Pure Gauge Compact QED in the Ice Limit

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NIC
Introduction.

issue: pure gauge compact QED has served as a toy model of confinement and the development of techniques


unfortunately we have not found any second order phase transition i.e., continuum field theory, in the $\beta - \gamma$ space of couplings

there are ideas for the further use of PQED: “Compact lattice U(1) and Seiberg Witten duality: a quantitative comparison”, D. Espriu and L. Tagliacozzo, Phys. Lett. B, 602, (2004), 137.
The action gap $\Delta w_p$ as a function of $\gamma$ in $\beta$-$\gamma$ PQED: Bunk, Arnold, Lippert, Neuhaus in very slow progress
2– loop finite size calculation of PQED partition function by B. Bunk in dimension $d$ on $V = L^d$ boxes with p.b.c.

\[
\log \frac{Z^{(2)}}{Z_{bulk}^{(2)}} = \frac{d - 1}{2} \log \frac{2\pi}{e^2 R} + \frac{(d - 1)(d - 4)}{2} \log L \\
+ \left( \frac{d}{2} - 1 \right) \left[ f_d + L^{-2} f_d^{(1)} + O(L^{-4}) \right] \\
- \frac{d - 1}{2d} \frac{e^2 R}{V}
\]

$f_4 = 1.701216$

$f_4^{(1)} = 3.122151$
“Ice Limit” of PQED.

there maybe academic interest in the ice limit

$$\frac{\gamma}{\beta} = \text{const}$$

$$\beta \rightarrow +\infty$$

$$\gamma \rightarrow -\infty$$

$$S = \sum_P \beta [1 - \cos(\Theta_P)] + \gamma [1 - \cos(2\Theta_P)]$$

where using flat histogram sampling i.e., Multicanonical, Wang Landau or Multiple Gaussian Modified Ensemble (T. Neuhaus and J.S. Hager, Phys. Rev. E 74, 036702 (2006)) simulations, on finds the ground state number density (degeneracy)

$$\frac{\log[n_{GS}]}{V} \approx \log[\Omega_{Group}] + \frac{3}{2}\log[2]$$

the situation is similar to the six vertex models in stat. Phys. However the Wilson loop correlation functions of the PQED ice limit are not yet understood
Wilson Loops expectation values in the iced ground state of two charge PQED at $\text{const} = -\frac{1}{2}$, that is at $\Theta_P(GS) = \pm \frac{\pi}{3}$, on a $8^4$ box.
Dual Superconductivity.

- The dual map of the PQED path integral interchanges electric and magnetic fields.

The Villain actions Boltzmann factor

\[
\exp[-S(\beta, \theta_P)] = \sum_{m=-\infty}^{m=+\infty} \exp[-\beta (\theta_P - 2\pi m)^2]
\]

maps onto

\[
\exp[-\frac{\kappa^2}{2} I_P^2]
\]

with the duality relation

\[
Z_{PQED}(\beta) \propto Z_{FZS}(\kappa = \frac{1}{\beta})
\]

\[
\beta_c = 0.643734(5), \quad \kappa_c = 1.553437(9)
\]
Dual Superconductivity..

\[ W_{R,T} = \langle \exp(\sum_{P \in W} i\theta_P) \rangle_{\text{PQED}} \]

\[ = \frac{1}{Z_{\text{FZS}}} \sum_{I_L} \prod_P \exp(-\frac{\kappa}{2}(I_P - M_P^{R,T,*})^2) \]

with world sheet

\[ \sum_\nu (M_{\bar{x},\mu\nu} - M_{\bar{x}-\hat{\nu},\mu\nu}) = - \sum_{L \in \partial W_{RT}} \delta_L, (\bar{x},\mu) \]
Dual Superconductivity...

world sheet solution for Wilson loop
Wilson loop closure on the boxes boundary yields charged world sheet free energy, \( q = n_W \)

\[
F_{WS}(\beta, q) = -\ln \left[ \frac{1}{Z_{FZS}} \sum_{I_L} \prod_{P} \exp \left( -\frac{1}{2\beta} (I_P - qM^*_P)^2 \right) \right]
\]

and the canonical fluctuating charge partition function of PQED at electric field \( E \)

\[
Z_{PQED}(\beta, E) = \sum_{q=-\infty}^{q=+\infty} \exp \left[ -F_{WS}(\beta, q) + qEN_xN_t \right]
\]
Dual Superconductivity.....

- the tension $T = \sigma$

$$T = \lim_{V \to \infty} \frac{F_{WS}(1)}{N_x N_t}$$

- the lower critical field value $E_{c1}$
  type II: $E_{c1} = T$
  type I: $E_{c1} < T$

from tangent construction to

$$P(q) = \frac{1}{\mathcal{N}} \exp [-F_{WS}(q) + qEN_xN_t]$$

- at $\beta_c$ note: $E_{c1}$ vanishes, while for type I $T$ is discontinuous
Results.

world sheet free energy on $14^4$ at $\beta = 0.6175, 0.6275, 0.6375$ with mildly attractive sheet sheet interactions
Results..

Tangent construction for $E_{c1}$ at $\beta = 0.62$ on $12^4$ cox: there is a type I “Meissner transition” with a nucleation barrier $B_0$.
the tension $T$ is discontinuous at $\beta_c$
the lower critical field $E_{c1}$ is continuous at $\beta_c$
Results.....

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<th>( T_b )</th>
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0.643734(5) | \( T^{-0.5} \) | \( \beta_c \) | \( \approx 6.5 \ a \)
Results......

dual photon correlation function in the confinement phase

\[ \beta = 0.6361, 0.6415, 0.6433 \]
Results

dual photon mass over root tension

\[ m_v T^{-0.5} \]
Conclusion.

- the deconfinement phase transition of pure gauge PQED is of first order in 2007.

- the “ice limit” of PQED defines an interesting model in statistical field theory, however consequences are not worked out. One does not expect a continuum field theory.

- dual superconductor treatment facilitates studies of phases in direction of the electric field. The construction is possible for theories with Gaussian (non-compact) gauge field actions.

- there is a type I “Meissner transition” at the lower critical electric field value $E_{c1}$ for pure gauge PQED with Villain action. It ends at the zero field $E$ first order deconfinement phase transition.