On the structure of QCD confining string

[a remark on the non-perturbative short distance physics]

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Introduction

There are accumulating evidences that short distance physics of YM fields is **not exhausted** by perturbation theory:

- Quadratic power corrections seen in high energy processes. [Zakharov’99]
- Ultraviolet renormalons (higher orders of perturbation theory). [Zakharov’92]
- “Fine tuning” of magnetic degrees of freedom (monopoles, vortices). [ITEP’01,02]
- Lower dimensionality of fermionic (near) zero modes. [Horvath’03, MILC’04, ITEP’05]

The common point is the explicit power-like mixing of InfraRed (IR) $\Lambda_{QCD}$ and UltraViolet (UV) $1/a$ scales.
IR/UV “mixing” in vacuum action density

Convention prediction (OPE): \( s = \text{Tr} F_{\mu\nu}^2 \)

\[ \langle s \rangle_0 = \frac{\alpha_0}{a^4} + \gamma_0 \Lambda_{QCD}^4 \] [up to logarithms]

However, it had long been discussed that this pattern is more involved [Burgio’97]

\[ \langle s \rangle_0 = \frac{\alpha_0}{a^4} + \frac{\beta_0}{a^2} \Lambda_{QCD}^2 + \gamma_0 \Lambda_{QCD}^4 \]

and includes explicit IR/UV “mixing” term.
IR/UV “mixing” in vacuum action density

\[ \langle s \rangle_0 \cdot a^2 \text{ versus } a^2 \]

Numerical subtraction of perturbative loops.

[ITEP’05]

Model dependent evaluation.

[ITEP’05]

The “mixing” term is known to be small

\[
SU(3) : \beta_0 \Lambda_{QCD}^2 \lesssim [40 \text{ MeV}]^2 ,
\]

\[
SU(2) : \beta_0 \Lambda_{QCD}^2 \lesssim [60 \text{ MeV}]^2 .
\]
Regardless of how small the “mixing” term is, it has rather dramatic consequences. Consider the difference:

\[ \Delta s = \langle s \rangle_0 - \langle s \rangle_W = \]

\[ = \langle s \rangle_0 - \lim_{T \to \infty} \frac{\langle s(h, r)W(R, T) \rangle_0}{\langle W(R, T) \rangle_0} \]

It follows that generically

\[ \Delta s = \frac{\beta \Lambda_{QCD}^2}{a^2} + \gamma \Lambda_{QCD}^4. \]

[As expected, leading divergence vanishes, see below]
String Width

Rigorous action sum rules

\[ \int d^3 x \Delta s = V(R) \quad \text{(up to logarithms)} \]

for \( R \gg \Lambda_{QCD}^{-1} \) allow to estimate squared string width \( \delta^2 \)

\[ \delta^2 \propto \sigma \cdot [\beta \Lambda_{QCD}^2/a^2 + \gamma \Lambda_{QCD}^4]^{-1} \xrightarrow{a \to 0} 0 \quad [!] \]

Compare with effective string theory prediction:

- Gaussian profile

\[ \Delta s(h = 0) = C(R) \exp\{-r^2/\delta^2(R)\} \]

- Infinitely long QCD string does not exist

\[ \delta^2(R) = \frac{1}{\pi \sigma} \ln[R/R_0] \xrightarrow{R \to \infty} \infty \]
Parameters

We considered the following lattices

- Range of lattice spacings
  \[ 0.041(1) \text{ fm} \leq a \leq 0.104(1) \text{ fm} \]

- Physical volumes
  \[ [1.5 \text{ fm}]^4 \lesssim V^{\text{phys}} \lesssim [2.5 \text{ fm}]^4 \]

generated with Wilson action. Standard tricks of APE smearing and multihit integration were used as well.
Ground state separation

Rigorous transfer matrix arguments imply (leading order):

\[ \Delta s(h, r, R, T) = \Delta s(h, r, R) + c(h, r, R) \cdot e^{-m(R)T} \]

We insist that \( m \) is an unknown function of \( R \) only and make cumulative fit in finite \((h, r)\) region, where significant signal is expected.

This gives rather stable values of \( \Delta s(h, r, R) \), \( \chi^2/\text{n.d.f} \) being always in the range \([0.5 : 0.9]\).

Note: the gap parameter \( m(R) \) is not determined precisely (due to the smearing \( c(h, r, R) \) is rather small).
Transverse profile at $h = 0$

Transverse profile is Gaussian for $R \gtrsim 0.3$ fm, width increases with rising $R$.

$\beta = 2.600, \ 40^4$

![Graph showing transverse profile](graph.png)

- $R=0.50$ fm
- $R=0.62$ fm
- $R=0.74$ fm
- $R=0.87$ fm
- $R=0.99$ fm
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Quadratic divergence I

Direct Approach
Longitudinal slice
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Summary

String width at $h = 0$

Squared string width $\delta^2(R)$ vs. $R$ at various spacings.

Note the systematic drop of $\delta^2$ for $a \lesssim 0.07$ fm
String widening with $R \to \infty$ (probably logarithmic) is observed [ok]

But this effect seems to be subleading: flux tube rapidly shrinks with $a \to 0$ [?!]

If this is caused by quadratic divergence $\beta \Lambda_{QCD}^2 / a^2$ then we could estimate its magnitude:

- Shrinkage starts at $a_{cr} \approx 0.07 \text{ fm}$
- Gluon condensate is roughly $\gamma \Lambda_{QCD}^4 \approx 0.02 \text{ GeV}^4$
- Hence

$$\beta \Lambda_{QCD}^2 \approx a_{cr}^2 \cdot \gamma \Lambda_{QCD}^4 \approx [50 \text{ MeV}]^2$$
On-axis \((r = 0)\) action density difference

Longitudinal \(r = 0\) slice (experimental finding) is best described by the Yukawa ansatz (\(T \to \infty\) limit already taken)

\[
\Delta s(h, R) = \Delta s + A \cdot e^{-MR/2} \cdot \cosh(Mh),
\]
which works nicely for all available data sets (\(\chi^2/n.d.f.\) is always in the range \([0.4 : 0.8]\)).
On-axis \((r = 0)\) action density difference

Sources separation \(R \approx 1.0\) fm.

\[\Delta s, \text{ GeV}^4\]

\[h, \text{ fm}\]

\(\beta = 2.45\)

\(\beta = 2.51\)

\(\beta = 2.55\)

\(\beta = 2.60\)

Note: data points are Y-shifted for readability.
Action density at the string center, $R \to \infty$

Plot of the product $a^2 \cdot \Delta s$ versus $a^2$. 

![Graph showing the relationship between $a^2 \cdot \Delta s$ and $a^2$.]
Action density at the string center point certainly diverges quadratically in the continuum limit:

\[ \Delta s = \frac{\beta \Lambda_{QCD}^2}{a^2} + \gamma \Lambda_{QCD}^4. \]

\[ \beta \Lambda_{QCD}^2 = [25(2) \text{ MeV}]^2, \quad \gamma \Lambda_{QCD}^4 = 0.019(1) \text{ GeV}^4. \]

Compare with vacuum values

\[ \beta_0 \Lambda_{QCD}^2 \lesssim [60 \text{ MeV}]^2 \quad \gamma_0 \Lambda_{QCD}^4 \approx 0.02 \text{ GeV}^4 \]

Hence the conventional gluon condensate vanishes on the string symmetry axis.

\[ \gamma = \gamma_0 \]
String widening is seen at finite UV cutoff and is compatible with logarithmic law, however, this is a subleading effect.

Width of the confining string shrinks almost linearly and its action density quadratically diverges in the limit $a \to 0$, so that the observable heavy quark potential remains physical:

$$
\begin{align*}
\delta &\sim a \\
\Delta s &\sim a^{-2}
\end{align*}
\rightarrow \delta^2 \cdot \Delta s \approx \sigma = const.
$$
Heavy quark potential

There is no sign whatsoever of UV cutoff dependence.