Cutoff effects for twisted mass Wilson fermions at tree-level of perturbation theory

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Pro-Fermion Wilson Actions

- Standard Wilson action (no bare quark mass):

\[ S_W^{00} = \sum_{x} \sum_{\sigma} \bar{\psi}(x) \left( \gamma^\sigma D^\sigma + m + \lambda \bar{\psi}(x) \right) \]

- Wilson Average (WM) and Mass Average (MA) (\( m_+ = m_- = m \)): (\( m_+ = m_- = m \), two-level, \( m_+ = m_0 \))

\[ S_W^{00} = \sum_{x} \sum_{\sigma} \bar{\psi}(x) \left( \gamma^\sigma D^\sigma + m_+ + \lambda \bar{\psi}(x) \right) \]

- Twisted mass Wilson (TM) (two-level, \( m_+ = m_0 \), twisted mass quark, \( m_0 \) has twisted mass):

\[ S_W^{00} = \sum_{x} \sum_{\sigma} \bar{\psi}(x) \left( \gamma^\sigma D^\sigma + m_0 + \lambda \bar{\psi}(x) \right) \]

- Twisted mass Wilson-Dirac operator:

\[ K^{ij}_{TM}(x, a) = \bar{\psi}^i(x) \gamma^\tau D^\tau \psi^j(x) + \bar{\psi}^i(x) \gamma^j D^j \psi^j(x) + m_0 \bar{\psi}^i(x) \gamma^i \psi^j(x) \]

Hadronic Masses Scaling Test - Standard Wilson Fermions

- WA at MA:

\[ N_{WM} = 2N_{WM} + \delta \phi \]

- WA (MA) \( \Rightarrow O(a) \) improved lattice quantities:

- Correlation, masses, decay amplitudes, renormalization constants...

- Consequences:
  - new masses
  - hypermasses

- O(a) scaling violations

- Chiral limit:

\[ \lim_{\chi \to 0} \]

- Finite volume limit:

\[ \lim_{V \to \infty} \]

- Continuous limits:

\[ \lim_{N \to \infty} \]

Pseudoscalar Decay Constants and Correlators

- Large time:

\[ C_\pi(t) = \frac{\langle 0 | \pi(t) \bar{\psi}(0) \psi(0) | 0 \rangle}{\langle 0 | \bar{\psi}(0) \psi(0) | 0 \rangle} \]

- Small time:

\[ C_\pi(t) \]

- Scaling test: Out of Maximal Twisted

Hadronic Masses Scaling Test - Wilson Average

- WA at MA:

\[ N_{WM} = 3N_{WM} + \delta \phi \]

- WA (MA) \( \Rightarrow O(a) \) improved lattice quantities:

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- Chiral limit:

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- Finite volume limit:

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- Continuous limits:

\[ \lim_{N \to \infty} \]

- Conclusion:

\[ \lim_{N \to \infty, \chi \to 0} \]

References

- D.H. Clogher and C.P. Bode
- K. Kamen, J. González López, J. Kamen, A. Kujawa, and A. Shindler

Conclusions

- Investigation at two-level of PT \( \Rightarrow \) demonstrating

- Standard Wilson regulation:

- WA and MA are equivalent

- Treat bare quark mass \( m_0 \) and twisted mass \( m_0 \)\( m \)

- Chiral limit:

\[ \lim_{\chi \to 0} \]

- Finite volume limit:

\[ \lim_{V \to \infty} \]

- Continuous limit:

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- Large lattice parameters are needed at larger values of \( N \)

- O(a) scaling violations

- WA (MA) \( \Rightarrow O(a) \) improved lattice quantities

- Consequences:

- WA and MA are equivalent

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