Lüscher-Weisz algorithm for excited states of the QCD flux tube

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Introduction

We look at the QCD flux tube between a static quark and an antiquark. The large-distance behavior of this flux tube is believed to be described by an effective string theory. The quantization of such a string leads to discrete energy-states. To leading order, the hamiltonian describing the transverse excitations of the string is given by:

\[ H = \sum_{l=0} b_l (3 - 2 b_l b_l) \]

where \( b_l \) and \( \bar{b}_l \) are the creation and annihilation operators. In 3 dimensions, these states can be classified by parity and charge-conjugation properties. \((|E, P \rangle)\). The states are shown in the following table:

<table>
<thead>
<tr>
<th>Parity</th>
<th>Charge Conjugation</th>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>C</td>
<td>( E )</td>
</tr>
<tr>
<td>-P</td>
<td>C</td>
<td>( E )</td>
</tr>
</tbody>
</table>

Goal: Measure the energy states on the lattice. The groundstate of the potential can be measured well with Polyakov-loop correlators, which can be written as:

\[ \langle P(R,T) P(0) \rangle = \sum_{\mathbf{L}} \delta_{\mathbf{L}} \langle \mathcal{E}(\mathbf{R}/T) \rangle \]

In this case the coefficients \( \delta_{\mathbf{L}} \) are integers and \( E(R) \) are the energy-states at quark-antiquark separation. From this formula, one can obtain the well known groundstate \( |0 \rangle = E(0) \).

We identify:

\[ |E_n \rangle = \frac{1}{\sqrt{2^n}} \sum \mathcal{E}(\mathbf{R}/T) \]

The \( 1/\mathbf{R} \) term is the well known Lüscher-terms, reproduced by all effective string theories. One way to distinguish between different string models is to measure the energy-differences of the excited states \([3]\).

Sources and excited states:

The Polyakov-loop correlators can be used to extract the groundstate of the potential. A better way of measuring the excited states is provided by Wilson-loops.

Wilson-loops with straight spatial lines at the ends, will again project strongly on the groundstate, but weakly on the excited states. In order to get a preferential coupling to the excited states, we use a set of wavefunctions at the ends of the loops called sources. These sources correspond to spatial lines on the lattice, that replace the spatial straight lines of the Wilson-loops.

Figure 1: Spatial lines that correspond to the sources on the lattice.

If we create superpositions of this sources with well defined parity and charge-conjugation, one can see \([2]\), that these channels couple directly to the excited states. The superpositions are shown in the table on the right.

To use the Polyakov-loops \( W(R,T) \) with channel \( \alpha \) and \( \mathcal{E}_z \) as the basis, we have the expansion:

\[ W(R,T) = \sum_{\mathcal{E}_z} \mathcal{E}_z \left( \mathcal{E}_z(R,T) \right) \]

The \( \mathcal{E}_z \) in this expression are due to the higher states in the channel and are exponentially damped with \( T_c T_1 \). This is why one would like to go to Wilson-loops with large time extent.

Problems:

Very small signal to noise ratio:

Even for SU(2) LQCD in d=3, conventional methods do not work. One way of reducing the error is provided by the Lüscher-Weisz algorithm \([4]\), which leads to an exponential error reduction for the time transporters of the Wilson-loops, putting the source on fixed-time-slices of the lattice and using the sublattice updates to reduce the fluctuations of the time-transporters produces good results and is practical for loops with time extents up to about a few fermi.

However this is not the best way to use the algorithm, because the fluctuations of the sources are not reduced with this method.

The new method:

To achieve further error reduction we now move the sources from the fixed lines to the middle of the time slices. Such a Wilson-loop with sources at the ends is shown in the picture on the right.

Main advantages:

- The fluctuations of the sources are reduced by the sublattice-updates as well.

- One can improve estimators for some links of the sources that also lead to a further error reduction.

Further it is useful to use different numbers of sublattice-updates for the time slices that contain the sources and the time-slices that contain only time-transporters. In this way it is possible to choose parameters for the algorithm to optimize the noise to signal ratio for the single parts of the Wilson-loops.

Several tests show that it is good for excited states to use more sub-lattice-updates for the sources than for the time-transporters.

Outlook:

A new method has an algorithm that allows one to use the advantages of the Lüscher-Weisz algorithm for excited states as well as for the sources and we have presented our first results. We were able to go much bigger Wilson-loops than was possible previously.

On the right we show preliminary results obtained from \( T = 4, 6, 8 \) and 12 Wilson-loops.

We see that the \( R = 9 \) points stay on the Avicz prediction both for the energy as well as the energy difference. We also observe that the corrections due to the higher states are much larger in this case. Nevertheless, the error reduction obtained with the naive states is not sufficient to give a signal for the third excited state beyond \( T = 6 \).

In our first runs we used \( B^{0,1,2,3}(5,6,7,8) \) from \( N_s = 32 \) and \( N_t = 6 \). Weisz 

First results:

In our first run we used \( B^{0,1,2,3}(5,6,7,8) \) from \( N_s = 32 \) and \( N_t = 6 \). We report 30000 total measurements and used the following scheme of sublattice-updates for the sources as well as for the time-transporters:

\[ T = 4, 6, 8 \rightarrow 4, 6, 8 \]

Compared to \([2]\) we were able to increase the time extent of the loops from \( T = 4 \) to \( T = 4, 6, 8 \).

The naive energies are calculated with the formula:

\[ E_n(R) = \frac{1}{T_c T_1} \langle \mathcal{E}_n(R) \rangle \]

with the parameters \( E_n, b, c \) to calculate the corrected energy \( \tilde{E}_n \).

\[ \tilde{E}_n(R) = E_n(R) + \Delta E_n(R) \]

where \( \Delta E_n(R) \) is the string-tension.

To get rid of the influence of the higher states in the channels, we use a fit to the form

\[ \frac{1}{T_c T_1} \langle \mathcal{E}_z(R) \rangle = E_z(R) + \Delta E_z(R) \]

With the fit parameters \( E_z, b, c \) to calculate the corrected energy \( \tilde{E}_n \).

Figure 3: The total energies and the prediction of the Avicz potential.

Figure 4: The energy difference \( \Delta E_z \) and the prediction of the Avicz potential.

Figure 5: Plot of the fit function for \( R = 7 \).

References


