Analytic continuation of the critical line in 2-color QCD at nonzero temperature and density

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**Introduction**

- It is known that the fermion determinant in QCD becomes complex in presence of a non-zero chemical potential, thus preventing us from performing standard Monte Carlo simulations. To circumvent this problem one may perform numerical simulations with *imaginary chemical potential* and then obtain physical results by *analytic continuation* to *real chemical potential*.

- It is very important to have some control on the method of *analytic continuation*. To this purpose it is useful to consider the case of *2-color QCD* where, for any value of the chemical potential, *the fermion matrix is real*.

We found, by looking at the behavior in both real and imaginary chemical potential of the Polyakov loop, the chiral condensate and the fermion density, that the method gives reliable results, within appropriate ranges of the chemical potential, and that a considerable improvement can be achieved if ratio of polynomials are used to interpolate data with imaginary chemical potential.

**[Cea-Cosmai-D'Elia-Papa, JHEP02(2007)066; Lattice 2006]**

In the present work we give a preliminary account on:

- finding the best interpolation of critical line at imaginary chemical potential
- testing the *analytic continuation* of the critical line from imaginary to real chemical potential
Theoretical background

Roberge and Weiss [NPB275(1986)734] showed that the partition function of any SU(N) gauge theory is periodic in the parameter $\theta = \mu/T$, where $T$ is the temperature and $\mu$ is the imaginary part of the chemical potential.

Above a certain temperature $T_E$, physical observables exhibit discontinuities for

$$\theta = 2\pi(k + 1/2)/N, \quad k = 0, 1, \ldots$$

Therefore, the window in imaginary chemical potential where to perform numerical simulations is limited to the interval $[0, \pi T/N]$, thus strongly limiting the region useful for the analytical continuation.

Expected phase diagram for SU(N) gauge theory

- vertical lines above $T_E$ are first order

- curved lines represent first/second/crossover transition depending on the model and on its parameters
Lattice simulations

- We consider SU(2) gauge theory with $N_f=8$ staggered fermions, $am=0.07$ on a $16^3 \times 4$ lattice.

- Numerical simulations have been performed using hybrid Monte Carlo with $dt=0.01$ (typical statistics: ~ 20k trajectories). Simulations have been done using apeNEXT.

- We locate the critical value of $\beta$ at fixed chemical potential $\mu$ (both real and imaginary) by looking at the susceptibilities of the following quantities: chiral condensate, Polyakov loop, and plaquette.
**The critical coupling (from susceptibilities)**

- **chiral condensate susceptibility ($\mu=0$)**
  - $\beta_{\text{crit}} = 1.4072(35)$

- **chiral condensate susceptibility ($\mu_I=0.15$)**
  - $\beta_{\text{crit}} = 1.4258(51)$

- **chiral condensate susceptibility ($\mu_I=0.20$)**
  - $\beta_{\text{crit}} = 1.4228(55)$

- **chiral condensate susceptibility ($\mu_I=0.10$)**
  - $\beta_{\text{crit}} = 1.4258(51)$

- **chiral condensate susceptibility ($\mu_I=0.15$)**
  - $\beta_{\text{crit}} = 1.5097(33)$

- **Polyakov loop plaquette**

- **plaquette**

<table>
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<th>$\mu_I$</th>
<th>chiral condensate</th>
<th>Polyakov loop</th>
<th>plaquette</th>
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<tbody>
<tr>
<td>0.30</td>
<td>1.5097(33)</td>
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<td>1.356(12)</td>
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<td>0.30</td>
<td>1.271(16)</td>
<td>1.267(26)</td>
<td>1.286(15)</td>
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</table>
Analytic continuation of the critical line (1)

\[ \beta_{\text{crit}}(\mu_{\text{I,RW}}) = 1.5829(81) \]

\[ \beta_{\text{crit}} = A + B \mu^2 \]

\[ A = 1.4071(27) \]
\[ B = -1.140(50) \]
\[ \chi^2/\text{d.o.f.} = 0.91 \]

From the peak of the susceptibility of the chiral condensate at \( \beta=1.30 \) and real \( \mu \)
$\beta_{\text{crit}}(\mu_{I,RW}) = 1.575(17)$

from the Polyakov loop

$\beta_{\text{crit}} = A + B\mu^2$

$A = 1.3931(55)$
$B = -1.18(10)$
$\chi^2$/d.o.f. = 0.85
Analytic continuation of the critical line (3)

\[ \beta_{\text{crit}}(\mu_{I,RW}) = 1.574(13) \]

\[ \beta_{\text{crit}} = A + B\mu^2 \]

\[ A = 1.4042(39) \]

\[ B = -1.104(83) \]

\[ \chi^2 / \text{d.o.f.} = 0.48 \]
**Conclusions**

We have determined the critical line $\beta_{\text{crit}}(\mu_I)$ in $N_c=2$ QCD.

We have found that the critical line can be interpolated by means of 2nd order polynomial $A + B\mu^2$ (similarly to [D’Elia-Lombardo, PRD67(2003)014505; deForcrand-Philipsen, NPB642(2002)290] in SU(3)).

If higher order polynomials $A + B\mu^2 + C\mu^4 + \cdots$ or ratio of polynomials

$$\frac{A + B\mu^2 + C\mu^4 + \cdots}{1 + A'\mu^2 + \cdots}$$

are used, all coefficients but $A$ and $B$ are put to zero by the fit.

The critical value of $\beta$ extrapolated to the Roberge-Weiss critical $\mu$ value is compatible with its direct determination.

The analytic continuation of the critical line to real chemical potential is in reasonable agreement(\textit{*}) with direct determinations at $\mu_R=0.20$ and $\mu_R=0.30$.

(\textit{*}) A more refined analysis with improved statistics will be necessary to further clarify this issue.