - 1) \pm i \epsilon_{ik} r_i e^{(\lambda)}_k ]
\]

Balitsky, Braun, Kolesnichenko ’89

**Chirality breaking:**

\[
\langle 0 | \bar{q}(0) \sigma_{\alpha\beta} q(x) | \gamma^{(\lambda)} (q) \rangle = \\
= i e_q \chi \langle \bar{q} q \rangle \left( e^{(\lambda)}_{\alpha} q_\beta - e^{(\lambda)}_{\beta} q_\alpha \right) \int_0^1 du \ e^{-iu(qx)} \phi_{\gamma}(u, \mu).
\]
Magnetic Susceptibility of the quark condensate
\[ \langle 0| \bar{q}\sigma_{\alpha\beta} q|0 \rangle_F = e_q \chi \langle \bar{q}q \rangle F_{\alpha\beta} \]

in hard processes, typically

<table>
<thead>
<tr>
<th>chirality breaking</th>
<th>( \sim )</th>
<th>chirality conserving</th>
<th>( \sim )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{4\pi^2 \chi \langle \bar{q}q \rangle}{N_c Q} )</td>
<td>( \frac{650\text{MeV}}{Q} )</td>
<td></td>
</tr>
</tbody>
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— large mass scale

asympotic (Balitsky et al)

instanton model

Petrov et al

instanton model

Praszalowicz et al

at 1 GeV

important for understanding of baryon magnetic moments etc.
chirality conserving (CC):
—large momentum \( \sim q_\perp \) can flow through the photon vertex

chirality violating (CV):
—large momentum \( \sim q_\perp \) cannot flow through the photon vertex

\[
2\pi \frac{d\sigma_{\gamma \rightarrow 2 \text{jets}}}{d\phi dq_\perp dt dz} \bigg|_{t=0} = \sum_q e_q^2 \frac{\alpha EM \alpha_s^2 \pi^2}{2 N_c q_\perp^6} \\
\times \left[ (1 - 4z\bar{z}\cos^2\phi) |J_{CC}|^2 + \frac{\pi^2 \alpha_s^2 \chi^2 \langle \bar{q}q \rangle^2}{N_c^2 q_\perp^2} |J_{CV}|^2 \right],
\]

- CC and CV contributions do not interfere
- CC contribution is of order \( 1/q_\perp^6 \), CV is \( 1/q_\perp^8 \)
- CC contribution is \( \sim 1 - \cos^2\phi \) where \( (e^{(\lambda)} \cdot q_\perp) \sim \cos\phi \)
Chirality Conserving Amplitude

**Leading Order**

\[ \mathcal{J}_{CC}^{LO} = -\frac{1}{\pi} \int_{-1}^{1} dy \, \mathcal{H}_g(y, \xi) \frac{\xi}{(y - \xi + i\epsilon)^2}, \]

\[ \mathcal{H}_g(y, \xi) = \mathcal{H}_g(-y, \xi) \text{ is the generalized gluon distribution} \]

- Notice an unusual double-pole

**Next-to-Leading Order**

\[ \text{Im} \mathcal{J}_{CC} \simeq \xi \mathcal{H}'_g(\xi, \xi) + \frac{\alpha_s N_c}{\pi} \int_{\xi}^{1} dy \frac{\mathcal{H}_g(y, \xi)}{y + \xi}, \]

- Both terms are of the same order in \( k_t \)-factorization
- Relative sign is negative
- First term dominates at large \( q_t \) in the Bjorken limit
- Second term dominates at large \( \log 1/x_B \) in the Regge limit

Chirality Violating Amplitude

**Calculation similar to \( \pi \rightarrow dijets \) (later)**
\[ d\sigma_{\gamma+N\rightarrow 2\text{jets}+N}/dq_{\perp}^2 \]

- Notice change of slope

**Dependence on longitudinal momentum fraction and azimuthal angle**

\[ dq_{\perp}, \text{ pb/GeV}^2 \]

- Clear signature

- \( q_{\perp} = 2 \text{ GeV} \)

- \( q_{\perp} = 5 \text{ GeV} \)

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Is it possible to calculate this amplitude as

$$M_{\pi \rightarrow 2\text{jets}} = \int_0^1 dz' \int_0^1 dx_1 \phi_{\pi}(z', \mu_F^2) T_H(z', x_1, \mu_F^2) F_\zeta^g(x_1, \mu_F^2)$$

where $F_\zeta^g(x_1, \mu_F^2)$ is the skewed gluon distribution

$\phi_{\pi}(z, \mu_F^2)$ only includes low transverse momenta $k_{\perp}^2 < \mu_F^2$

$T_H(z', x_1, \mu_F^2)$ is calculated with all partons on-shell
Subtlety: Glauber region

On the light-cone

\[
\frac{F_{\xi}^g(x)}{x - \zeta + i\varepsilon} \cdot \left[ \frac{A}{x - \zeta + i\varepsilon} + \frac{B}{x - \zeta - i\varepsilon} \cdot (x - \zeta) \right]
\]

? how to cancel the singularity

\[
\frac{(x - \zeta)}{(x - \zeta + i\varepsilon)(x - \zeta - i\varepsilon)} \quad \text{or} \quad \frac{(x - \zeta)}{(x - \zeta + i\varepsilon)(x - \zeta - i\varepsilon)}
\]

Making the (first) choice implicitly assumes factorization

Step back:

(1, k_t, \ldots) \cdot \frac{1}{x - \zeta + i\varepsilon} \cdot \frac{1}{x - \zeta + k_t^2/M^2 \pm i\varepsilon}

No simple prescription possible

(More later)
\[ iM = \frac{4\pi^2 \alpha_s^2 s f_p}{N_c^2 q_\perp^4} \bar{u}(q_1) \gamma_5 \frac{p_2}{s} v(q_2) \int_0^1 dz' \phi_\pi(z') \int_0^1 dx_1 \mathcal{F}_\zeta^g(x_1) H(z, z', x_1, x_2) \]

where

\[ H(z, z', x_1, x_2) = \]
\[ = C_F \left( \frac{\bar{z} \bar{z}'}{z' + \bar{z}'} + \frac{z}{z' + \bar{z}} \right) \left( \frac{\zeta}{[x_1 - i \epsilon]^2} + \frac{\zeta}{[x_2 + i \epsilon]^2} - \frac{\zeta}{[x_1 - i \epsilon][x_2 + i \epsilon]} \right) \]
\[ + \left( \frac{z \bar{z}}{z' \bar{z}'} + 1 \right) \left[ C_F \left( \frac{z \bar{z}}{z' \bar{z}'} + 1 \right) + \frac{1}{2N_c} \left( \frac{z}{z'} + \frac{\bar{z}}{\bar{z}'} \right) \right] \]
\[ \times \left( \frac{1}{[(z - z')x_1 - z \bar{z}' \zeta + i \epsilon]} + \frac{1}{[(z' - z)x_2 - z \bar{z}' \zeta + i \epsilon]} \right) \]
\[ - \left[ C_F \frac{z \bar{z}}{z' \bar{z}'} \left( \frac{\bar{z}}{z'} + \frac{z}{\bar{z}'} \right) + \frac{1}{2N_c z' \bar{z}'} \left( \frac{z \bar{z}}{z' \bar{z}'} + 1 \right) \right] \frac{\zeta}{[x_1 + i \epsilon][x_2 - i \epsilon]} \]

\[ \text{The } i \epsilon \text{ prescriptions in the last line are different} \]
\[ \text{from Chernyak '01} \]

\[ \text{Two important regions:} \]

- \[ \zeta \ll |z' - z| \ll 1 \quad \text{Logarithmic enhancement} \]
- \[ z' \to 1, \quad z' \to 0 \quad \text{End-point singularity} \]

\[ \text{Bad news: Factorization is broken} \]
The $\zeta \ll |z' - z| \ll 1$ region

$$M \bigg|_{z' \approx z} = 4N_c \phi_\pi(z) \int \frac{dz'}{z' - z} F_\zeta(\zeta, \frac{z' \bar{z}}{z' - z}, q^2_\perp) \sim 4N_c \phi_\pi(z) \int \frac{dy}{\zeta} F_\zeta(y, q^2_\perp)$$

Color factors combine to produce $C_A = N_c$ and the factor $2N_c/y$ is the low-$y$ limit of the DGLAP splitting function

$$q^2_\perp \frac{\partial}{\partial q^2_\perp} F_\zeta(x = \zeta, q^2_\perp) = \frac{\alpha_s}{2\pi} \int_{\zeta}^{1} dy \frac{P^{gg}_\zeta(\zeta, y) F_\zeta(y, q^2_\perp)}{2\pi} \simeq \frac{\alpha_s}{2\pi} \int_{\zeta}^{1} dy \frac{2N_c}{y} F_\zeta(y, q^2_\perp)$$

Interpretation (NSS): hard gluon exchange can be viewed as a part of the unintegrated gluon distribution

The end-point region

The end-point singularity

$$M \bigg|_{\text{end-points}} = \left( N_c + \frac{1}{N_c} \right) z(1 - z) \int_{0}^{1} dz' \frac{\phi_\pi(z', \mu^2)}{z' r^2} F_\zeta(\zeta, \mu^2)$$

imitates the contribution of the asymptotic pion DA

Hope that on a heavy nucleus

$$\int_{\Lambda^2/Q^2}^{1} dz' \frac{\phi_\pi(z')}{z' r^2} \rightarrow \int_{\Lambda^2 A^{2/3}/Q^2}^{1} dz' \frac{\phi_\pi(z')}{z' r^2}$$

and/or Sudakov suppression

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**E791:** $1.5 \leq q_\perp \leq 2.5$ GeV

![Graph showing events vs. z with different curve styles and annotations.](image)

- **Black** — asymptotic pion DA
- **Red** — Chernyak-Zhitnitsky pion DA at low scale
- **Blue-dashed** — CZ pion DA at $\mu = 2$ GeV

The overall normalization (the same for all curves) is arbitrary.
E791 kinematics, LO MRST2001 parametrization for pdf’s

- The real part of the amplitude is significant.
- The quark-singlet contribution is sizable.
**Sudakov parametrization**

\[
k_1^{\mu} = \alpha p_1^{\mu} + x p_2^{\mu} + k_\perp \\
k_2^{\mu} = \alpha p_1^{\mu} + (x - \zeta) p_2^{\mu} + k_\perp
\]

\[
\int d^4 k = \frac{1}{2} s \int d\alpha \int d x \int d^2 k_\perp
\]

**Parton model interpretation** arises by choice to take the integral over \(\alpha\) first. After this integration one finds \(0 < x < 1\).

\(\checkmark\) Factorization implies that in the calculation of \(H\) one can neglect all components of \(k^{\mu}\) except for \(xp_2^{\mu}\).

\(\checkmark\) This is delicate because in the imaginary part one has a contribution of \(x = \zeta\), so the “large” component of the momentum is in fact zero!
Hard Electroproduction of vector mesons

\[ M \sim \int_0^1 \frac{dz}{z} \phi_V(z) \int_0^{k_\perp^4} \frac{d^2 k_\perp}{k_\perp^4} \int_0^1 dx \left[ \frac{2z}{[\bar{z}(x - \zeta) - k_\perp^2/s + i\epsilon]} + \Theta(0 < x < \zeta) \frac{2(x - \zeta) + 2k_\perp^2/s}{(x - \zeta)\zeta} \right]. \]

To expand in \( k_\perp^2 \) deform the contour:

\[ \begin{array}{c}
\longrightarrow \\
\downarrow \\
\underset{(0, \xi)}{\text{x}} \quad \underset{(1)}{\text{x}}
\end{array} \quad \begin{array}{c}
\longrightarrow \\
\uparrow \\
\underset{(0, \xi)}{\text{x}} \quad \underset{(1)}{\text{x}}
\end{array} \]

\[ \text{\textbullet\quad In Feynman gauge, deformation of the contour is possible for each diagram, but the light-cone dominance is only valid for the sum of all diagrams.} \]

\[ \text{\textbullet\quad In light-cone gauge, the light-cone dominance is valid for each diagram unless the contour deformation is obstructed by factors } 1/(x - \zeta) \text{ in the gluon propagators.} \]
For the present case, the simplest is to choose \( \frac{1}{(x - \zeta + i\epsilon)} \)

\[
D_{\mu\nu}(k^2) = \frac{-i}{k^2 + i\epsilon} \left[ g_{\mu\nu} - \frac{k_{\mu}p_{1,\nu} + k_{\nu}p_{1,\mu}}{(k \cdot p_1 + i\epsilon)} \right]
\]

Any gauge choice yields the same result, but only in this case one can calculate the hard scattering kernel as scattering from on-shell (transverse) gluons.

The final answer reads: Radyushkin

\[
M = \frac{4\pi e Q_q \alpha_s f_V}{N Q} \int_0^1 dz \frac{\phi_V(z)}{z\bar{z}} \int_0^1 dx \frac{\sqrt{1 - \zeta} F_{\zeta}^g(x)}{x(x - \zeta + i\epsilon)}
\]

where

\[
F_{\zeta}^g(x) = \frac{\alpha_s}{2\pi} C_F \left( \frac{1 + (1 - x)^2 - \zeta}{1 - \zeta} - \Theta(0 < x < \zeta) \frac{(\zeta - x)(2 - x - \zeta)}{\zeta(1 - \zeta)} \right) \int \frac{dk_{\perp}^2}{k_{\perp}^2}
\]

can be identified with the (perturbative) non-forward gluon distribution of a quark.

The problem is familiar from studies of Drell-Yan production, see e.g. Collins, Soper, Sterman, NPB261(1985)104 and is usually referred to as contribution of Glauber region.
Claim: For dijet production the $x$-integration is pinched by the singularities corresponding to the initial and the final soft (Glauber) gluons. In particular, it is not possible to choose an axial gauge in such a way that $x$-integral can be deformed in the upper half-plane for each diagrams separately.

Example:

\[ M_{a)} \sim \int_0^1 dx \frac{\text{Numerator}}{[x - \zeta + i\epsilon][-\bar{z}'(x - \zeta)s - k^2_{\perp} + i\epsilon]} \]
\[ M_{b)} \sim \int_0^1 dx \frac{\text{Numerator}}{[x - \zeta + i\epsilon][+\bar{z}(x - \zeta)s - k^2_{\perp} + i\epsilon]} \]

Due to the pinching, the contribution of diagrams with initial state soft interaction reverses its sign compared to the “naive” light-cone calculation.

\[ \diamond \text{QCD factorization is broken because both initial and final soft interactions are present. In the gauge-invariant sum of all Feynman diagrams the integration contours are pinched in the Glauber region.} \]
Conclusions

- Very interesting physics
- Can provide direct evidence for chirality violation in hard processes and measurement of magnetic susceptibility

Caveat:
- Collinear factorization is broken by soft gluon interactions in the Glauber region.
- Results by Nikolaev et al. correspond to the double-logarithmic approximation $\log q^2 \cdot \log x$ but do not extend beyond it.
- Further work needed. Other type of QCD factorization can be useful in this case.