Vortex topology and the continuum limit of LGT

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WARNING!!!

This is a
GAUGE INVARIANT
talk
Plan of the talk:

- Introduction
- Bulk transitions: SO(3) vs. SU(2)
- Bulk transitions: 4d, 3d and 2d
- $\mathbb{Z}_2$ Monopoles and Vortices
- 3d at T=0: a new bulk transition
- Order and critical exponents?
- Conclusions & Outlook
Introduction

Original motivations:

- **Universality**: $\text{SU}(N) \simeq \text{SU}(N)/\mathbb{Z}_N$ for pure YM?
- **4d SO(3)**: Vortex free energy $F'(T) < 0$ below $T_c$
  [GB, Fuhrmann, Kerler, Müller-Preussker (2006)]
- **Crosscheck**: what happens in lower dim?

Start from $\beta_A - \beta_F$ phase diagram at $T=0$...
Believed to be simpler...
  - **Unexpected result**: more complex than up to now assumed!!

Forget finite $T$! Important consequences:

- **For the continuum limit of 2, 3d SU(2)**
- **Some caveat about 4d should be learned**
Bulk transitions: SO(3) vs. SU(2)

\[ S = \beta_A \sum_P \left( 1 - \frac{1}{3} Tr_A U_P \right) + \beta_F \sum_P \left( 1 - \frac{1}{2} Tr_F U_P \right) \]

\[ \frac{1}{g^2} = \frac{1}{4} \beta_F + 2 \frac{3}{3} \beta_A \]

[Bhanot, Creutz (1981); Greensite (1981)]; similar for SU(N), N\geq 3
\[ \sigma_P = \text{sign} U_P \in \mathbb{Z}_2 \text{ for each plaquette} \]

\[ M = 1 - \left\langle \frac{1}{N_c} \sum_c \sigma_c \right\rangle \quad \sigma_c = \prod_{P \in \partial c} \sigma_P \in \text{SO}(3) \]

\[ E = 1 - \left\langle \frac{1}{N_l} \sum_l \sigma_l \right\rangle \quad \sigma_l = \prod_{P \in \partial l} \sigma_P \in \text{SU}(2) \]

1\text{st order bulks} caused by \( \mathbb{Z}_2 \) local artifacts;

[Halliday, Schwimmer (1981)]
Bulk transitions: 3d...

$Z_2$ gauge theory

3d SU(2)

[Baig, Cuervo (1986)]
... and 2d

2d SU(2)
**$\mathbb{Z}_2$ Monopoles and Vortices**

*Pure YM at $T \neq 0$ in continuum allows large gauge transf.*

- Represented by so-called twist matrices $\Omega$
- Define topological sectors ($\mathbb{Z}_N$ vortices)
- (Evolving) lines in 3+1, points in 2+1.
- Instanton-like objects in 1+1
- Lattice quenched theory only allows one sector (fixed by b.c).
- Adjoint theory with p.b.c. compatible with all sectors
- (maximal) 't Hooft loop measures $\mathbb{Z}_N$ magnetic flux
extended vortex ↔ maximal ’t Hooft loop ↔ temporal twist

\[ z_i = \frac{1}{N_s^{d-1}} \sum_{j,k \neq i} \prod_{x \in (i,t) \text{ plane}} \text{sign}(\text{Tr}_f U_{i,t}(x)) \]

\( z_i \neq 1 \) only with t.b.c. (in any dim)!

SO(3) in phase II automatically includes all t.b.c.!

[Mack, Petkova (1979); Tomboulis (1981); de Forcrand, Jahn (2003)]

\[ \sum_{\text{b.c.}} Z_{\text{SU}(2)} = \int (DU) e^{-S_{\text{SO}(3)}} \prod_c \delta(\sigma_c - 1) = Z_{\text{SO}(3)} \big|_{\text{phase II}} \]
$\mathbb{Z}_N$ magnetic monopoles:

- source of open $\mathbb{Z}_N$ magnetic vortices
- particles in 3+1, “instantons” in 2+1
- Abelian monopole of charge $n$ carries $\mathbb{Z}_N$ monopole mod$_N(n)$
  [Lubkin (1962), Coleman (1982)]

Only abelian monopoles of charge $\propto N$ compatible with closed $\mathbb{Z}_N$ vortices [Fröhlich, Marchetti 2000]

In a $\mathbb{Z}_N$ monopole background stable vortices submerged in open vortex background.

$z_i$ follow monopoles... undefined below transition/crossover!!!

What happens in 2, 3d? Simulations hard! Concentrate on $\beta_F$
\[ z = \frac{1}{d} \sum_i |z_i| \]
Order and critical exponents?

How come never seen?
$C_s$ bounded and small!
Need much higher statistics...
Not 1st, not “standard”
2nd, $C_s \propto |\beta - \beta_c|^{-\alpha/\nu}$

Cannot exclude discontinuous 2nd ($\alpha = 0$) or higher (3nd etc.)
Discontinuous transition implies $\nu = n/d = 2/3$
see e.g. [Janke, Johnston, Kenna (2006)]
Fit $\chi_{\text{max}}(L) \propto L^\gamma/\nu$, cross fingers and try your luck with FFS...
Works reasonably well with 
\( \beta^c_F = 7.26(9) \)
\( \gamma = 1.57(7) \)

Next scaling relation for \( z \), assuming 2\(^{nd} \) order and \( \alpha = 0 \)
\( \beta = 1 - \frac{\gamma}{2} \). Again, try your luck...
Works even better!!!

\[ \nu = \frac{2}{3} \]

\[ \beta = 1 - \gamma/2 = 0.21(3) \]
$Z_2$ gauge theory

3d SU(2)
2d at T=0

Simulations up to $\beta_{A,F} = O(100)$, $V = 1024^2$ still give $\langle z \rangle = 0$...

Expected for $\beta_A$; $\beta_F$ should have $\langle z \rangle = 1$...

$\Rightarrow$ No continuum limit?!?!

Transition caused by breaking of discrete symmetry?

Gauge freedom: 2d gauge $\Rightarrow$ 1d Mermin-Wagner!!!

More work needed... In SO(3) is easy to conceive symmetries for which $z$ is order parameter. In SU(2) not so trivial...
care needed in 4d

$Z_2$ gauge theory

4d SU(2)
Conclusions

• Bulk transition in 3d SU(2) $\beta_A - \beta_F$
• Data allow discontinuous 2$^{nd}$ along $\beta_F$
• $\beta_F^c = 7.27(9)$: quite high!
• Consequences for continuum limit!
• Might need reconsider many calculations!
• 2d SU(2) always disordered phase?
• Does it have a continuum limit?
Outlook

- End point for $\beta_A < 0$?
- Bad critical slowing down for $z$. Strategies?
- Action doesn’t see slow modes: reweighting possible
- Fisher zeros? Better determination of $C_s$?
- 2d Results need further work!
- Extension to SU(3)!!