The Kaon B-parameter from Domain Wall
Fermions

hep-ph/0702042

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People

Outline

Introduction

Calculation of $B_K$ — hep-ph/0702042

Chiral Extrapolation Error
Introduction

• Important conversion: lattice number $\rightarrow$ phenomenologically relevant number

• Lattice quantities are calculated
  • At the wrong quark mass
  • On a femto taurus
  • With a finite lattice spacing

• Careful consideration of the systematic errors to ensure that the correct error is quoted.
  • Chiral extrapolation errors
  • Finite volume effects
  • Discretisation errors
  • Renormalisation errors
Simulation Details

- Volume = $16^3 \times 32$, $\beta = 2.13$, Iwasaki action.
- 2+1 flavour DWF fermions, $L_s = 16 \implies a m_{\text{res}} = 0.0031$

<table>
<thead>
<tr>
<th>$a m^\text{input}_l$</th>
<th>$a m^\text{input}_s$</th>
<th>$M_\pi (\text{MeV})$</th>
<th>$M_K (\text{MeV})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.04</td>
<td>400</td>
<td>580</td>
</tr>
<tr>
<td>0.02</td>
<td>0.04</td>
<td>530</td>
<td>620</td>
</tr>
<tr>
<td>0.03</td>
<td>0.04</td>
<td>630</td>
<td>680</td>
</tr>
</tbody>
</table>

- Scale calculated in hep-lat/0701013: $a^{-1} = 1.62(4)$ GeV
  - from $M_\rho$ and $r_0 = 0.5$ fm
Kaon B-Parameter defined as the ratio of matrix elements

\[ B_{K}^{lat} = \frac{\langle K^0 | O_{VV+AA} | K^0 \rangle}{\frac{8}{3} \langle K^0 | A_0 \rangle \langle A_0 | K^0 \rangle} \]

- Use periodic + anti-periodic boundary conditions
- removes around the world propagation \( \Rightarrow \) long plateaux
Chiral Extrapolation

- We extrapolate to the light quark limit using continuum $SU(3) \times SU(3)$ NLO partially quenched chiral form [Van de Water and Sharpe, hep-lat/0507012]

**Figure:** $B_K(am_x, am_y)$ fitted to NLO partially quenched $SU(3)$ form.

$am_l = 0.01$, $am_s = 0.04$
Chiral Extrapolation: mass cut

A cut in mean bare valence mass required to obtain a good fit

- \[ \frac{1}{2}(m_{x}^{\text{val}} + m_{y}^{\text{val}}) \leq m^{\text{cut}} \in \{0.02, 0.025, 0.03\} \]

- Insensitive to mass cut
- Chiral logs contained in $SU(3)$ form
- Choose $m^{\text{cut}} = 0.03$
- $B_{K}^{\text{lat}}(am_{l}^{\text{phys}}, am_{s}^{\text{phys}}) = 0.606(13)_{\text{stat}}$

Input from hep-lat/0701013

- $am_{s}^{\text{phys}} = 0.0390(21)$, $am_{l}^{\text{phys}} = 0.00162(8)$
- $f_{\pi} = 0.077(4)$, $B_{0} = 2.22(7)$
Volume Dependence & Scaling

Finite volume ChPT suggests that corrections are negligible

- Van de Water & Sharpe, Becirevic & Villadoro
- quenched CP-PACS hep-lat/0105020 — $\sim 1\%$
- We estimate a volume systematic error of 2%

Estimate scaling effects from CP-PACS quenched DWF result.

- Fit points to form $f(a^2) = a + ba^2$.
- Most likely scaling error (central value) estimated to be 4%.
### Scaling Estimate

**Figure:** estimate scaling by fitting CP-PACS points to the form $f(a^2) = a + ba^2$
Non-Perturbative Renormalisation

Use the RI-MOM [Martinelli et al] renormalisation scheme

- NPR paper in progress

2% continuum PT error & 2% lattice error [Peter Boyle plenary] — $Z_{B_K}^{MS}(2\,\text{GeV}) = 0.919(7)_{\text{stat}}(26)_{\text{sys}}$

$$B_K(\mu) = Z_{B_K}(a\mu)B_{K}^{\text{lat}}(a) + \sum_i Z_i(a\mu)B_i^{\text{lat}}(a)$$

- $m_{\text{res}}$ small enough that we can ignore mixing [Aoki et al, Christ, Sharpe]
Final Result

\( B_K \) at fixed volume and lattice spacing

\[
B_K^{\overline{\text{MS}}} (2 \text{ GeV}) = 0.557(12)_{\text{stat}}(16)_{\text{ren}}
\]

Estimate \( B_K \) in the continuum using volume and scaling systematics

\[
B_K^{\overline{\text{MS}}} (2 \text{ GeV}) = 0.557(12)_{\text{stat}}(16)_{\text{ren}}(22)_{\text{scale}}(11)_{\text{vol}}
\]

\[
= 0.557(12)(29)
\]
Final Result

Figure: Result at fixed volume and lattice spacing compared to others
Final Result

\( B_K \) at fixed volume and lattice spacing

\[
B_K^{\overline{MS}} (2 \text{ GeV}) = 0.557(12)_{\text{stat}}(16)_{\text{ren}} \quad B_K^{\overline{MS}} (2 \text{ GeV})_{243} = 0.522(10)_{\text{stat}}(15)_{\text{ren}}
\]

Estimate \( B_K \) in the continuum using volume and scaling systematics

\[
B_K^{\overline{MS}} (2 \text{ GeV}) = 0.557(12)_{\text{stat}}(16)_{\text{ren}}(22)_{\text{scale}}(11)_{\text{vol}}
\]

\[
= 0.557(12)(29)
\]
Chiral Extrapolation Error

- Can $SU(3)$ ChPT be safely applied?
- Linear fits and NLO $SU(3)$ fits give similar answers.
  - Intricate cancellation of non-analyticity?—seems unlikely
- Points outside region of convergence constrain the fit and weaken $\ln(M_\pi^2)$ coefficient

\[ B_{K}^{\text{NLO fit}}(m_1,0.043) = B_{K}^{\text{NLO fit}}(m_1^{\text{phys}},m_s^{\text{phys}}) \]

\[ B_{K}^{\text{lat}}(am_l^{\text{phys}},0.04) = 0.612(8) \]

\[ B_{K}^{\text{NLO fit}}(am_l^{\text{phys}},0.04) = 0.616(8) \]
• \(SU(3)\) ChPT cannot be applied in the region of the data

• Use \(SU(2) + \text{heavy meson}\) to estimate error (valid with sufficiently heavy strange)

• Form from heavy quark ChPT [Sharpe & Zhang, hep-lat/9510037]
  • errors \(\propto \mathcal{O} \frac{M_\pi^4}{M_K^2} \log(M_\pi^2)\)

\[
B_K = B_K^0 \left[ 1 - \frac{M_\pi^2}{2(4\pi f)^2} \log \left( \frac{M_\pi^2}{\Lambda_Q^2} \right) + c_0 M_\pi^2 \right]. \tag{1}
\]

• Correct chiral behaviour if \(M_\pi << M_K, \Lambda_{QCD}\)
  • \(SU(2)\) ChPT will have correct \(M_\pi \to 0\) form
  • Consistent with \(SU(3)\) formula used previously
- Extrapolate data linearly until $m_q^{\text{min}}$, then match to $SU(2)$ ChPT
  - $m_q^{\text{min}} = 0$: linear extrapolation is correct
  - $m_q^{\text{min}} = 0.013$: data lies just outside region of validity of $SU(2)$
  - Can push up $m_q^{\text{min}}$ providing we maintain consistency with data
- Depending on level of pessimism, estimate chiral extrapolation error to be 2-4%
Comparison with $24^3 \times 64$ volume

Large volume data light enough to fit to $SU(2)$ ChPT

- data beginning to show curvature
Comparison with $24^3 \times 64$ volume

Large volume data light enough to fit to $SU(2)$ ChPT

- Both measurements consistent
Comparison with $24^3 \times 64$ volume

Large volume data light enough to fit to $SU(2)$ ChPT

- Favours 4% chiral extrapolation error
conclusion

- Underestimated systematic error from $SU(3)$ chiral extrapolation
- Matching linear + $SU(2)$ provides a simple and sound method of estimating error
- With the benefit of the large volume result we can see that a $4\%$ systematic error is more reasonable
- Care must be taken: try many extrapolations to understand systematic