The strong-coupling expansion is given by

\[ Z = \sum_{\phi} \exp \left( \beta \int d^d x \left( -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \lambda \phi^4 \right) \right) \]

with

\[ F_{\mu \nu} = \partial_\mu \phi \partial_\nu \phi - \partial_\nu \phi \partial_\mu \phi. \]

This leads to the following phase diagram:

- **Critical exponents for the anisotropic SU(3) model**

\[ \chi \propto L^{\nu - \delta} \]

with

\[ \nu = 2.6 \quad \text{and} \quad \delta = 0.5 \]

A Monte-Carlo simulation with the modified Wolff cluster algorithm leads to the following critical exponents:

\[ \chi = 3.0 \text{ for } D = 2 \]

\[ \chi = 4.0 \text{ for } D = 3 \]

**Application of IMC and results**

We have simulated the underlying YM theory with well-known heat methods on different lattices around the critical coupling. Afterwards we have computed the DMC and obtained corresponding coupling constants for the effective models. The computations and programming codes were checked by simulating effective actions with fixed coupling and reproducing them consistently with the YM procedure. In these tests the algorithm and geometric Ward identities were compatible and the quality of reproduction was only limited by the statistical accuracy.

By simulating the full YM theory we compared the SD expansion arising from geometric and algebraic Ward identities on an N \times N lattice which resulted in a strong coupling expansion up to order C^2(N^4) in the effective coupling. Here the algebraic identities outperformed the geometric ones by reproducing a comparable critical coupling.

**Geometric Ward identities for invariant group integrals**

We define the left derivative of a function f on the Group G by

\[ L_f (x) = \lim_{\epsilon \to 0} \frac{f(x + \epsilon z) - f(x)}{\epsilon} \]

where \( z \) denotes the generator of the Lie group. Invariance of the Haar measure implies that

\[ \int G f(x) \, d\mu(x) = 0 \quad \text{for} \quad \int G 1 \, d\mu(x) = 1 \]

where \( 1 \) denotes the identity of the group.

For the special choice \( f = f_{\mu \nu} \), with a class function \( f_\mu \) and fundamental character \( \lambda_\mu \), holds

\[ L_{\mu \nu} = f_{\mu \nu} (x) = f_{\mu \nu} (x, \chi, \lambda) \]

and the Ward identity reduces to

\[ \partial_{\lambda} S = \sum_{\mu \nu} \frac{\partial f_{\mu \nu}}{\partial \lambda} \]

Making use of \( f_{\mu \nu} = \delta_{\mu \nu} \) and

\[ (\lambda_\lambda) \]

with constant values, and Cliboich Gordon coefficients \( c_{\lambda \mu} \) the equation can be specialized to the case of SU(3). For a particular choice of the function \( f \) with Schwinger-Dyson equations

\[ \frac{1}{2} \sum_{\mu \nu} \left( \lambda_{\mu \nu} - \delta_{\mu \nu} \right) = 0 \]

\[ \chi_{\mu \nu} = \lambda_{\mu \nu} - \delta_{\mu \nu} \]

\[ \lambda_{\mu \nu} = \lambda_{\mu \nu} (x, \chi, \lambda) \]

where \( \lambda \) is the parameter of the group.

**Algebraic Ward identities**

For the SU(3) case (generalizations for SU(N) are possible) we use the general identity

\[ \int [dG] \rho (x) f (x, \chi, \lambda) = 0 \]

for a function \( f \) vanishing on \( \partial \Sigma \). Now we choose

\[ f (x, \chi, \chi, \lambda) = \frac{1}{2} \sum_{\mu \nu} \left( \lambda_{\mu \nu} - \delta_{\mu \nu} \right) \]

and analogously with a system of Schwinger-Dyson equations

\[ \lambda_{\mu \nu} = \lambda_{\mu \nu} (x, \chi, \lambda) \]

\[ \lambda_{\mu \nu} = \lambda_{\mu \nu} (x, \chi, \lambda) \]

for reasonable predictions.

**References**


