High loop renormalization constants by NSPT: a status report

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Numerical Stochastic Perturbation Theory is a viable tool to compute high loop renormalization constants for Lattice QCD\(^a\).

A couple of issues are the natural steps forward:

- **Finite volume effects** can be quite important on quantities having an anomalous dimension (i.e. displaying log-divergencies). These have to be carefully taken care of.

- **Taking into account different actions** (both for gluons and for fermions) does not imply a real overhead (no Feynman rules to derive!)

We will report on these items, adding an interesting aside\(^b\):

- **Gluon and ghost propagators**.

As a matter of fact, they had not yet been looked at by NSPT.


\(^b\)In collaboration with M. Ilgenfritz, M. Mueller-Presker, A. Schiller. See their poster as well.
A convenient scheme to compute RC by NSPT is RI-MOM’. A useful example is the computation of quark bilinears. First of all one computes them (in Landau gauge) between external quark states at fixed (off-shell) momentum $p$

$$\int dx \langle p| \bar{\psi}(x) \Gamma \psi(x) |p\rangle = G_{\Gamma}(pa)$$

This step is taken in NSPT much the same way as in non-perturbative simulations. One then amputates ($S(pa)$ is the quark propagator) and projects out the tree-level structure

$$\Gamma_{\Gamma}(pa) = S^{-1}(pa) \ G_{\Gamma}(pa) \ S^{-1}(pa) \ \ \ \ O_{\Gamma}(pa) = Tr \left( \hat{P}_{O_{\Gamma}} \ \ G_{\Gamma}(pa) \right)$$

Renormalization conditions now read ($Z_q$ is the quark field renormalization constant)

$$Z_{O_{\Gamma}}(\mu a, g(a)) \ \ Z_q^{-1}(\mu a, g(a)) \ \ O_{\Gamma}(pa) \bigg|_{p^2=\mu^2} = 1$$ (1)
In NSPT one would like to take anomalous dimensions for granted. For RI-MOM’ this is the case: thanks to J. Gracey\textsuperscript{c}!

Let us write our master formula for the scalar current

\[
\left(1 - \frac{z_q^{(1)}}{\beta} + \ldots\right)\left(1 + \frac{z_s^{(1)} - \gamma_s^{(1)} \log(\hat{p}^2)}{\beta} + \ldots\right)\left(1 + \frac{O_s^{(1)}(\hat{p}^2)}{\beta} + \ldots\right) \bigg|_{p^2=\mu^2} = 1
\]

We explicitly wrote both the constant and the logarithmic contributions to renormalization constants (at first order the only log comes from $Z_S$ since one-loop quark-field anomalous dimension is zero in Landau gauge). $O_s^{(1)}(\hat{p}^2)$ is what is actually numerically measured. At one-loop order we can solve the previous relation to

\[
z_q^{(1)} - z_s^{(1)} = O_s^{(1)}(\hat{p}^2) - \gamma_s^{(1)} \log(\hat{p}^2). \tag{2}
\]

The quantity on l.h.s. is now finite and the only dependence on $pa \equiv \hat{p}$ is an irrelevant one, which can be wiped out by extrapolating to zero by means of what we call a **Hypercubic-symmetric Taylor expansion**.

Unfortunately this program results in a failure ...

Figure 1: Computation of one loop renormalization constant for the scalar current. With respect to Eq. (2), upper points are the unsubtracted \( \mathcal{O}_s^{(1)}(\hat{p}^2) \), while lower (circled crosses) stand for the subtracted \( \mathcal{O}_s^{(1)}(\hat{p}^2) - \gamma_s^{(1)} \log(\hat{p}^2) \). Analytic result (darker symbol) is missed: looking at the figure you would suspect IR effects.
Failure does not come as a surprise. By simply inspecting the previous figure one would suspect IR effects. As a matter of fact, points away from deep IR are ”pointing at the right result”. Let us put it this way: in the IR region points are doing their best to mimic a log-divergence on a finite volume!

One can understand that this is indeed a finite volume effect by looking at another figure that we always like to display. By taking the ratio of $Z_s$ and $Z_p$ one obtains a finite quantity which is safely computable: results on a $32^4$ lattice and on a $16^4$ lattice perfectly agree. On the other hand, the smaller the lattice, the bigger IR effects one gets. By looking at the following figure it is obvious that by performing the subtraction of Eq. (2) on the $16^4$ data points one misses the analytical result even more than in the previous figure. These are one loop results, but the picture stays much the same at higher loops.

This is why we have not yet given results for log-diverging quantities, while for finite ones we even got to four loop ...
Figure 2: Computations of $O_p^{(1)}(\hat{p}^2)$ (the equivalent of Eq. (2) for the pseudoscalar current) (top) and $O_s^{(1)}(\hat{p}^2)$ (bottom) on $32^4$ (circles) and $16^4$ (diamonds). In the middle the ratio $\frac{O_s^{(1)}(\hat{p}^2)}{O_p^{(1)}(\hat{p}^2)}$, which appears safe with respect to finite-size effects.
Define $L = Na$. Let us now write down for the quantity at hand the momentum sum $I(p, a, L)$ of conventional Lattice Perturbation Theory\(^d\) and split it as

$$I(p, a, L) = I(0, a, L) + (I(p, a, L) - I(0, a, L)) \equiv I(0, a, L) + J(p, a, L).$$  \[(3)\]

The divergence is logarithmic so that by subtracting $I(0, a, L)$ we make $J(p, a, L)$ **UV finite**. IR divergences will pop up and cancel with those in $I(0, a, L)$:

$$I(0, a, L) = c_1 + \gamma \log(a/L) + H(a/L)$$

$$J(p, a, L) = c_2 + \gamma \log(pL) + G(pa, a/L, pL).$$  \[(4)\]

Now we look for $pL = \hat{p}N$ **effects**. These should be looked for in $G(pa, a/L, pL) \to \tilde{G}(pL)$, which we computed from the formal continuum limit\(^e\) of our sum $J(p, a, L)$.

We call this contribution a **tamed-log**.

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\(^d\)With the same regularization of zero modes that we use in NSPT: zero momentum removed from the sum.

\(^e\)This means $a \to 0$ with $L = Na$ fixed. As we pointed out, $J(p, a, L)$ is UV finite. Again, we need the same ad hoc regularization of zero modes.
It is supposed to resemble the expected log, but with $pL = \hat{p}N$ effects on top of it: it indeed **approaches a log for $p >> 1$**. By subtracting this *tamed-log* we got the right one loop result!
So, we are happy with our one loop results! Still, the above method has got obvious drawbacks:

- One has to go back to diagramatic computations one would have liked to get rid of by means of NSPT.
- While one is happy enough with the one loop picture, at higher loops the situation is less clear and it is for sure much more cumbersome.
- One is not actually making use of the computations on different lattice sizes, but has to revert instead to a continuum computation.

Can we do better? We think YES! In the following we do not yet display results, but we present a strategy which is currently under investigation. It can be better understood having in mind figure 2.

This method is a very close relative of the one which is employed in an NSPT 3-d application with a mass in play\textsuperscript{f}.

\textsuperscript{f}C. Torrero, M. Laine, Y. Schroeder, F. Di Renzo, V. Miccio. See poster by Torrero.
An obvious way of looking at figure 2 is the following: on $32^4$ and $16^4$ one gets results at the same physical momentum $p$ from points affected by different $pL = \hat{p}N$ effects. Another point to make is that one can go to sufficiently high momenta in order to be substantially free of finite volume effects ($pL$ large enough).

Supposing you have measurements on at least two different sizes, this suggests to proceed as follows:

- Go to a momentum high enough to have the big lattice momentum substantially safe.
- Read the deviation of the result you got on the little lattice.
- This deviation defines the $pL = \hat{p}N$ effect which you have to correct for at the relevant point on the big lattice (i.e. at the point having the same value of $pL = \hat{p}N$).
- Repeat ...

The strategy is promising and first results are encouraging!
Changing the action can be quite cumbersome for Lattice Perturbation Theory: deriving Feynman rules for propagators and (both relevant and irrelevant!) vertices can require hard work (this is usually a computer-aided task). Life is by far easier in NSPT, so that it is worth exploiting this opportunity! We are working on the different combinations resulting from having

- Plain Wilson or tree-level Symanzik improved gauge action
- Plain Wilson or Clover fermionic action

From the computational point of view, notice that all the combinations are fairly well implemented on apeNEXT (as compared to APEmille) because of the larger available memory.

In particular, we are in a very advanced stage for the tree-level Symanzik improved gauge action, which became quite popular as a choice in recent times\(^9\).

\(^9\)See presentations by the European Twisted Mass Collaboration.
Figure 3: Computations of the Wilson fermions one loop critical mass for tree-level Symanzik improved gauge action. Result from the measurement on a single configuration: to keep in mind when moving to the Gluon Propagator ...
Figure 4: Computations of the Wilson fermions one loop self-energy for tree-level Symanzik improved gauge action. Points come in "families" corresponding to different violations of Lorentz symmetry, depending on the length of momentum along the gamma matrix one traces with. Result from the measurement on a single configuration: to keep in mind when moving to the Gluon Propagator ...
We report one loop results as benchmarks. Simulations are ready for higher orders and the quenched case is starting collecting statistic at three loop. You need instead a precise value for the critical mass for \( n_f \neq 0 \). Apart for this, one of the good point with NSPT is that everything is implemented order by order, so that once you get first order right you are standing on a quite safe ground.

We are still in a bit embarassing situation, since we are (slightly) off the only results which are explicitely quoted in the literature\(^h\). There is anyway the possibility to cross-check results and also to get the second loop for the critical mass\(^i\), which enables one to run at three loops.

Apart from the actual results, notice that plots are measurements from a single configuration. Fermionic measurements are remarkably stable (curves are smooth enough): keep this in mind now that we move to the Gluon Propagator.

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\(^{\text{i}}\)M. Constantinou, H. Panagopoulos, A. Skouroupathis, Phys. Rev. D74 (2006) 074503. Hironically, results for the Iwasaki action are explicitely written down, while those for Symanzik gauge action are not.
The **gluon propagator** in NSPT is a straightforward computation:

- Fix the gauge: once again, Landau is the obvious choice (with FFT acceleration).
- Go to momentum space.
- Take the product of fields and trace.

This is true, but in a sense to a minor extent than expected ... 

Gluon and ghost propagators are the subject of a new collaboration\(^\text{j}\). The framework is the qualitative/quantitative investigation of confinement via Schwinger-Dyson equations for the fundamental degrees of freedom of the theory.

While the gluon propagator is measured as stated just above, the ghost is measured much the same as the fermion propagator, by inverting order by order the Faddeev-Popov matrix on a (momentum space) source. The technology for the ghost was mainly derived in the context of another application\(^\text{k}\).

\(^{\text{j}}\) F. Di Renzo, C. Torrero, M. Ilgenfritz, M. Mueller-Presker, A. Schiller.

Figure 5: Tree level computation of the gluon propagator. Notation is: \( \Pi_{\mu\nu}(k) = D(k) \left( \frac{\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}}{k^2} \right) \).

The trivial constant \( D(k) \rightarrow 1 \) is recovered in the continuum limit. Also in this case data arrange themselves in families according to different patterns of violation of Lorentz symmetry. Results still to be extrapolated in stochastic time.
Figure 6: One loop computation of the gluon propagator. Notation is: $\Pi_{\mu\nu}(k) = D(k) \left( \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right)$. Once the log dictated by anomalous dimension is subtracted, the analytically known constant is recovered in the continuum limit. Also in this case data arrange themselves in families according to different patterns of violation of Lorentz symmetry. Results still to be extrapolated in stochastic time.
Notice that this time we present as a benchmark also the tree level computation. The gluon propagator is a bit different from the fermion (or ghost) propagator.

In fact, this time there is no inversion on a source, but simply the computation of a correlator (in momentum space). While this is extremely easy from a computational point of view, one now gets much more noise. Even tree level comes from an actual measurement (first fluctuations around the vacuum are the Lie algebraic free fields whose correlator gives the free Feynman propagator).

It is funny to compare the previous plots (which come out of O(100) configurations) with the (one configuration) measurements of figures 3 and 4!

Still, this is a doable computation. Both for the gluon and for the ghost, we provide measurements from fairly big lattices: $16^4$ and $32^4$, coming from the same configurations stored to measure the QCD fermionic quantities (work is in progress). Our German collaborators provide instead data from smaller lattices.
• We presented a new method to correct for finite volume effects in NSPT renormalization constants computations.

• Computations for tree level Symanzik improved gauge action are on their way. We still need to carefully cross check our first preliminary results with analytic ones (literature is not so well established).

• We also reported on the computation of gluon (and ghost) propagator. It is funny that, while the gluon propagator is the most straightforward to measure, it is not at all the best one with respect to the statistical fluctuations.