B meson excitations with chirally improved light quarks

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for the Bern-Graz-Regensburg Collaboration

Abstract

We present our latest results for the excitations of static-light mesons on both quenched and unquenched lattices, where the light quarks are simulated using the chirally improved (CI) lattice Dirac operator.
Chirally Improved Light Quarks

A general lattice Dirac operator:

\[
\begin{align*}
\text{Wilson} & \\
&S_1 + S_2 + S_3 + S_4 + \ldots \nonumber \\
+ \gamma_\mu (v_1 - + + v_2 - + + v_3 - + + \ldots) \\
+ \gamma_\mu \gamma_\nu (t_1 + + + \ldots) + \gamma_\mu \gamma_\nu (a_1 - + + \ldots) + \gamma_5 (p_1 - + + \ldots)
\end{align*}
\]

Choose a (large) set of “most relevant” paths and set coefficients to satisfy the Ginsparg–Wilson relation:

\[
\gamma_5 D + D \gamma_5 = D \gamma_5 D
\]

Approximately chiral quarks in the massless limit: more reliable simulations at low quark mass.

Domain-Decomposition Improvement

Propagator ($P$) between regions 1 and 2 estimated using $N$ random sources ($\chi$):

$$P_{12} = -M_{11}^{-1} M_{12} P_{22}$$

$$\approx -M_{11}^{-1} M_{12} \frac{1}{N} \chi_{2}^{n} \chi_{2}^{n\dagger} P$$

$$\approx -\frac{1}{N} \left( M_{11}^{-1} M_{12} \chi_{2}^{n} \right) \left( \gamma_{5} P \gamma_{5} \chi_{2}^{n} \right)^{\dagger}$$

$$\approx -\frac{1}{N} \psi_{1}^{n} \phi_{2}^{n\dagger}.$$

Note: No sources needed in region 1 and those in region 2 should reach region 1 with one application of $M$.

Random sources surrounding one boundary:

Static-Light Correlators

Using different “wavefunctions” for the light-quark source and sink, we construct the following correlators:

\[
C_{ij}(t) = \langle 0 \left| (\bar{Q} O_j q)_t (\bar{q} \bar{O}_i Q)_0 \right| 0 \rangle \\
= \left\langle \sum_x \text{Tr} \left[ \frac{1 + \gamma_4}{2} \prod_{i=0}^{t-1} U_4(x + i\hat{4}) O_j P_{x+4,x} \bar{O}_i \right] \right\rangle,
\]

where \( x \) is in one domain and \( x + t\hat{4} \) is in the other.

We then solve the generalized eigenvalue problem:

\[
\sum_j C_{ij}(t) \nu_j^k = \lambda^k(t, t_0) \sum_j C_{ij}(t_0) \nu_j^k.
\]

For sufficiently large \( t \), the eigenvalues are

\[
\lambda^k(t, t_0) \propto e^{-M_k^t} \left[ 1 + O(e^{-\Delta M_k^t}) \right],
\]

where \( \Delta M_k \) is the energy difference to the closest state.

We use bilinears of the form:

\[ \bar{Q} O(\Gamma, D_i) (\bar{D}^2)^n S_J(\kappa, N_{sm}) q, \]

where \( S_J \) is a gauge-covariant (Jacobi) smearing function and we apply \( n = 0, 1, \) or 2 Laplacians. We also include the local source to obtain a \( 4 \times 4 \) correlator matrix for each set of quantum numbers, determined by \( O(\Gamma, D_i) \):

<table>
<thead>
<tr>
<th>oper.</th>
<th>( J^P )</th>
<th>( O(\Gamma, D_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S )</td>
<td>( 0^-, 1^- )</td>
<td>( \gamma_5 )</td>
</tr>
<tr>
<td>( P_- )</td>
<td>( 0^+, 1^+ )</td>
<td>( \sum_i \gamma_i D_i )</td>
</tr>
<tr>
<td>( P_+ )</td>
<td>( 1^+, 2^+ )</td>
<td>( \gamma_1 D_1 - \gamma_2 D_2 )</td>
</tr>
<tr>
<td>( D_{\pm} )</td>
<td>( 1^-, 2^-, 3^- )</td>
<td>( \gamma_5(D_1^2 - D_2^2) )</td>
</tr>
</tbody>
</table>

We present results from three sets of (Lüscher-Weisz gauge) configurations; two quenched (Hyp-blocked links), one \( N_f = 2 \) dynamical (Stout links):

| \( N_S^3 N_T \) | \( a \) (fm) | \( M_{\pi,\text{sea}} \) | \( N_{\text{conf}} \) | \( (n, \kappa, N_{sm}) \) |
|-----------------|************|************|**********|************|
| \( 12^324 \) | 0.20 | \( \infty \) | 200 | \( (0\,0,\,0.2,\,0\,8,\,12\,16) \) |
| \( 16^332 \) | 0.15 | \( \infty \) | 100 | \( (0\,0,\,0.2,\,0\,12,\,18\,24) \) |
| \( 16^332 \) | 0.16* | 450 MeV* | 33 | \( (0\,0,\,0.2,\,0\,12,\,18\,24) \) |

* For more details, see talk of C. B. Lang and poster of M. Joergler.
$B_s$ S- and D-waves (quenched):

$B_s$ P-waves (quenched):
**Effective Masses II**

$B_s$ S-waves ($N_f = 2$, $M_{\pi, \text{sea}} \approx 450$ MeV):

- Unlike quenched case, significant change for $2S$ results by “pruning” basis to first 3 operators.
- Statistics (33 configs.) are too small here.

$B_s$ P-waves ($N_f = 2$, $M_{\pi, \text{sea}} \approx 450$ MeV):
Chiral Extrapolations

\( a = 0.15 \text{fm} \) quenched and \\
\( a = 0.16 \text{fm} \) dynamical (black symbols at \( m_s \)):

Green symbols are experimental values from PDG \( (J^P \text{ needs confirmation}) \).

$B_s$ Mass Splittings

<table>
<thead>
<tr>
<th>state</th>
<th>$M - M_{1S}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_{\pi,\text{sea}} = \infty$</td>
</tr>
<tr>
<td></td>
<td>$a \approx 0.20$ fm</td>
</tr>
<tr>
<td>$2S$</td>
<td>684(14)</td>
</tr>
<tr>
<td>$1P_-$</td>
<td>395(4)</td>
</tr>
<tr>
<td>$2P_-$</td>
<td>945(20)</td>
</tr>
<tr>
<td>$1P_+$</td>
<td>400(7)</td>
</tr>
<tr>
<td>$2P_+$</td>
<td>985(20)</td>
</tr>
<tr>
<td>$1D_{\pm}$</td>
<td>730(12)</td>
</tr>
<tr>
<td>$2D_{\pm}$</td>
<td>1210(30)</td>
</tr>
</tbody>
</table>

Errors are only statistical, scale set using $r_0 = 0.5$ fm.

$a = 0.20$ fm quenched data:

![Graph showing mass splittings](image-url)
Conclusions / Outlook

- Quark propagator estimation between regions is greatly improved.
- Successful isolation of excited static-light mesons via variational method.
- A number of excited states “found”: $2S$, $1P$, $2P$, $1D$, $2D$.
- Assuming orbital excitation for $B_{sJ}^*$, our $1P-1S$ splittings are too low.
- Mass splittings appear to rise as we approach the (quenched) chiral limit (?).
- No discernible difference between our quenched and dynamical $B_s$ mesons... yet.
- $1P_+ - 1P_-$ splitting quite small; barely resolved on $16^3$ quenched lattice: $\sim 20$ MeV.
- Better statistics are needed to settle matters...

For the future:

- Propagators on one more quenched lattice at $L \approx 2.4$ fm are in progress ($20^3 \times 40$).
- More dynamical CI lattices are on the way...

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