The complete lowest order chiral Lagrangian from a little box

T. DEGRAND
Department of Physics, University of Colorado, Boulder, CO 80309, USA
S. SCHAEFER
NIC, DESY Zeuthen, Platanenallee 6, D-15738 Zeuthen, Germany

Abstract
Recent advances in Random Matrix Theory enable one to determine the pseudoscalar decay constant from the response of eigenmodes of quenched fermions to an imaginary isospin chemical potential. We perform a pilot test of this idea, from simulations with two flavors of dynamical overlap fermions.

1 Statement of the Problem
The leading-order chiral effective Lagrangian is
\[ \mathcal{L}_\text{eff} = \frac{g^2}{4} \text{Tr} \left( \frac{D^2}{G^2} \right) - \frac{g^2}{4} \text{Tr} \left[ \Psi \left( \partial^2 + m^2 \right) \Psi \right] \]
\( \Sigma \) and \( F \) are lattice targets:
- \( \Sigma \) can be computed using eigenmodes of \( \Psi \) – this can be done cheaply in small volumes
- \( F \) controls rise

2 Beautiful Theory
Connected correlation function between eigenvalues of the Dirac operator computed in the presence of a quenched chemical potential \( \lambda_q \) and ordinary eigenvalues computed at zero chemical potential, \( \Lambda \)
\[ \rho^{(1)}_{\text{bare}}(\varepsilon, \lambda) = \frac{1}{2} \sum_{\lambda' \neq \lambda} \delta(\varepsilon - \lambda + \lambda') \]

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Horrible RMT formula, too long for a poster, gives \( \rho^{(1)}_{\text{bare}} \) in terms of the usual rescaled variables \( \varepsilon' = \frac{\varepsilon}{\Sigma V}, \lambda' = \frac{\lambda}{\Sigma V} \), the rescaled mass \( m = m \Sigma V \), and the the rescaled isospin chemical potential \( \mu = \mu' \Sigma V \) is the volume.
To deal with statistics, finite number of eigenmodes, etc, we integrate
\[ C_n(\varepsilon_n, \lambda_n) = \int_{0}^{\infty} d\rho(\varepsilon + \rho, \lambda) \]
\[ f(\varepsilon, \lambda) = \int_{0}^{\infty} d\rho(\varepsilon + \rho, \lambda) \]
Hampered by statistics, we’ll fit \( f(\varepsilon) \).

3 Procedure
- Simulate QCD in a small volume, hopefully in epsilon regime (we are not)
- Compute eigenmodes \( \alpha, \beta \) of \( \Psi \)
- Introduce imaginary isospin \( \mu \)
- Compute eigenmodes \( \psi_i, \phi_i \) of \( \Phi = \psi \gamma \mu \)
- Correlator of \( \alpha, \beta \) gives a \( R_x, \alpha \Sigma \) in finite volume

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4 \( N_f = 2 \) Simulations
(Yes, we know, it’s not physical…)
Overlap fermions–
- Easy to simulate
  - Fodor-Katz-Szabo reflection/refraction
  - \( n \)FPT links for the fermions
  - multiple Hasenbusch-preconditioning
- Easier to analyse
  - Continuum-like chiral symmetry (with anomaly)
  - Obvious assignment of topology (which RMT needs)
  - Straightforward connection of lattice eigenmodes to continuum (Mohsen)

5 Tests and Examples
A typical fit – errors from bootstrap – \( \nu = 1, \langle \Sigma \rangle = 0.0 \)

6 Finite Volume Corrections
In ChPT, all quantities exhibit finite volume corrections
\[ \Delta Q = \frac{g \psi(\Sigma V) = m \Sigma V}{\Sigma V} \]
where \( \Delta \Sigma \) is the sum of image corrections to the tadpole.
In the regime \( \| \Delta \| = \frac{g^2}{\Sigma V} \ll 1 \), we can replace \( \psi \) and \( \Sigma V \) by \( \psi \Sigma^2 V \) and \( \Sigma V \)

7 Pulling Results Out
- Lattice spacing from Sommer parameter
- \( T \Sigma \) needs \( \mu \) (from IFSM analysis) \( T \Sigma = (3.71(5)) \)
- \( \Sigma = \Sigma_{\text{NLO}} \)
- \( \mu = \mu_{\text{NLO}} \)
Note: no extrapolation in \( m \) needed

8 Summary
With our statistics, we find (combining results from \( \nu = 0 \) and \( \nu = 1 \))
\[ n_{\Sigma}^{(0)} \rightarrow 1 = 0.00(0.1) \]
\( n_{\mu}^{(0)} = 0.2 \pm 0.1 \)
or
\[ \langle \Sigma \rangle_{\text{phys}} = (3.71(5)) \]
\[ \langle \mu \rangle_{\text{phys}} = (3.71(5)) \]
Not bad – we expect 8 MeV.
An independent (though of course correlated) determination of \( \Sigma \)– quite consistent with eigenmode cumulants.
Our \( \mu \) not really competitive with big simulations but we used \( 10^4 \) their resources.
All lattice numbers are ephemeral – if you have a bigger box, you can beat us easily. (Please reference us!)

9 The Bottom Lines
- This is really cheap! But–
  - You need “good RMT” eigenmodes to even try to do this
  - More statistics than we have would help, too
- So would an RMT prediction for INDIVIDUAL mode correlators
  - Are there more RMT tricks out there, for other LEC’s?