Topological defects and equation of state of gluon plasma

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in collaboration with

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Plan

- Thermodynamics of Yang–Mills theory
- Models of color confinement at $T < T_c$:
  - Abelian monopoles
  - Center vortices
- In deconfinement (gluon plasma at $T > T_c$):

  **Question:** Are they (still) alive as real* objects?
  *“Real” means REAL gluonic objects in plasma.
  
  **Check:** Contribution to (trace of) energy–momentum tensor
  - from Abelian monopoles (found in simulations!)
  - from Center vortices (found in simulations!)

  **Answer:** Yes! They are still alive at $T > T_c$

  **Proposal:** As plasma quickly cools down, vortices and monopoles
  must coherently decay into white particles

  **Dream:** Detectable at RHIC, LHC/ALICE, GSI experiments (?)
Thermodynamics

- **Free Energy** \((T\text{ is temperature and } V\text{ is spatial volume})\)

\[
F = -T \log Z(T, V)
\]

- **Pressure**

\[
p = \frac{T \partial \log Z(T, V)}{V} = \frac{-F}{V} = \frac{T}{V} \log Z(T, V)
\]

- **Energy density**

\[
\varepsilon = \frac{T \partial \log Z(T, V)}{V} \partial \log T
\]

- **Entropy density**

\[
s(T) = \frac{\varepsilon + p}{T} = \frac{\partial p(T)}{\partial T}
\]
Thermodynamics: Trace Anomaly

• Trace anomaly of the energy–momentum tensor $T_{\mu\nu}$

$$\theta(T) = \langle T^{\mu}_{\mu} \rangle \equiv \epsilon - 3p = T^5 \frac{\partial p(T)}{\partial T} \frac{T}{T^4}$$

• Trace anomaly is vanishing, $\theta = 0$,

(i) if excitations are massive, $M \gg T$, since $\epsilon \sim p \sim \exp\{-M/T\}$
(ii) if particles are massless and non-interacting, $\epsilon = 3p$

• Pressure via trace anomaly

$$p(T) = T^4 \int_{T_1}^{T} \frac{dT_1}{T_1} \frac{\theta(T_1)}{T_1^4}$$

• Energy density via trace anomaly

$$\epsilon(T) = 3T^4 \int_{T_1}^{T} \frac{dT_1}{T_1} \frac{\theta(T_1)}{T_1^4} + \theta(T)$$

• Trace anomaly is a key quantity
Trace Anomaly for pure gluons

- Partition Function

\[ Z(T, V) = \int DU \exp\{-\beta \sum_P S_P[U]\}, \quad S_P[U] = (1-\frac{1}{2} \text{Tr} U_P) \]

- Trace Anomaly

\[ \theta(T) = T^5 \frac{\partial}{\partial T} \frac{\log Z(T, V)}{T^3 V} \]

- Asymmetric \( N_s^3 N_t \) lattice:

\[ T = \frac{1}{N_t a}, \quad V = (N_s a)^3 \]

- Trace anomaly on the lattice

\[ \frac{\theta(T)}{T^4} = 6 N_t^4 \left( \frac{\partial \beta(a)}{\partial \log a} \right) \cdot (\langle S_P \rangle_T - \langle S_P \rangle_0) \]
Mechanisms of color confinement, $T < T_c$

- **Dual superconductor picture**
  \([’t\ Hooft, Mandelstam, Nambu, ’74–’76]\)
  - Based on existence of special gluonic configurations, called “magnetic monopoles”
  - Monopoles are classified with respect to the Cartan subgroup \([U(1)]^{N-1}\) of the \(SU(N)\) gauge group
  - Confinement is due to monopole condensation

- **Center vortex mechanism**
  \([Del\ Debbio, Faber, Greensite, Olejnik, ’97]\)
  - A realization of spaghetti (Copenhagen) vacuum
  - Center strings are classified with respect to the center \(\mathbb{Z}_N\) of the \(SU(N)\) gauge group
  - Confinement is due to vortex percolation
Center/Monopole mechanisms are linked

- non-oriented half-flux of magnetic field
- monopoles are at points at which the flux alternates
- vortices are chains of monopoles
  
  [Ambjorn, Giedt, Greensite, ’00]
- a similar string–monopole structure appears also in SUSY models
  
  [Hanany, Tong, ’03; Auzzi et al ’03]
  and in non-SUSY theories
  
  [Feldmann, Ilgenfritz, Schiller & Ch. ’05]
  [Gorsky, Shifman, Yung, ’04 ... ’07]
- required analytically [Zakharov ’05]
Confinement \((T < T_c)\) and plasma \((T > T_c)\)

- The vortices and monopoles are percolating in confining vacuum
- Must emerge as a real (thermal) component of deconfinement plasma
  
  \[ \text{[V.I. Zakharov, M.N.Ch., Phys. Rev. Lett. 98, 082002 (2007)]} \]
  
  \[ \text{[Related ideas were also suggested by Liao and Shuryak, ’06–’07]} \]

- Similar to electrically neutral electron–positron plasma:
  - individual particles exist at high temperatures in a heat bath
  - annihilate at low temperatures, but still present in the vacuum

- **Check**: if the suggestion is true, then the monopoles and the vortices must contribute to the equation of state of the gluon plasma
Trace Anomaly from monopoles

- Fix Maximal Abelian gauge $D^\text{diag}_\mu A^\text{off}_\mu = 0$
- Define particular singular gluon objects (monopoles) $k_\mu = \partial_\nu \tilde{F}^\text{diag}_{\mu\nu}$
- Determine the monopole action by inverse Monte Carlo algorithm [Shiba, Suzuki '95]

- Partition function and the Trace of energy–momentum tensor:

$$Z = Z^{\text{mon}} Z^{\text{rest}} , \quad \theta = \theta^{\text{mon}} + \theta^{\text{rest}}$$

- Monopole partition function:

$$Z^{\text{mon}} = \sum_{\text{monopoles}, k} \exp \left\{ - \sum_{x} \sum_{i=1}^{n} f_i(\beta) S^{\text{mon}}_i(k) \right\}$$

- Contribution of monopoles into the trace anomaly:

$$\theta^{\text{mon}} = N_t^4 \left( a \frac{\partial \beta}{\partial a} \right) \sum_i \left( \frac{\partial f_i(\beta)}{\partial \beta} \right) \left[ \langle S^{\text{mon}}_i \rangle_T - \langle S^{\text{mon}}_i \rangle_0 \right]$$
Are monopoles thermodynamically important?

- Trace anomaly of energy momentum tensor

pure SU(3) glue

contribution from monopoles

\[
\frac{(\epsilon-3\rho)}{T^4}
\]

\begin{itemize}
  \item \(16^3 \times 4\)
  \item \(32^3 \times 6\)
  \item \(32^3 \times 8\)
\end{itemize}


Yes, there is a contribution
Trace Anomaly from vortices

• Fix Maximal Center gauge $\min_{\Omega} (\text{Tr} U_{x\mu}^{(\Omega)})^2$

• Define the singular links $Z_l = \text{sign} U_l$

• Define singular string-like gluon objects (vortices) with the worldsheet current $\sigma_P = \prod_{l \in \partial P} Z_l$

• Separate all plaquettes into two sets:
  i) $\sigma_P = -1$ (belong to the vortices)
  ii) $\sigma_P = +1$ (outside the vortices)

• Action splits trivially:
  $$\sum_P S_P = \sum_{P \in \text{vort}} S_P + \sum_{P \notin \text{vort}} S_P$$

• Trace anomaly splits as well:
  $$\theta = \theta^{\text{vort}} + \theta^{\text{outside}}$$
Are vortices thermodynamically important?

- Trace anomaly of energy momentum tensor

pure SU(2) glue

contribution from vortices


from [A.Gorsky, V.I.Zakharov, '07]
• Vortex–like and monopole–like magnetic gluonic configurations are present in the gluon plasma.

• Found: strong contributions from the Abelian monopoles and from the Center vortices to the trace anomaly, and, consequently, to the pressure and to the energy density of the gluon plasma.

Proposal: As plasma quickly cools down, vortices and monopoles must coherently decay into white particles

Dream: Detectable at RHIC, LHC/ALICE, GSI experiments (?)