The art of smearing

Can one reach $M_\pi = 140\text{ MeV}$ (quenched) with clover quarks?

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Introduction: Why smear?

$4^4, \beta = 5.4$

- $c_{SW} = 0$, none
- $c_{SW} = 1$, 3HYL
Overview: APE, stout=EXP, n-APE, LOG

- **APE**: [Michael/Albanese et al. 1987]

\[
U^\text{APE}_\mu(x) = P_{SU(3)} \left\{ (1-\alpha)I + \frac{\alpha}{2(d-1)} \sum_{\pm \nu \neq \mu} U_\nu(x)U_\mu(x+\hat{\nu})U_\nu^\dagger(x+\hat{\mu})U_\mu^\dagger(x) \right\} U_\mu(x)
\]

- **EXP**: [Morningstar_Peardon 2004]

\[
U^\text{EXP}_\mu(x) = \exp \left( \frac{\alpha}{2} \sum_{\pm \nu \neq \mu} \{ [U_\nu(x)U_\mu(x+\hat{\nu})U_\nu^\dagger(x+\hat{\mu})U_\mu^\dagger(x) - \text{h.c.}] - \frac{1}{3} \text{Tr}[.] \} \right) U_\mu(x)
\]

- **n-APE**: [Hasenfratz_Hoffmann_Schaefer 2007] ... back-project to \( U(3) \) only

- **LOG**: [forthcoming] ... like EXP, but average taken consistently in Lie Algebra

\[
U^\text{LOG}_\mu(x) = \exp \left( \frac{\alpha}{2(d-1)} \sum_{\pm \nu \neq \mu} \text{tflog}[U_\nu(x)U_\mu(x+\hat{\nu})U_\nu^\dagger(x+\hat{\mu})U_\mu^\dagger(x)] \right) U_\mu(x)
\]
LOG: principal versus trace-free logarithm

Problem: on arbitrary background $\text{Tr} \log(U_{\mu\nu}(x))$ is not zero, just zero (mod $2\pi i$).

Solution_1:

$$U'_\mu(x) = \exp\left(\frac{\alpha}{2(d-1)} \sum_{\pm \nu \neq \mu} \left\{ \log[U_{\nu}(x)U_{\mu}(x+\hat{\nu})U_{\nu}^\dagger(x+\hat{\mu})U_{\mu}^\dagger(x)] - \frac{1}{3} \text{Tr} \log[.] \right\} \right) U_{\mu}(x)$$

Solution_2:

$$U''_{\mu}(x) = \exp\left(\frac{\alpha}{2(d-1)} \sum_{\pm \nu \neq \mu} c_{\pm \nu} \left\{ \log[U_{\nu}(x)U_{\mu}(x+\hat{\nu})U_{\nu}^\dagger(x+\hat{\mu})U_{\mu}^\dagger(x)] - \frac{1}{3} \text{Tr} \ldots \right\} \right) U_{\mu}(x)$$

Solution_3:

$$U'''_{\mu}(x) = \exp\left(\frac{\alpha}{2(d-1)} \sum_{\pm \nu \neq \mu} \text{tflog}[U_{\nu}(x)U_{\mu}(x+\hat{\nu})U_{\nu}^\dagger(x+\hat{\mu})U_{\mu}^\dagger(x)] \right) U_{\mu}(x)$$

Solution_4:

Add constraint to $S_G$ to guarantee that the principal logarithm of all plaquettes is trace-free; a sufficient condition is $\text{Re} \text{Tr}(U_{\mu\nu}(x)) > -1$
The hypercubic nesting trick with APE as core recipe [Hasenfratz_Knechtli_2001]

\[ \tilde{V}_{\mu,\nu}(x) = P_{SU(3)} \left\{ (1 - \alpha_3) I + \frac{\alpha_3}{2} \sum_{\pm \sigma \neq \mu, \nu, \rho} U_\sigma(x) U_\mu(x + \sigma) U_\sigma^\dagger(x + \mu) U_\mu^\dagger(x) \right\} U_\mu(x) \]

\[ \tilde{V}_{\mu,\nu}(x) = P_{SU(3)} \left\{ (1 - \alpha_2) I + \frac{\alpha_2}{4} \sum_{\pm \rho \neq \mu, \nu} \tilde{V}_{\rho,\nu}(x) \tilde{V}_{\nu,\rho}(x + \rho) \tilde{V}_{\rho,\nu}^\dagger(x + \mu) U_\mu^\dagger(x) \right\} U_\mu(x) \]

\[ U_\mu^{HYP}(x) = P_{SU(3)} \left\{ (1 - \alpha_1) I + \frac{\alpha_1}{6} \sum_{\pm \nu \neq \mu} \tilde{V}_{\nu,\mu}(x) \tilde{V}_{\nu,\mu}(x + \nu) \tilde{V}_{\nu,\mu}^\dagger(x + \mu) U_\mu^\dagger(x) \right\} U_\mu(x) \]

can be extended to the EXP [cf. CDH_2006] and LOG recipes

\[ \tilde{V}_{\mu,\nu}(x) = \exp \left\{ \frac{\alpha_3}{2} \sum_{\pm \sigma \neq \mu, \nu, \rho} \text{tflog} \left[ U_\sigma(x) U_\mu(x + \sigma) U_\sigma(x + \mu)^\dagger U_\mu(x)^\dagger \right] \right\} U_\mu(x) \]

\[ \tilde{V}_{\mu,\nu}(x) = \exp \left\{ \frac{\alpha_2}{4} \sum_{\pm \rho \neq \mu, \nu} \text{tflog} \left[ \tilde{V}_{\rho,\nu}(x) \tilde{V}_{\nu,\rho}(x + \rho) \tilde{V}_{\rho,\nu}^\dagger(x + \mu) U_\mu^\dagger(x) \right] \right\} U_\mu(x) \]

\[ U_\mu^{HYL}(x) = \exp \left\{ \frac{\alpha_1}{6} \sum_{\pm \nu \neq \mu} \text{tflog} \left[ \tilde{V}_{\nu,\mu}(x) \tilde{V}_{\nu,\mu}(x + \nu) \tilde{V}_{\nu,\mu}^\dagger(x + \mu) U_\mu^\dagger(x) \right] \right\} U_\mu(x) \]

and we use the names APE $\rightarrow$ HYP, EXP $\rightarrow$ HEX, LOG $\rightarrow$ HYL.
Plaquette distribution

Wilson gauge action, $32^4$, $\beta = 5.8$

Distribution resembles black-body radiation: $\rho \sim s^p$ and $\rho \sim e^{-\text{const} \cdot s}$ at low/high $s$

$32^4$, $\beta = 5.8$, perturbative equivalents of $\alpha_{\text{APE}} = 0.6$

$32^4$, $\beta = 5.8$, perturbative equivalents of $\alpha_{\text{HYP}} = (0.75, 0.6, 0.3)$

$\rightarrow$ LOG at least as good as APE, while EXP is less effective than these recipes
Creutz ratios with APE/EXP/LOG smearing

Wilson gauge action, $16^3 \times 32, \beta = 6.0$

Observable: $-\log(\chi(r, r))$ versus $r$ after 1 step of APE/EXP/LOG/HYP/HEX/HYL

$16^3 \times 32, \beta = 6.0$, perturbative equivalents of $\alpha_{\text{APE}} = 0.6$

$16^3 \times 32, \beta = 6.0$, perturbative equivalents of $\alpha_{\text{HYP}} = (0.75, 0.6, 0.3)$

LOG and APE equivalent, while EXP is less effective than these recipes
Interlude: technical equipment ...

All data presented in this talk together take $O(1 \text{ PC-year})$. 
Chiral symmetry breaking

Summary of [eg. CDH 2006]: *combining* clover-improvement and link-fattening yields low-cost “chirally improved” fermions. How well does LOG/HYL perform?

Idea: measure $a m_{\text{PCAC}}$ at $a m_0 = 0$.

$$a m_{\text{PCAC}} = \frac{(\partial_4 + \partial_4^\ast)\langle A_4(x)P(0)\rangle}{4\langle P(x)P(0)\rangle}$$

- **Action and algorithm details** ($N_f = 0$)

  Wilson gauge action; APE/EXP/LOG fat links, w/out hypercubic nesting, $c_{SW} = 1$; CGNE, CGH, BCG$_5 \gamma_5$ [kind$=16$ sum in $(.\gamma_5.)$], w/out EO-preconditioning, $||r|| \leq 10^{-8}$. 

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Explicit tests with APE/EXP/LOG smeared links
• **Summary of preliminary** $a m_{\text{res}}$ **values** (defined as $a m_{\text{PCAC}}$ at $a m_0 = 0$)

<table>
<thead>
<tr>
<th>$(\beta, L/a)$</th>
<th>$(5.6, 08)$</th>
<th>$(5.8, 12)$</th>
<th>$(6.0, 16)$</th>
<th>$(6.2, 20)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L/r_0$</td>
<td>3.48</td>
<td>3.27</td>
<td>2.98</td>
<td>2.98</td>
</tr>
<tr>
<td>n_conf</td>
<td>256</td>
<td>128</td>
<td>64</td>
<td>32</td>
</tr>
<tr>
<td>1 APE</td>
<td>$0.4354(23)$</td>
<td>$0.2615(12)$</td>
<td>$0.1923(08)$</td>
<td>$0.1582(12)$</td>
</tr>
<tr>
<td>1 HYP</td>
<td>0.1895(14)</td>
<td>0.0934(09)</td>
<td>0.0611(04)</td>
<td>0.0479(06)</td>
</tr>
<tr>
<td>1 EXP</td>
<td>0.5461(27)</td>
<td>0.3413(14)</td>
<td>0.2563(09)</td>
<td>0.2131(13)</td>
</tr>
<tr>
<td>1 HEX</td>
<td>0.3125(21)</td>
<td>0.1584(11)</td>
<td>0.1025(06)</td>
<td>0.0780(07)</td>
</tr>
<tr>
<td>1 LOG</td>
<td>$0.4296(23)$</td>
<td>$0.2579(12)$</td>
<td>$0.1897(07)$</td>
<td>$0.1564(12)$</td>
</tr>
<tr>
<td>1 HLO</td>
<td>0.1905(15)</td>
<td>0.0928(09)</td>
<td>0.0603(04)</td>
<td>0.0472(05)</td>
</tr>
</tbody>
</table>

LOG at least as good as APE (with perturbatively equivalent parameter)

• **Degradation towards lighter quark masses** (always $a m_0 = 0$)
• **My lightest pion** $22^3 \times 44$, $\beta = 6.3$, 3 HYL: $m_{PCAC} = 0.0059 \cdot 3.4 \text{ GeV} \simeq 20 \text{ MeV} \ (M_\pi \simeq 310 \text{ MeV})$

• **With cheating** ... $20^3 \times 40$, $\beta = 6.2$, 7 HYL: $m_{PCAC} = 0.0019 \cdot 2.9 \text{ GeV} \simeq 5.5 \text{ MeV} \ (M_\pi \simeq 160 \text{ MeV})$
Serious (1 LOG and 1 HYL) data: compare with 1-loop LPT

\[ m_{\text{res}} = \left| m_{\text{crit}} \right| \frac{Z_P}{Z_A} \frac{Z_P}{Z_S} \quad \text{for } c_{\text{SW}} = 1 \text{ and } \alpha_{\text{APE}} = 0.6, \alpha_{\text{HYP}} = (0.75, 0.6, 0.3) \text{ is known:} \]

\[ \left| m_{\text{crit}} \right| = \frac{g_0^2}{16\pi^2} C_F S = \frac{g_0^2}{12\pi^2} \begin{cases} 4.90876 & \text{1 APE/EXP/LOG} \quad \text{[CDH 2006]} \\ 1.98381 & \text{1 HYP/HEX/HYL} \quad \text{[CDH 2006]} \end{cases} \]

\[ \frac{1}{Z_A} \frac{Z_P}{Z_S} = 1 + O(g_0^2) \quad \text{...suggesting} \quad f(g_0^2) = \begin{cases} 4.90876 \quad \frac{g_0^2}{12\pi^2} \frac{1 + f_1 g_0^2}{1 + f_2 g_0^2} & \end{cases} \]

\[ \alpha = 0, \, c_{\text{SW}} = 0 \quad \alpha = 0, \, c_{\text{SW}} = 1 \]

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Summary

- Smearing is medicine against UV-noise: neither under- nor over-dose!

- Criteria one may ask for (whenever possible use hypercubic nesting):

<table>
<thead>
<tr>
<th>Smears efficiently</th>
<th>APE</th>
<th>n-APE</th>
<th>LOG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Link differentiable</td>
<td>EXP</td>
<td>n-APE</td>
<td>LOG</td>
</tr>
<tr>
<td>Link in $SU(3)$</td>
<td>APE</td>
<td>EXP</td>
<td>LOG</td>
</tr>
</tbody>
</table>

- Comparison of smearing recipes and/or parameters via plaquette distribution or via signal-to-noise ratio of a physical correlation function.

- Combination of clover term and HYL-smearing yields Wilson fermions with much reduced chiral symmetry breaking [staggered: ... Naik term ... taste breaking].

⇒ LOG/HYL promising to build UV-filtered actions suitable for full QCD via HMC