Vector and axial vector current in Wilson ChPT

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Lattice 2007 - The XXV International Symposium on Lattice Field Theory
University of Regensburg, July 30 - August 4, 2007
Suppose: 2 massless flavours

\[ \mathcal{L}_{\text{QCD}} \text{ is invariant under } SU(2)_L \otimes SU(2)_R \]

\[ \text{Existence of conserved currents } \partial_\mu j^a_\mu = 0 \]

- Vector current
  \[ j^a_\mu = V^a_\mu = \bar{\psi} \gamma_\mu \frac{\sigma^a}{2} \psi \]

- Axial vector current
  \[ j^a_\mu = A^a_\mu = \bar{\psi} \gamma_\mu \gamma_5 \frac{\sigma^a}{2} \psi \]

2 massive but degenerate flavours \( \rightarrow \) vector current is still conserved

The conserved currents ...

- ... are the Noether currents associated with the chiral symmetries
- ... satisfy chiral Ward identities (“current algebra”)

\[ \partial_\mu j^a_\mu = 0 \]
Currents in continuum ChPT

- Chiral Perturbation theory (ChPT)
  - SSB: $SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_V$
  - Existence of (pseudo) Goldstone bosons: pions
  - Described by an effective low-energy theory:

$$\mathcal{L}_{\text{eff}} = \frac{f^2}{4} \langle \partial_\mu \Sigma (\partial_\mu \Sigma)^\dagger \rangle + \frac{f^2 B}{2} m \langle \Sigma^\dagger + \Sigma \rangle + \ldots$$

- Construction principle:
  Most general Lagrangian compatible with the symmetries (chiral, P, C)

- Vector and axial vector currents are the Noether currents

$$V_\mu^a = \frac{f^2}{4} \langle \sigma^a (\Sigma^\dagger \partial_\mu \Sigma + \Sigma \partial_\mu \Sigma^\dagger) \rangle + \ldots$$

$$A_\mu^a = \frac{f^2}{4} \langle \sigma^a (\Sigma^\dagger \partial_\mu \Sigma - \Sigma \partial_\mu \Sigma^\dagger) \rangle + \ldots$$

Weinberg 1979
Gasser, Leutwyler 1985
Currents in lattice QCD with Wilson fermions

- Various currents ($N_f = 2$, degenerate mass)
  - Vector current
    - Local current
      \[ V_{\mu,\text{Loc}}^{a}(x) = \bar{\psi}(x)\gamma_{\mu} \frac{\sigma^{a}}{2} \psi(x) \]
    - Conserved current (Noether current)
      \[ V_{\mu,\text{Con}}^{a}(x) = \frac{1}{2} \left[ \bar{\psi}(x)\gamma_{\mu} \frac{\sigma^{a}}{2} U_{\mu}(x) \psi(x + a\hat{\mu}) \right] + \ldots \]
  - Axial vector current
    - Local current
      \[ A_{\mu,\text{Loc}}^{a}(x) = \bar{\psi}(x)\gamma_{\mu}\gamma_{5} \frac{\sigma^{a}}{2} \psi(x) \]
    - No (partially) conserved current due to Wilson term

- Local currents need non-trivial renormalization factors $Z_V$, $Z_A$

\[ \ldots \rightarrow \text{Karsten, Smit 1980} \]
Currents in Wilson ChPT

- Wilson ChPT: chiral effective theory at non-zero lattice spacing

- Slightly unusual:
  - Construction of the effective currents corresponding to the local lattice currents
    ('Noether link' does not hold)
  - Correct normalization \(\rightarrow\) Impose the proper renormalization conditions

Sharpe, Singleton 1998
Symanzik effective theory
Symanzik 1983

- Locality + symmetries + dimensional analysis

\[ S_{\text{Sym}} = S_{\text{QCD}} + a \bar{c}_{sw} \int d^4x \overline{\psi}(x)i\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x) + O(a^2) \]

Pauli term

- Most general Lorentz scalar compatible with the symmetries*
- Noether currents are derived from this effective action

Local currents → same procedure as for the action

\[ V_{\mu,\text{Sym},\text{Loc}} = V_{\mu,\text{QCD}} + a \bar{c}_V \partial_\nu \overline{\psi}\sigma_{\mu\nu}T^a\psi + O(a^2) \]

\[ A_{\mu,\text{Sym},\text{Loc}} = A_{\mu,\text{QCD}} + a \bar{c}_A \partial_\mu \overline{\psi}\gamma_\mu\gamma_5T^a\psi + O(a^2) \]

- Most general (axial) vector current compatible with the symmetries*

*EOM have been used
The currents in WChPT

Write down the most general expressions compatible with the symmetries (chiral, P, C) which transform as a vector and axial vector

\[ V^a_\mu = V^{a,\text{ChPT}}_\mu \left( 1 + \frac{4}{f^2} a W^V_\mu \bar{c}_V \langle \Sigma + \Sigma^\dagger \rangle \right) + \text{higher orders} \]

\[ A^a_\mu = A^{a,\text{ChPT}}_\mu \left( 1 + \frac{4}{f^2} a W^{A,1}_\mu \bar{c}_A \langle \Sigma + \Sigma^\dagger \rangle \right) + \ldots \]

\[ \ldots 4a W^{A,2}_\mu \bar{c}_A \partial_\mu \langle \sigma^a (\Sigma - \Sigma^\dagger) \rangle + \text{higher orders} \]

- Set \( a = 0 \) \( \rightarrow \) obtain the continuum ChPT currents
- \( O(a) \) terms parameterized by new low-energy coefficients \( W^V, W^{A,1}, W^{A,2} \)
- higher orders \( \rightarrow \) corrections of order \( am, ap^2, \ldots, a^2 \)

Conserved vector current

\[ V^{a,\text{Con}}_\mu = V^{a,\text{ChPT}}_\mu \left( 1 + \frac{4}{f^2} a W^{45}_\mu \bar{c}_{SW} \langle \Sigma + \Sigma^\dagger \rangle \right) + \text{higher orders} \]
Renormalization

- Local currents require non-trivial renormalization factors

\[ V_{\mu,\text{ren}}^a = Z_V V_{\mu,\text{Loc}}^a \quad A_{\mu,\text{ren}}^a = Z_A A_{\mu,\text{Loc}}^a \]

- Fix Z-factors by imposing the chiral Ward identities

\[ Z_V = \frac{\langle f | V^a_{\text{Con}}(0) | i \rangle}{\langle f | V^a_{\text{Loc}}(0) | i \rangle} \]

or

\[ Z_V \langle \pi^a(p) | V^b_{0,\text{Loc}}(0) | \pi^c(p) \rangle = 2E \]

- Impose the same condition in the chiral effective theory

\[ Z_V = 1 - a \frac{16}{f^2} (W_V \bar{c}_V - W_{45} \bar{c}_SW) + O(a^2) \]

\[ \Rightarrow \text{At LO: local renormalized current = conserved current} \]
Renormalization

- In order to determine $Z_A$ use the chiral Ward identity

$$\int_{\partial R} d\sigma_\mu(x) \epsilon^{abc} \langle f | A^a_{\mu, \text{ren}}(x) A^b_{\nu, \text{ren}}(y) | i \rangle = 2i \langle f | V^c_{\nu, \text{ren}}(y) | i \rangle$$

- holds for $m_{PCAC} = 0$ → numerically feasible with SF boundary conditions

- Impose the same Ward identity in the effective theory with pseudo scalar states

$$Z_A = 1 - \frac{8}{f^2} a \left( [2W_{A,1} + W_{A,2}] \bar{c}_A - W_{45} \bar{c}_{SW} \right) + O(a^2)$$

- no problems with setting $m_{PCAC} = 0$ in the effective theory

Lüscher, Sint, Sommer, Wittig 1997
Jansen et.al. 1995
Application: \( f_\pi \) to one loop including \( O(a) \)

\[
f_\pi = f \left( 1 - \frac{1}{16\pi^2 f^2} [1 + aC_1] M_\pi^2 \ln \frac{M_\pi^2}{\mu^2} + \frac{8}{f^2} M_\pi^2 [L_{45} + aC_2] \right)
\]

- \( O(a) \) corrections parameterized by two unknown coefficients \( C_1, C_2 \)
  (combinations of the low-energy coefficients in the current)

- Setting \( a = 0 \) recovers the continuum ChPT result

- Leading correction is \( O(am) \), not \( O(a) \)
  ➞ consequence of using the renormalized current and \( Z_A \)

- Coefficient of the chiral log is modified by \( O(a) \)
  ➞ chiral log is not an ‘acid test’ for being in the chiral regime

- Rupak & Shoresh have obtained a different result
  (They used the Noether current and ignored the renormalization)

\( f_\pi = f \left( 1 - \frac{1}{16\pi^2 f^2} [1 + aC_1] M_\pi^2 \ln \frac{M_\pi^2}{\mu^2} + \frac{8}{f^2} M_\pi^2 [L_{45} + aC_2] \right) \)
Matching local lattice currents to WChPT needs to take into account:

- Local currents are not Noether currents
  - Construction of effective currents is based on power counting and symmetries
- Finite renormalization of the currents
  - Impose the same renormalization condition that is used in the lattice theory
  - Implies non-trivial $Z$-factors $\neq 1$ in the effective theory

Explicit example: $f_\pi$

- Leading $O(a)$ correction is “taken care off” by the renormalization
  - Similar to pion mass: leading $O(a)$ contributes to the additive mass renormalization only

Sharpe, Singleton 1998