Lattice supersymmetry in 1D with two supercharges

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Abstract

A consistent formulation of a fully supersymmetric lattice theory on the lattice has been a longstanding challenge. In recent years there has been renewed interest on the problem with different approaches [1-12]. At the basis of the formulation we presented in this talk, there is the Dirac-Fkeller moving procedure, which was proposed in the continuum for a number of theories, including QCD in four dimensions [9]. Following the formulation developed in recent papers [13,14], we use supersymmetry theory with two supercharges on a one dimensional lattice and realize using a matrix formal model the matrix Dirac field theory. The matrix structure reproduces on a non-dissipative lattice the expected fermionic LahaLika. Various limits of consistency of the formalism [15] are discussed and effects not to be relevant.

Matrix representation of a Grassmann algebra

In the context of a supersymmetric one dimensional lattice theory with two supercharges, we can represent the odd Grassmann variables in terms of elements of a matrix algebra.

\[
\begin{align*}
\alpha_1 &= 1 - \sigma_3 - \partial_\theta, \\
\alpha_2 &= 0, \\
\alpha_3 &= 0, \\
\alpha_4 &= -\sigma_3 - \partial_\theta.
\end{align*}
\]

Theorems of the calculus of Grassmann variables

\[
\begin{align*}
\alpha_1 + \alpha_2 &= 0, \\
\alpha_1 + \alpha_3 &= 0, \\
\alpha_1 + \alpha_4 &= 0.
\end{align*}
\]

Supercharges and supertransformations

In our lattice formulation, the 1D action is described by a set of two supercharges, \(\alpha_1, \alpha_4\), which are related to a set of two Grassmann variables, \(\theta, \bar{\theta}\). The supercharges are defined as follows:

\[
\begin{align*}
\alpha_1 &= \theta, & \alpha_4 &= \bar{\theta},
\end{align*}
\]

The matrices \(\alpha_1, \alpha_4\) are Grassmann odd.

The supersymmetric transformation law of \(\theta\) is given by

\[
\alpha_{1} \rightarrow \alpha_{1} + \bar{\Xi}_{\theta} \partial_\theta \alpha_{1},
\]

where \(\bar{\Xi}_{\theta}\) is the supercharge of \(\theta\) and \(\alpha_{1}\) is the Grassmann variable.

The supersymmetric transformation law of \(\bar{\theta}\) is given by

\[
\alpha_{4} \rightarrow \alpha_{4} + \Xi_{\theta} \partial_\theta \alpha_{4},
\]

where \(\Xi_{\theta}\) is the supercharge of \(\bar{\theta}\) and \(\alpha_{4}\) is the Grassmann variable.

The supersymmetric transformation law of \(\bar{\theta}\) is given by

\[
\bar{\theta} \rightarrow \bar{\theta} + \Xi_{\bar{\theta}} \partial_{\bar{\theta}} \bar{\theta},
\]

where \(\Xi_{\bar{\theta}}\) is the supercharge of \(\bar{\theta}\) and \(\bar{\theta}\) is the Grassmann variable.

The supersymmetric transformation law of \(\theta\) is given by

\[
\theta \rightarrow \theta + \bar{\Xi}_{\bar{\theta}} \partial_{\bar{\theta}} \theta,
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