Fourier Accelerating RHMC for the Overlap Top-Higgs Yukawa Model

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Abstract: For the purpose of studying the top-Higgs Yukawa model [1-4], we employ the RHMC algorithm [5] with chiral overlap formulation [6]. We obtain substantial performance gains by implementing the overlap operator in Fourier space and applying Fourier acceleration [7-8]. Computing the overlap in Fourier space is 10-20 times faster than the usual Zolotarev-based method required when working with a gauge field. Furthermore, in some regions of parameter space, the Fourier acceleration (FA) technique reduces autocorrelations by a factor of roughly 100. These performance enhancements are crucial for the top-Higgs study which we are attempting.

The 1-Component Top-Higgs Yukawa Model

\[ S = \sum \left[ -\kappa \sum \phi_x (\phi_{x+m} + \phi_{x-m}) + \phi_y (\phi_{y+1} - \phi_{y-1})^2 + \phi_z (1 - D_2) \right] \]

The Higgs sector is essential to theoretical progress but not well understood.
- Is this model trivial? Is the effective potential stable?
- Currently modeling dynamical top quark and one-component Higgs without gluons, but top-bottom doublet, four-component Higgs, and gluons being (or have been) developed.
- Interesting extensions: higher dimensional operators, intermediate energy scale insertion.

Rational Hybrid Monte Carlo (RHMC)

HMC is a powerful tool to simulate dynamical fermions - global updating and exact.
- A conjugate momentum field \( \pi(x) \) is introduced, and a kinetic term, \( \exp(-\pi(x)^2/2) \), is added to the action to form a fictitious Hamiltonian.
- Detailed balance and equivalence to the original system are straightforward.
- The fermion field \( \psi \) is replaced with a pseudofermion field \( \chi = \frac{1}{\sqrt{A}} \phi ; \)
- The root is approximated by Zolotarev's rational polynomial method.
- The \( \phi \) and \( \pi \) fields are evolved according to classical equations of motion.
- Finally, the resulting field \( \phi \) field is accepted or rejected based on the fictitious \( \Delta H \).

Overlap Operator in Fourier Space

- The overlap fermion operator is given, in coordinate space, by:
  \[ D_{ol} = \frac{1}{2} \left( 1 - \frac{A}{\sqrt{A}} \right), \quad A = \frac{1}{2} - D_W \]
- Where \( D_W \) is the usual Wilson-Dirac operator.
- The root is usually evaluated using Zolotarev's method, requiring a nested multishift inversion.
- With no gauge, the overlap operator can be evaluated in Fourier space; each inner inversion is replaced with a much cheaper pair of (multithreaded FFTW3) Fourier transforms.

Autocorrelation

- Measure of independence of subsequent field configurations, defined as:
  \[ \tau = \frac{\sum_{i=1}^{N} \sum_{j=0}^{k} Q(i)Q(j) - Q^2}{(Q^2) - (Q)^2} \]
- For significant regions of the model's parameter space, the autocorrelations are huge, hundreds of unit updates.

Vev History Without FA

- Vev history vs M/D time for \( m_{FA} = 5.0 \) with \( \tau = 111.6 \)
- Vev history vs M/D time for \( m_{FA} = 1.0 \) with \( \tau = 9.2 \)
- Vev history vs M/D time for \( m_{FA} = 0.2 \) with \( \tau = -0.5(7) \)

Vev Histories With FA

- Vev history vs M/D time for \( m_{FA} = 5.0 \) with \( \tau = 152.7 \)
- Vev history vs M/D time for \( m_{FA} = 1.0 \) with \( \tau = 13.0 \)
- Vev history vs M/D time for \( m_{FA} = 0.2 \) with \( \tau = 12.3 \)

Fourier Acceleration

- Reduce autocorrelation by "boosting" update \( d\phi/dt \) for slowly evolving modes, i.e. long wavelength Fourier modes.
- Introduce "conjugate mass" along with conjugate momentum, making the update \( d\phi/dt = \pi(k/m(k)) \) and kinetic term \( \exp(\pi(k^2/m(k))) \).
- Assign a small "mass" to the slow modes. The inverse of the propagator (scaled for convenience), with a tunable mass parameter, is a reasonable starting point:
  \[ m(k) = (m_{FA}^2 + k^2)/(m_{FA}^2 + k^2_{max}) \]

Fourier mode dependence of \( d\phi/dt \)

- For the case studied, FA reduces \( \tau \) by up to 2 orders of magnitude.
- Decreasing \( m_{FA} \) improves decorrelation, but instability arises for too small \( m_{FA} \).
- It will be interesting to test the FA on larger lattices and see how the related critical exponent is affected as in [7].

References

Talks presented at Lattice 2007
[4] J. Kuti and K. Holland. A trivial Higgs lower bound ...
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