Symposium

Hofstadter Butterfly

Scientific Coordination: Klaus Richter, Dieter Weiss

Programme and Abstracts

9:00  Opening (K. Richter)

9:05  D. Hofstadter (Bloomington):
      Bumping into a Butterfly, Back in my Youth, in Regensburg

9:35  P. Wiegmann (Chicago):
      Bethe Ansatz solution to the Hofstadter butterfly problem

10:05 D. Weiss (Regensburg):
      Hofstadter’s butterfly in lateral semiconductor superlattices

10:35 Coffee Break

11:15 M. Koshino (Osaka):
      Hofstadter’s butterfly in 2D and 3D

11:45 V. Falko (Manchester):
      Magnetic minibands in Moiré superlattices in graphene-hBN
      heterostructures

Abstracts see next page
Bumping into a Butterfly, Back in my Youth, in Regensburg

Douglas Hofstadter, Indiana University, Bloomington

I will recount the main events that led to my discovery of the so-called “Hofstadter butterfly” when I was a doctoral student in physics at the University of Regensburg in the years 1974 - 75. A key moment in the tale was when, after years of futile struggle, I finally abandoned particle physics, and chose, with much trepidation, to try solid-state physics instead, a field of which I knew nothing at all. I was instinctively drawn to a long-standing classic unsolved problem in the field – What is the nature of the energy spectrum of Bloch electrons in a magnetic field? – when Professor Gregory Wannier told me that it involved a weird distinction between so-called “rational” and “irrational” magnetic fields, which neither he nor anyone else understood. This mystery allured me, as I was sure that this rational/irrational distinction could not possibly play a role in physical phenomena. I tried manipulating equations for a long time but was unable to make any headway, and so, as a last resort, I wound up using brute-force calculation instead. I programmed a small desktop computer, nicknamed “Rumpelstilzchen”, to give me energy eigenvalues that I then plotted by hand on paper – and one fine day, to my shock, my eyes suddenly recognized a remarkable type of visual pattern that I had discovered twelve years earlier, when I was an undergraduate mathematics major exploring number theory. All at once, I realized that the theoretical energy spectrum that I’d plotted by hand consisted of infinitely many copies of itself, nested infinitely deeply, and it looked a little bit like a butterfly, whence its name. This unanticipated discovery eventually led to many new insights into the behavior of quantum systems featuring two competing periodicities. I will briefly describe some of the consequences that I found back then of the infinitely nested spectrum, and in particular how the formerly baffling rational/irrational distinction melted away totally, once the butterfly’s number-theoretical and topological nature had been sufficiently clearly understood.

Bethe-Ansatz Solution of the Hofstadter butterfly problem

Paul Wiegmann, University of Chicago

I review the solution of the Hofstadter problem by means of the Bethe Ansatz. The solution has been obtained quite some time ago together with Anton Zabrodin and further developed together with Alexander Abanov and Joop Talstra. Solution is possible due to realization of the group of magnetic translations by means of the cyclic representation of the quantum group \( U_q(SL_2) \). It equates the Hofstadter problem to the Heisenberg magnetic chain on just few sites but with a large spin.
Hofstadter's butterfly in lateral semiconductor superlattices

Dieter Weiss, Universität Regensburg

In my presentation I will briefly review our experiments to probe Hofstadter’s peculiar energy spectrum of Bloch-electrons in a magnetic field. By imposing a two-dimensional periodic potential by means of a patterned gate electrode onto a high mobility two-dimensional electron system we succeeded in generating a two dimensional “crystal” with periods of order 100 nm [1]. In these systems the Hofstadter butterfly describes the internal structure of each Landau band. The most prominent gaps in the butterfly were heralded in experiment by a splitting of the Shubnikov-de Haas peak at corresponding magnetic flux through the unit cell and associated Hall conductance [2]. The values of the Hall conductance tend to take the quantized values predicted by Thouless et al. [3].

“Fermiology of two-dimensional lateral superlattices”

“Evidence of Hofstadter’s fractal energy spectrum in the quantized Hall conductance”

“Quantized Hall conductance in a two-dimensional periodic potential”

Hofstadter's butterfly in 2D and 3D

Mikito Koshino, Osaka University

I will talk on two different topics selected from our theoretical researches closely related to the Hofstadter physics. First I present our recent studies on the electronic properties in two-dimensional moiré superlattice, or the slightly misoriented two-dimensional materials. Using the effective continuum theory, I will demonstrate that the energy bands are strongly modified by the long-period moiré pattern. We also argue about the Hofstadter butterfly spectrum under a magnetic field and its experimental observation in the transport measurement. In the latter half, I will present our theoretical attempt to realize the Hofstadter butterfly in three-dimensions. We consider an anisotropic 3D crystal under a tilted magnetic field, and show that the Hofstadter butterfly is caused by a completely different mechanism from 2D, due to the interference of the Landau quantizations on different crystal planes.
Magnetic minibands in moiré superlattices in graphene-hBN heterostructures

Vladimir Falko, University of Manchester

When graphene lattice is aligned with the hBN lattice, a long-wavelength periodic moiré pattern forms due to a weak incommensurability of the two lattice structures, leading to a long-range superlattice affecting properties of electrons in graphene. We shall discuss the effects produced by such moiré superlattice on electrons at low and strong magnetic field. At weak fields, electron states form minibands with peculiar properties [1,2], and also moiré patterns acts as a magnifying glass for the deformation of the 2D crystal [3]. At high magnetic fields, electron spectrum transforms into a fractal sequence of Brown-Zak magnetic minibands [4], which emerge from broadening of Landay levels at low magnetic field and develop into spectra [5] resembling the 'Hofstadter butterfly' [6]. For graphene-hBN heterostructures, the experimentally available magnetic fields are sufficient to provide flux BS through the moiré superlattice cell comparable to the magnetic flux quantum \( \Phi_0 \) and to observe the manifestation of magnetic minibands in magneto-transport and magneto-capacitance measurements [2,7]. As a result, a single device can offer a multiplicity of two-dimensional electron systems, realized at rational flux values BS= \( \Phi_0 \), \( \Phi_0/2 \), \( \Phi_0/3 \), etc., each with its own intricate topological properties, including quantum Hall effect physics related to the effective Landau levels emerging from these magnetic minibands at the nearby range of magnetic fields [5,7]. These minibands display a hierarchy of their band properties, such as typical group velocity in the band, which is reflected in the electronic transport in the form of 1/B magneto-oscillation persistent to high temperatures [8].