The nature of the finite temperature QCD transition as a function of the quark masses

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  - chiral susceptibility
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Introduction, motivation

- Zero $\mu$ area is relevant for - the early Universe
  - high energy collisions

- Nature of QCD transition at zero $\mu$ is found to be a crossover [Y. Aoki, G.Endrődi, Z. Fodor, S.D. Katz, K.K. Szabó]
The phase diagram

- Exploring the phase diagram:
  QCD effective models for $m = 0$
  lattice results for $m = m_{phys}$ and $m \to \infty$

- Aims:
  locate the second order line
determine the universality class
How to locate a second order line?

- Effect of the line can be "felt" in a surrounding area
- Statistical physical approach: order parameter is $\bar{\psi}\psi$
  reduced temperature is $t \equiv \frac{T-T_C}{T_C}$
  external "magnetic" field is $m$
- Definition of critical exponents:
  $\bar{\psi}\psi \sim |t|^\beta$, $\chi\bar{\psi}\psi \sim |t|^{-\gamma}$, $\bar{\psi}\psi^\delta|_{t=0} \sim m$
- Examine the properties of $\chi\bar{\psi}\psi \equiv \frac{\partial \bar{\psi}\psi}{\partial m}$
  height of susceptibility peak: $\chi\bar{\psi}\psi|_{t=0} \sim m^{\frac{1}{\delta}-1}$
  position of the peak: $|t| \sim m^{\frac{1}{\beta\delta}}$
Behaviour of the susceptibility

- Using Symanzik improved gauge and stout improved fermion action

- $m_s/m_{u,d}$ ratio is kept fixed

- Where does the dominant region begin?

- Limitations - smaller mass costs much more
  - correlation length: $C_{\psi\psi} \sim \frac{1}{\sqrt{m}} \ll N_S$
Results - the susceptibility peak $16^3 \times 4$

- Critical exponents:

<table>
<thead>
<tr>
<th>$1/\beta \delta$</th>
<th>3D Ising</th>
<th>$1/\delta - 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.633</td>
<td></td>
<td>-0.785</td>
</tr>
<tr>
<td>0.598</td>
<td>3D $O(2)$</td>
<td>-0.794</td>
</tr>
<tr>
<td>0.537</td>
<td>3D $O(4)$</td>
<td>-0.794</td>
</tr>
</tbody>
</table>
Results - the susceptibility peak $18^3 \times 6$
Results for the second order point \( m_0 \)

- Estimates:
  - height of peak gives: \( \frac{m_0}{m_{phys}} \lesssim 0.05 \)
  - position rather unstable, gives \( \frac{m_0}{m_{phys}} \lesssim 0.12 \)
- Height of peak is more suitable to determine \( m_0 \)
Theory of the Binder cumulant

- A quantity to distinguish between 1st order and crossover transitions
- Defined as $B_{\bar{\psi}\psi} = \frac{\langle (\delta\bar{\psi}\psi)^4 \rangle}{\langle (\delta\bar{\psi}\psi)^2 \rangle^2}$, where $\delta\bar{\psi}\psi \equiv \bar{\psi}\psi - \langle \bar{\psi}\psi \rangle$
- The distribution of $\bar{\psi}\psi$ at $T_C$ determines its value
- Some interesting cases:
  - 1st order transition $\iff B = 1$
  - crossover $\iff B \approx 3$
  - 2nd order transition depends on univ. class: $B|_{Z(2)} = 1.604$, $B|_{O(2)} = 1.242$, $B|_{O(4)} = 1.092$
Results - Binder cumulant $N_T = 4$

$N_S = 16$

$N_S = 24$

- Results are consistent with $\mathbb{Z}(2)$ universality class
- Taking this assumption we have
  
  $m_0/m_{\text{phys}} < 0.07$ ($N_T = 4$)
  
  $m_0/m_{\text{phys}} < 0.12$ ($N_T = 6$, next slide)
Results - Binder cumulant \( N_T = 6 \)

\[ N_S = 18 \]

- \( 24^3 \times 6 \) is in progress
Based on $N_T = 4$ and 6 simulations, the 2nd order point should be below $0.07 \cdot m_{phys}$

Its universality class is found to be consistent with $Z(2)$
Thank you for your attention!