Motivation \( q^2 \lesssim 0.1 \text{ GeV}^2 \): ChPT \( q^2 \ll m_N^3/m_\pi \): LET \( q^2 \gg m_N^3/m_\pi \): pQCD

Moving away from threshold

Summary

Threshold Pion Production at Large Momentum Transfers

V. M. Braun

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based on


PANIC 2008, Eilat, 13.11.08
Electroproduction with $Q^2$ in a few GeV$^2$ range:

- **Tradition**: excitation of nucleon resonances (transition form factors)

  $$
e(l) + p(P) \to e(l') + \Delta(1232)(P')$$
  $$e(l) + p(P) \to e(l') + N(1440)(P')$$

- **Proposal**: pion electroproduction close to threshold $W \to W_{\text{th}}$

  $$e(l) + p(P) \to e(l') + \pi^+(k) + n(P')$$
  $$e(l) + p(P) \to e(l') + \pi^0(k) + p(P')$$

  \[
  W^2 = (P' + k)^2 \\
  W_{\text{th}} = m_N + m_\pi \\
  Q^2 = -q^2 = -(\ell - \ell')^2
  \]

**Hard** ($pQCD$) and **soft** ($ChPT$) physics meet together!
Generalized Form Factors = S-wave Multipoles at Threshold

at the threshold

\[ \langle \pi N | f^\text{em}_\mu | p \rangle = - \frac{i}{f_\pi} \tilde{N}(P_2) \gamma_5 \left\{ \left( \gamma_\mu q^2 - q_\mu \not{q} \right) \frac{1}{m_N^2} G_1^{\pi N}(Q^2) - \frac{i\sigma_{\mu\nu}q^\nu}{2m_N} G_2^{\pi N}(Q^2) \right\} N(P_2) \]

related to S-wave multipoles in the PWA, e.g. for \( m_\pi = 0 \)

\[ E_{0+}^{\pi N}(Q^2, W_{\text{th}}) = \frac{\sqrt{4\pi\alpha_{\text{em}}}}{8\pi} \frac{Q^2 \sqrt{Q^2 + 4m_N^2}}{m_N^3 f_\pi} G_1^{\pi N} \]

\[ L_{0+}^{\pi N}(Q^2, W_{\text{th}}) = \frac{\sqrt{4\pi\alpha_{\text{em}}}}{32\pi} \frac{Q^2 \sqrt{Q^2 + 4m_N^2}}{m_N^3 f_\pi} G_2^{\pi N} \]

e.g. the differential cross section at threshold is given by

\[ \frac{d\sigma_{\gamma^*}}{d\Omega_\pi} \bigg|_{\text{th}} = \frac{2|\vec{k}_f|W}{W^2 - m_N^2} \left[ (E_{0+}^{\pi N})^2 + \epsilon \frac{Q^2}{(\omega_{\gamma})^2} (L_{0+}^{\pi N})^2 \right] \]
Motivation \[ Q^2 \lesssim 0.1 \text{ GeV}^2 : \text{ChPT} \quad Q^2 \ll m_N^3 / m_\pi : \text{LET} \quad Q^2 \gg m_N^3 / \]

Chiral rotation

Spontaneous Breaking of Chiral Symmetry

- In the chiral limit, \( m_\pi / m_N \to 0 \), the pion can be “rotated” away:

\[
\begin{align*}
|p \uparrow \rangle &= \frac{\phi_s(x)}{\sqrt{6}} |2u\uparrow d\downarrow u\uparrow - u\uparrow u\downarrow d\uparrow - d\uparrow u\downarrow u\uparrow\rangle + \frac{\phi_a(x)}{\sqrt{2}} |u\uparrow u\downarrow d\uparrow - d\uparrow u\downarrow u\uparrow\rangle \\
|p \uparrow \pi^0 \rangle &= \frac{\phi_s(x)}{2\sqrt{6}f_\pi} |6u\uparrow d\downarrow u\uparrow + u\uparrow u\downarrow d\uparrow + d\uparrow u\downarrow u\uparrow\rangle - \frac{\phi_a(x)}{2\sqrt{2}f_\pi} |u\uparrow u\downarrow d\uparrow - d\uparrow u\downarrow u\uparrow\rangle \\
|n \uparrow \pi^+ \rangle &= \frac{\phi_s(x)}{\sqrt{12}f_\pi} |2u\uparrow d\downarrow u\uparrow - 3u\uparrow u\downarrow d\uparrow - 3d\uparrow u\downarrow u\uparrow\rangle - \frac{\phi_a(x)}{2f_\pi} |u\uparrow u\downarrow d\uparrow - d\uparrow u\downarrow u\uparrow\rangle
\end{align*}
\]

Pobylitsa, Polyakov, Strikman; PRL87(2001)022001

- allows one to “look” at the proton from a different “angle”

The relevant degrees of freedom change with \( Q^2 \)
\[ \Rightarrow \text{rich physics} \]
\[ \Rightarrow \text{rich theory} \]
Motivation \( Q^2 \lesssim 0.1 \text{ GeV}^2 \): ChPT
\( Q^2 \ll m_N^3/m_\pi \): LET
\( Q^2 \gg m_N^3 / m_\pi \): pQCD

\( Q^2 < 0.1 \text{ GeV}^2 \): Chiral Perturbation Theory

- local effective low-energy theory
- systematic expansion in powers of \( m_\pi \) and \( |q| \)
- applicable for \( |q| \sim m_\pi < 300 \text{ MeV} \)

Kroll, Ruderman

\[
E_{0^+}^\pi (q^2 = 0, W_{th}) = \sqrt{2} \frac{eg_{\pi N}}{8\pi m_N} \left[ 1 - \frac{3}{2} \mu + O(\mu^2 \ln \mu^2) \right] = 26.6 \cdot 10^{-3} / m_\pi
dep: \ 27.9 \pm 0.5; 28.8 \pm 0.7
\]

\[
E_{0^+}^\pi (q^2 = 0, W_{th}) = -\frac{eg_{\pi N}}{8\pi m_N} \mu \left\{ 1 - \mu \left[ \frac{1}{2} (3 + \kappa_p) + \left( \frac{m_N}{4f_\pi} \right)^2 \right] \right\}
\]

- Subtlety: \( q \to 0 \) and \( m_\pi \to 0 \) limits do not commute

\( \mu = m_\pi / m_N \simeq 1/7 \)

Chiral Perturbation Theory – continued

Nambu, Lurié, Shrauner

\[ E_{0+}^{(-)}(m_\pi = 0, q^2) = \frac{e g_A}{8 \pi f_\pi} \left\{ \frac{q^2}{6} r_A^2 + \frac{q^2}{4 m_N^2} \left( \kappa_v + \frac{1}{2} \right) + \frac{q^2}{128 f_\pi^2} \left( 1 - \frac{12}{\pi^2} \right) \right\} \]

\[ G_A(q^2) = g_A \left( 1 + \frac{q^2}{6} r_A^2 + \ldots \right) \]

Experiment: \( r_A = 0.65 \pm 0.03 \) (elastic ep); \( r_A = 0.59 + 0.04 \pm 0.05 \) (pion el.prod)

S-wave cross section

\[ \gamma^* p \rightarrow \pi^0 p \]

\( W = 1074, \epsilon = 0.58 \)
Motivation \( Q^2 \lesssim 0.1 \text{ GeV}^2 \) ChPT \( Q^2 \ll m_N^3/m_{\pi} \) LET \( Q^2 \gg m_N^3 \) \( \Lambda_{QCD} \)

\( Q^2 \ll m_N^3/m_{\pi} \): Low-Energy Theorems

- **predate ChPT and QCD**

  **Chiral symmetry:**
  1. pion mass \( m_\pi \to 0 \)
  2. pion coupling \( \sim |k| \to 0 \)

- **Pion emission from external legs**

- **Chiral Rotation**

  \[ \langle \pi^a N | j^{\text{em}}_{\mu} | N \rangle \sim \frac{i}{f_\pi} \langle N | [j^{\text{em}}_{\mu}, Q_5^a] | N \rangle \]

  Kroll, Ruderman '54

V. M. Braun

Threshold Pion Production in QCD
Motivation \( Q^2 \lesssim 0.1 \text{ GeV}^2 \): ChPT \( Q^2 \ll m_N^3/m_{\pi^+} \): LET
\( Q^2 \gg m_N^3 \): pQCD

Low-Energy Theorems – continued

PCAC + current algebra:

\[
\frac{Q^2}{m_N^2} G_1^{\pi^0 p} = \frac{g_A}{2} \frac{Q^2}{(Q^2 + 2m_N^2)} G_M^p, \quad G_2^{\pi^0 p} = \frac{2g_A m_N^2}{(Q^2 + 2m_N^2)} G_E^p, \\
\frac{Q^2}{m_N^2} G_1^{\pi^+ n} = \frac{g_A}{\sqrt{2}} \frac{Q^2}{(Q^2 + 2m_N^2)} G_M^n + \frac{1}{\sqrt{2}} G_A, \quad G_2^{\pi^+ n} = \frac{2\sqrt{2} g_A m_N^2}{(Q^2 + 2m_N^2)} G_E^n,
\]

\textbf{Derivation does not imply } Q^2 \sim m_{\pi^+}^2 \textbf{ !}

- Threshold photoproduction of \( \pi^0 \) is suppressed compared to \( \pi^+ \)
- The \( \pi^0/\pi^+ \)–ratio is rapidly increasing with \( Q^2 \)

- The \( O(m_{\pi}) \) corrections can be added
- but, no systematic way to treat \( O(m_{\pi}^2) \) terms (ChPT)
• expected to fail for \( Q^2 \sim \frac{m_N^3}{m_{\pi}} \)

since \( \pi \) cannot have small momentum w.r.t. the initial and final state protons simultaneously

at threshold

\[
m_N^2 - (P - k)^2 = \frac{m_{\pi}}{m_N} \left[ Q^2 + 2m_N^2 \right]
\]

⇒ phenomenological Lagrangians to take into account nucleon resonances
⇒ or go over to quark-gluon description
Motivation: $Q^2 \lessgtr 0.1 \text{ GeV}^2$: ChPT  $Q^2 \ll m_N^3/m_\pi$: LET  $Q^2 \gg m_N^3$.

$Q^2 \gg m_N^3/m_\pi$: Perturbative QCD

QCD factorization for $Q^2 \gg \Lambda_{\text{QCD}}^3/m_\pi$

Pobylitsa, Polyakov, Strikman, PRL87(2001)022001:

$$|p \uparrow\rangle = \frac{\phi_s(x)}{\sqrt{6}} |2 u^\uparrow d^\downarrow u^\uparrow - u^\uparrow u^\downarrow d^\uparrow - d^\uparrow u_\downarrow u^\uparrow\rangle + \frac{\phi_a(x)}{\sqrt{2}} |u^\uparrow u^\downarrow d^\uparrow - d^\uparrow u_\downarrow u^\uparrow\rangle$$

$$|p \uparrow \pi^0\rangle = \frac{\phi_s(x)}{2\sqrt{6}f_\pi} |6 u^\uparrow d^\downarrow u^\uparrow + u^\uparrow u^\downarrow d^\uparrow + d^\uparrow u_\downarrow u^\uparrow\rangle - \frac{\phi_a(x)}{2\sqrt{2}f_\pi} |u^\uparrow u^\downarrow d^\uparrow - d^\uparrow u_\downarrow u^\uparrow\rangle$$

$$|n \uparrow \pi^+\rangle = \frac{\phi_s(x)}{\sqrt{12}f_\pi} |2 u^\uparrow d^\downarrow u^\uparrow - 3 u^\uparrow u^\downarrow d^\uparrow - 3 d^\uparrow u_\downarrow u^\uparrow\rangle - \frac{\phi_a(x)}{2f_\pi} |u^\uparrow u^\downarrow d^\uparrow - d^\uparrow u_\downarrow u^\uparrow\rangle$$

◇ Only for $G_1^{\pi N}(E_{0+})$

◇ Probably unrealistic at reachable momentum transfers
Motivation

$q^2 \lesssim 0.1 \text{ GeV}^2$: ChPT
$q^2 \ll m_N^3/m_\pi$: LET
$q^2 \gg m_N^3$:

\[ Q^2 \sim m_N^3/m_\pi : \text{ Light–Cone Sum Rules: } Q^2 \sim 1 - 10 \text{ GeV}^2 \]

\[ \diamond \text{ normalized to the dipole formula } G_D = 1/(1 + Q^2/0.71)^2 \]

\[ E_{0+}^{\pi^0 p}/G_D \]

\[ E_{0+}^{\pi^+ n}/G_D \]

\[ L_{0+}^{\pi^0 p}/G_D \]

\[ L_{0+}^{\pi^+ n}/G_D \]

green dots: MAID; ■: $W = 1074 \text{ MeV}$, ▲: $W = 1084 \text{ MeV}$, ★: $W = 1094 \text{ MeV}$

solid curves: LCSRs using experimental elastic EM formfactors as input

dashed curves: pure LCSRs, no experimental input
Motivation \( Q^2 \lesssim 0.1 \text{ GeV}^2 \): ChPT \( Q^2 \ll m_N^3 / m_\pi \): LET \( Q^2 \gg m_N^3 / m_\pi \): pQCD

Light Cone Sum Rules – continued

Deviation from LET:

\[
\frac{(G_1^{\pi 0 p})_{\text{QCD}}}{(G_1^{\pi 0 p})_{\text{LET}}}
\]

\[
\frac{(G_2^{\pi 0 p})_{\text{QCD}}}{(G_2^{\pi 0 p})_{\text{LET}}}
\]

- \( Q^2 \sim 1 \text{ GeV}^2 \)
- \( Q^2 \to \infty \) (part of the NNLO \( \alpha_s^2 \) contribution)
- No double counting of “soft” and “hard” contributions
- Tested: Electromagnetic and axial form factors, heavy meson decays, pion form factors
Moving away from threshold

- Higher partial waves
  - \( P \)-wave dominated by pion emission from the final state

- Energy dependence
  - \( E_{0+}(W), L_{0+}(W), \text{etc.} \)

- due to final state interactions
Figure: The structure function $F_2^p(W, Q^2)$ as a function of $W^2$ scaled by a factor $10^3$ compared to the SLAC E136 data at the average value $Q^2 = 7.14$ GeV$^2$ (left panel) and $Q^2 = 9.43$ GeV$^2$ (right panel).
Motivation

\[ Q^2 \lesssim 0.1 \text{ GeV}^2: \text{ ChPT} \]

\[ Q^2 \ll m_N^3/m_{\pi}^3: \text{ LET} \]

\[ Q^2 \gg m_N^3/m_{\pi}^3: \text{ pQCD} \]

CLAS (preliminary) \( ep \rightarrow e\pi^+n \)

Kijun Park; DNP 2008
Oakland, CA (Oct. 23-26)

MAID07

Blue circle = assumption of \( G_F^n = 0 \)
Red cicle = \( G_F^n(Q^2) = -\alpha_{\mu}^2 G_\rho(Q^2)/(1 + b\tau) \)
with assumption \( \text{Im}(G_\rho^{\mu\nu}) = 0 \)

Stat. Err. ONLY in data

Red Solid line =
LCSR using experimental
EM form factors as input

Red Dash line = pure LCSR
Motivation

\[ Q^2 \lesssim 0.1 \text{ GeV}^2 : \text{ChPT} \quad Q^2 \ll \frac{m_N^3}{m_\pi} : \text{LET} \quad Q^2 \gg \frac{m_N^3}{m_\pi} : \text{pQCD} \]

Moving away from threshold

Summary

- Use pion as a handle to “rotate” the nucleon wave function
  - a novel object: generalized form factor; an overlap between usual and rotated WF
  - check Low Energy Theorems (Nambu, ...) and transition to QCD
  - new scale in QCD: \( Q^2 \sim \frac{m_N^3}{m_\pi} \)
  - measure nucleon axial form factor (requires \( \pi^+ \))
  - theory progress feasible, large community (ChPT, pQCD, PWA)
  - an (almost) untouched terrain...

! no data at \( Q^2 \sim 0.1 - 1 \text{ GeV}^2 \) MAMI?

Perfectly suited for the JLab 12 GeV upgrade physics program
Supplementary Material
\( Q^2 \sim 1 - 10 \, \text{GeV}^2: \text{Light Cone Sum Rules} \)

Balitsky, V.B., Kolesnichenko '86–'88

- **Motivation**

\[ Q^2 \lesssim 0.1 \, \text{GeV}^2: \text{ChPT} \quad Q^2 \ll m_N^3/m_\pi: \text{LET} \quad Q^2 \gg m_N^3. \]

- **Q^2 \sim 1 - 10 \, \text{GeV}^2:**

1. Consider

\[
T^\pi_N(P',q) = i \int d^4x \ e^{-iqx} \langle N(P')\pi^a(k)|T\{f^\text{em}_\nu(x)\bar{\eta}_p(0)\}|0\rangle
\]

\[ \eta_p(x) = \epsilon^{ijk} [u^i(x)C\gamma_\mu u^j(x)] \gamma_5 \gamma^\mu d^k(x), \quad \langle 0|\eta_p|N(P)\rangle = \lambda_pm_NN(P) \]

2. Take \( P = P' + q - k, \quad P^2 \sim -1 \, \text{GeV}^2 \) and make a matching between

- **(a) The Operator Product Expansion in terms of pion-nucleon DAs**

\[
\langle N(P')\pi^a(k)|T\{f^\text{em}_\nu(x)\bar{\eta}_p(0)\}|0\rangle = \sum_{\text{twist}} H_\nu(x^2, px) \otimes \langle 0|q(x_1)q(x_2)q(x_3)|N(P')\pi^a(k)\rangle^+\]

- **(b) The dispersion representation in terms of hadronic states**

\[ \Box \quad \text{Borel transformation to improve convergence} \]

V. M. Braun

Threshold Pion Production in QCD
Good things:

♥ Reproduce LET for $Q^2 \sim 1$ GeV$^2$
♥ Reproduce pQCD for $Q^2 \to \infty$ (part of the NNLO $\alpha_s^2$ contribution)
♥ No double counting of “soft” and “hard” contributions
♥ Tested: Electromagnetic and axial form factors, heavy meson decays, pion form factors

Not–so–good things:

♦ Use nucleon distribution amplitudes as input — not so well known
♦ Calculation rather demanding, especially in NLO

Bad things:

♠ Approximation for the continuum contribution not improvable — irreducible error of order 20% for all $Q^2$
Motivation \( q^2 \lesssim 0.1 \text{ GeV}^2 \): ChPT \( q^2 \ll m_N^3/m_\pi^3 \): LET \( q^2 \gg m_N^3/m_\pi^3 \): pQCD

Light–Cone Sum Rules: Nucleon Electromagnetic form factors

choice of nucleon DAs:
- solid: BLW model
- long dashes: asymptotic
- short dashes: CZ model

Braun, Lenz, Wittmann; PRD73(2006)094019
A. Lenz; arXiv:0708.0633v1
● New: Semidisconnected pion-nucleon contributions in the intermediate state

\begin{align*}
\langle 0 | \eta_p(0) | N(P' - k)\pi(k) \rangle &= \frac{i\lambda_1^p m_N}{2f_\pi} \left[ 1 - \frac{g_A}{P'^2 - m_N^2} (P' - \gamma + m_N) \gamma \right] \gamma_5 N(P' - k).
\end{align*}

In the threshold kinematics, with $\delta = m_\pi / m_N$

\begin{align*}
T^{\pi^0 p}_{\gamma \nu}(P, q) &= \frac{i\lambda_1^p m_N}{f_\pi} \left\{ \frac{(1 + \delta) (P - \gamma + m_N) \gamma_5}{m_N^2 - P'^2} \right. \\
&\quad \times \left. \left[ (\gamma_\nu q^2 - q_\nu \gamma) \frac{G^{\pi^0 p}_1}{m_N^2} - \frac{i\sigma_\nu q^\mu}{2m_N} G^{\pi^0 p}_2 \right] \right. \\
&\quad + \frac{1}{2} \frac{(1 + \delta) \gamma_5 (P - \gamma + m_N)}{m_N^2 (1 + \delta)^2 + \delta Q^2 - P'^2} \left[ \gamma_\nu F^p_1 - \frac{i\sigma_\nu q^\mu}{2m_N} F^p_2 \right] \\
&\quad \left. - \frac{(1 + \delta) g_A (P - \gamma + m_N) \gamma_5}{m_N^2 (1 + \delta)^2 + \delta Q^2 - P'^2} \right] \left[ (\gamma_\nu q^2 - q_\nu \gamma) G^p_M - \frac{i\sigma_\nu q^\mu}{2m_N} 4m_N^2 G^p_E \right] \right\} N(P)
\end{align*}

**Figure:** Schematic structure of the pole terms in the correlation function

b) and c) correspond to $\pi N$ coupling to the Ioffe current
**Light–Cone Sum Rules: Pion–Nucleon Intermediate States — cont.**

- The semidisconnected $\pi N$ contributions can be included in the continuum if
  \[
  m_\pi Q^2 > m_N(s_0 - m_N^2) \ \Rightarrow \ \ Q^2 > 7 \text{ GeV}^2 \quad [\sim \Lambda_{QCD}^3/m_\pi]
  \]

- Otherwise they have to be taken into account explicitly

\[
\begin{align*}
\frac{Q^2}{m_N^2} G_1^{\pi 0p} &= \frac{e^{m_N^2/M^2}}{2 \chi^p_1} B_{P'2} \{A_0^{\pi 0p}(M^2, Q^2) - \frac{1}{2} e^{-\delta(2m_N^2+Q^2)/M^2} \left[ F_1^p(Q^2) - \frac{g_A Q^2}{Q^2 + 2m_N^2} G_M^p(Q^2) \right] \}
\end{align*}
\]

\[
\begin{align*}
\frac{Q^2}{m_N^2} G_2^{\pi 0p} &= -\frac{e^{m_N^2/M^2}}{\chi^p_1} B_{P'2} \{B_0^{\pi 0p}(M^2, Q^2) + e^{-\delta(2m_N^2+Q^2)/M^2} \left[ \frac{1}{2} F_2^p(Q^2) + \frac{2g_A m_N^2}{Q^2 + 2m_N^2} G_E^p(Q^2) \right] \}
\end{align*}
\]

\[
\begin{align*}
\frac{Q^2}{m_N^2} G_1^{\pi 0n} &= \frac{e^{m_N^2/M^2}}{2 \chi^p_1} B_{P'2} \{A_0^{\pi 0n}(M^2, Q^2) - \frac{1}{\sqrt{2}} e^{-\delta(2m_N^2+Q^2)/M^2} \left[ F_1^n(Q^2) - \frac{g_A Q^2}{Q^2 + 2m_N^2} G_M^n(Q^2) \right] \}
\end{align*}
\]

\[
\begin{align*}
\frac{Q^2}{m_N^2} G_2^{\pi 0n} &= -\frac{e^{m_N^2/M^2}}{\chi^p_1} B_{P'2} \{B_0^{\pi 0n}(M^2, Q^2) + e^{-\delta(2m_N^2+Q^2)/M^2} \left[ \frac{1}{\sqrt{2}} F_2^n(Q^2) + \frac{2\sqrt{2}g_A m_N^2}{Q^2 + 2m_N^2} G_E^n(Q^2) \right] \}
\end{align*}
\]

Where $A(P'^2, Q^2)$ and $A(P'^2, Q^2)$ are the invariant functions defined as

\[
z^\nu \Lambda^+ T_{\nu}^{\pi N}(P, q) = \frac{i}{f_\pi} (pz + k\bar{z}) \gamma_5 \left\{ m_N A(P'^2, Q^2) + h_{\perp} B(P'^2, Q^2) \right\} N^+(P)
\]

where $A(P'^2, Q^2)$ and $A(P'^2, Q^2)$ are the invariant functions defined as

\[
z^\nu \Lambda^+ T_{\nu}^{\pi N}(P, q) = \frac{i}{f_\pi} (pz + k\bar{z}) \gamma_5 \left\{ m_N A(P'^2, Q^2) + h_{\perp} B(P'^2, Q^2) \right\} N^+(P)
\]
\( Q^2 \lesssim 0.1 \text{ GeV}^2 \): ChPT
\( Q^2 \ll \frac{m_N^3}{m_\pi} \): LET
\( Q^2 \gg \frac{m_N^3}{m_\pi} \): pQCD

\( \pi N \) scattering phases

Figure from Nozawa et al., PRC41(1990)213
Including P-waves:

\[ \langle N(P')\pi(k)|j_{\mu}^{em}(0)|p(P)\rangle = \]
\[ = -\frac{i}{f_\pi}\bar{N}(P')\gamma_5\left\{ \left(\gamma_\mu q^2 - q_\mu q\right) - \frac{1}{m_N^2}G_{1}^{\pi N}(Q^2) - \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}G_{2}^{\pi N}(Q^2) \right\}N(P) \]
\[ + \frac{ic_\pi g_A}{2f_\pi[(P'+k)^2-m_N^2]}\bar{N}(P')\not{\gamma_5}(P'+m_N)\left\{ F_{1}^{p}(Q^2)\left(\gamma_\mu - \frac{q_\mu q}{q^2}\right) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_{2}^{p}(Q^2) \right\}N(P) \]

\begin{itemize}
  \item S–wave: generalized form factors from LCSR
  \item P–wave: pion emission from the final state nucleon; exact in chiral limit
  \item Eventually can take into account the final state interactions
    \[ G_{1}^{\pi N}(Q^2) \rightarrow G_{1}^{\pi N}(Q^2, W) \equiv G_{1}^{\pi N}(Q^2)[1 + it_{\pi N}] \]
\end{itemize}
Motivation \( Q^2 \lesssim 0.1 \text{ GeV}^2 \): ChPT \( Q^2 \ll \frac{m_N^3}{m_\pi} \): LET \( Q^2 \gg \frac{m_N^3}{m_\pi} \): pQCD

Structure Functions at \( x_B \to 1 \)

\[
F_1(W, Q^2) = \frac{\beta(W)}{(4\pi f_\pi)^2} \sum_{\pi^0, \pi^+} \left\{ \frac{Q^2 + 4m_N^2}{2m_N^4} |Q^2 G_{1N}^\pi|^2 + \frac{c_\pi^2 g_A^2 W^2 \beta^2(W)}{8(W^2 - m_N^2)^2} Q^2 m_N^2 G_M^2 \right\}
\]

\[
F_2(W, Q^2) = \frac{\beta(W)}{(4\pi f_\pi)^2} \sum_{\pi^0, \pi^+} \left\{ \frac{Q^2}{m_N^4} \left( |Q^2 G_{1N}^\pi|^2 + \frac{m_N^2}{4} Q^2 |G_{2N}^\pi|^2 \right) + \frac{c_\pi^2 g_A^2 W^2 \beta^2(W) Q^2 m_N^2}{4(W^2 - m_N^2)^2} \left( \frac{Q^2 G_M^2 + 4m_N^2 G_E^2}{Q^2 + 4m_N^2} \right) \right\}
\]

\[
g_1(W, Q^2) = \frac{\beta(W)}{(4\pi f_\pi)^2} \sum_{\pi^0, \pi^+} \left\{ \frac{Q^2}{2m_N^4} \left[ |Q^2 G_{1N}^\pi|^2 - m_N^2 \text{Re}(Q^2 G_{1N}^N G_{N}^{\pi^*}) \right] + \frac{c_\pi^2 g_A^2 W^2 \beta^2(W)}{8(W^2 - m_N^2)^2} Q^2 m_N^2 G_M F_1^p \right\}
\]

\[
g_2(W, Q^2) = -\frac{\beta(W)}{(4\pi f_\pi)^2} \sum_{\pi^0, \pi^+} \left\{ \frac{Q^2}{2m_N^4} \left[ |Q^2 G_{1N}^\pi|^2 + \frac{1}{4} Q^2 \text{Re}(Q^2 G_{1N}^N G_{N}^{\pi^*}) \right] + \frac{c_\pi^2 g_A^2 W^2 \beta^2(W)}{32(W^2 - m_N^2)^2} Q^4 G_M F_2^p \right\}
\]

\[
\beta(W) = \frac{|\vec{k}_f|}{W}, \quad x_B = \frac{Q^2}{Q^2 + W^2 - m_N^2}
\]
Differential Cross Section

For unpolarized protons, the virtual photon cross section is

$$d\sigma_{\gamma^*} = \frac{\alpha_{\text{em}}}{8\pi} \frac{k_f}{W} \frac{d\Omega_\pi}{W^2 - m_N^2} |\mathcal{M}_{\gamma^*}|^2$$

with

$$|\mathcal{M}_{\gamma^*}|^2 = M_T + \epsilon M_L + \sqrt{2\epsilon(1 + \epsilon)} M_{LT} \cos(\phi_\pi) + \epsilon M_{TT} \cos(2\phi_\pi) + \lambda \sqrt{2\epsilon(1 - \epsilon)} M'_{LT} \sin(\phi_\pi)$$

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\begin{align*}
 f^2 M_T &= \frac{4k_i^2 Q^2}{m_N^2} |G_1^{\pi N}|^2 + \frac{c_{\pi}^g g_A k_f^2}{(W^2 - m_N^2)^2} Q^2 m_N^2 G_M^2 + \cos \theta \frac{c_{\pi} g_A |k_i||k_f|}{W^2 - m_N^2} 4Q^2 G_M \text{Re} G_1^{\pi N} \\
 f^2 M_L &= \frac{k_i^2}{k_f^2} |G_2^{\pi N}|^2 + \frac{4c_{\pi}^g g_A k_f^2}{(W^2 - m_N^2)^2} m_N^2 G_E^2 - \cos \theta \frac{c_{\pi} g_A |k_i||k_f|}{W^2 - m_N^2} 4m_N^2 G_E \text{Re} G_2^{\pi N} \\
 f^2 M_{LT} &= -\sin \theta \frac{c_{\pi} g_A |k_i||k_f|}{W^2 - m_N^2} Qm_N \left[ G_M \text{Re} G_2^{\pi N} + 4G_E \text{Re} G_1^{\pi N} \right] \\
 f^2 M_{TT} &= 0, \\
 f^2 M'_{LT} &= -\sin \theta \frac{c_{\pi} g_A |k_i||k_f|}{W^2 - m_N^2} Qm_N \left[ G_M \text{Im} G_2^{\pi N} - 4G_E \text{Im} G_1^{\pi N} \right]
\end{align*}
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\(\diamond\) \(M_{TT} = 0\): no D–wave; tests quality of the approximation

\(\diamond\) \(M'_{LT}\): single–spin asymmetry; arises because of FSI, calculable
Motivation \( Q^2 \lesssim 0.1 \text{ GeV}^2 \): ChPT \( Q^2 \ll \frac{m_N^3}{m_\pi} \): LET \( Q^2 \gg \frac{m_N^3}{m_\pi} \): pQCD

Miscellaneous Results

**Figure:** The fraction of \( \pi^0p \) in \( F_2^p(W, Q^2) \) for \( Q^2 = 3 \text{ GeV}^2 \) (upper curve) and \( Q^2 = 9 \text{ GeV}^2 \) (lower curve)

**Figure:** Differential cross section \( d\sigma_{\gamma^*p \rightarrow \pi^0p}/d\Omega_\pi \), \( \mu \text{ b/ster} \) for \( \phi_\pi = 135 \text{ grad} \), \( Q^2 = 4.2 \text{ GeV}^2 \) and \( W = 1.11 \text{ GeV} \)

**Figure:** S-wave (solid) vs. P-wave (dashed) for \( F_2^p(W, Q^2) \) at \( Q^2 = 7.14 \text{ GeV}^2 \)

**Figure:** Integrated cross section \( Q^6 \sigma_{\gamma^*p \rightarrow \pi^0p} \) for \( W = 1.11 \text{ GeV} \) (lower curve) and \( W = 1.15 \text{ GeV} \) (upper curve)