The glue content of the pion

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Deep Inelastic Scattering experiments:
- the momentum of a proton carried by gluons $\langle x \rangle_g$ is about 50% 

Operator Product Expansion:
- $\langle x \rangle_g$ is related to a diagonal matrix element of the energy-momentum tensor $T_{\mu \nu}$ on the proton

Lattice QCD:
- ...but gluonic observables are notoriously noisy. first attempt

[Horsley et al. LATTICE 96]
quark and gluon distribution functions $f(x)$, $\bar{f}(x)$ and $g(x)$:

\[
\begin{align*}
\langle x \rangle_f(q^2) &\equiv \sum_{f=u,d,s} \int_0^1 x \, dx \{ \bar{f}(x, q^2) + f(x, q^2) \} \\
\langle x \rangle_g(q^2) &\equiv \int_0^1 x \, dx \, g(x, q^2)
\end{align*}
\]

- $\langle x \rangle_f(q^2)$ and $\langle x \rangle_g(q^2)$ are scale (and scheme) dependent

- **Momentum Sum Rule:** $\langle x \rangle_f(q^2) + \langle x \rangle_g(q^2) = 1$
The energy-momentum tensor in QCD

Separating the traceless part $\overline{T}_{\mu \nu}$ from the trace part $S$ for gluons, denoted ‘g’, and quarks, denoted ‘f’,

\[
T_{\mu \nu} \equiv \overline{T}^g_{\mu \nu} + \overline{T}^f_{\mu \nu} + \frac{1}{4} \delta_{\mu \nu} (S^g + S^f),
\]

\[
\overline{T}^g_{\mu \nu} = \frac{1}{4} \delta_{\mu \nu} F^a_{\rho \sigma} F^a_{\rho \sigma} - F^a_{\mu \alpha} F^a_{\nu \alpha},
\]

\[
\overline{T}^f_{\mu \nu} = \frac{1}{4} \sum_f \bar{\psi}_f \overleftrightarrow{D}_\mu \gamma_\nu \psi_f + \bar{\psi}_f \overleftrightarrow{D}_\nu \gamma_\mu \psi_f - \frac{1}{2} \delta_{\mu \nu} \bar{\psi}_f \overleftrightarrow{D}_\rho \gamma_\rho \psi_f,
\]

\[
S^g = \beta(g)/ (2g) \; F^a_{\rho \sigma} F^a_{\rho \sigma}, \quad S^f = [1 + \gamma_m(g)] \sum_f \bar{\psi}_f m \psi_f
\]

- $\overleftrightarrow{D}_\mu = \overrightarrow{D}_\mu - \overleftarrow{D}_\mu$
- $\beta(g)$ is the beta-function
- $\gamma_m(g)$ is the anomalous dimension of the mass operator
- all expressions are written in Euclidean space.
The connection between $T_{\mu\nu}$ and $\langle x \rangle_{f,g}$

- for an on-shell particle with four-momentum $p = (iE_p, p)$, $E_p^2 = M^2 + p^2$,

\[
\langle \psi, p | \int d^3z \, \bar{T}^{f,g}_{00}(z) | \psi, p \rangle = [E_p - \frac{1}{4} M^2 / E_p] \langle x \rangle_{f,g},
\]

\[
\langle \psi, p | \int d^3z \, S^{f,g}(z) | \psi, p \rangle = (M^2 / E_p) b_{f,g},
\]

\[
1 = \langle x \rangle_f + \langle x \rangle_g = b_f + b_g,
\]

- states are normalized according to $\langle p | p \rangle = 1$.

in the infinite momentum frame:
- $\langle x \rangle_g$ represents the momentum fraction carried by gluons

in the rest frame:
- the gluon contribution to the hadron mass is $\frac{3}{4} M \langle x \rangle_g$
- the contribution of the trace anomaly $S^g$ to the hadron mass is $\frac{1}{4} b_g M$ [X.D. Ji, '95]
Discretizations of $\bar{T}_{\mu\nu}^g$ and $S^g$

**body-centered definition:**

$$a^3 \sum_\mathbf{x} \bar{T}_{00}^{bp}(x_\odot) = \frac{2Z_g(g_0)}{ag_0^2} \sum_\mathbf{x} \mathrm{Re} \mathrm{Tr} \left[ \sum_k P_{0k}(x) - \sum_{k<l} \frac{1}{2} [P_{kl}(x) + P_{kl}(x + a\hat{0})] \right],$$

- breaking of continuous translation invariance $\Rightarrow$ finite renormalization, $Z_g(g_0) = \frac{\partial \xi_0(\beta,\xi)}{\partial \xi}$
- analog of the axial current normalization factor $Z_A(g_0)$.

**site-centered definition:**

$$\bar{T}_{00}^{bc}(x) \equiv \frac{\chi^{bc}(g_0)Z_g(g_0)}{g_0^2} \mathrm{Re} \mathrm{Tr} \left[ \sum_k (\hat{F}_{0k})^2 - \sum_{k<l}(\hat{F}_{kl})^2 \right]$$

- we calculated the relative normalization factors $\chi^{bc}(g_0)$
- for bare links and for HYP-smeared links.
On an $L_0 \times L^3$ lattice,

$$\begin{cases} 
\epsilon - 3P = \langle S \rangle_T - \langle S \rangle_0 \\
\epsilon + P = \frac{4}{3} \langle T_{00} \rangle.
\end{cases}$$

- $\epsilon =$ energy density, $P =$ pressure
- $\langle L_0^4 \sum_x T_{00}(x) \rangle$ is an RGI quantity
- it can serve to normalize the different discretizations
- $\Rightarrow$ fix $\chi(g_0)$
- we chose $1/L_0 \equiv T = 1.21 T_c$
- corresponds to $L_0/a = 6$ at $\beta = 6.0$. 

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Non-perturbative Normalization Factors

Normalization of the traceless energy-momentum tensor $Z(g_0)$

- 1-loop [Karsch '82]
- Data [Karsch et al. '99]
- Non-pert. fit

Wilson gauge action $\beta=6.0$, $(L_0/a) \times 10^3$ lattice

- Bare-clover (x 1/4)
- Hyp-plaq.
- Hyp-clover

The glue content of the pion
• huge differences between operators
• HYP-smearing the plaquette saves a factor 40 in CPU time ⇒ our choice.
How is this possible?

- the matrix elements of $\overline{T}_{00}(x)$ are physical and therefore independent of the discretization (up to $O(a^2)$ effects).

- **Lattice Sum Rules**: [HM, in preparation]

\[
\langle a^4 \sum_x \overline{T}_{00}(x)T_{00}(0) \rangle = \frac{-1}{2b_0 g_0^2} \langle S \rangle + \text{finite terms } \sim a^{-4}
\]

- this Eq. shows that the variance is not a physical quantity
- it is proportional to the quartically divergent “gluon condensate”
- $\Rightarrow$ it depends strongly on the discretization
- $\Rightarrow$ the discretization can be optimized to minimize the variance.
The glue momentum fraction in a heavy pion quenched Wilson action simulations

- $\beta = 6.0$, $a = 0.093\text{fm}$ if $r_0 = 0.5\text{fm}
- \kappa = 0.1515, 0.1530, 0.1550
- \begin{array}{c}
600\text{MeV} < M_\pi < 1060\text{MeV}
\end{array}
- 32 \cdot 12^3, 32 \cdot 16^3, 48 \cdot 16^3 \text{ and } 24^4

- define the effective momentum fraction

$$
\langle x \rangle^{(\pi)}_{g,\text{eff}}(x_0^{\text{min}}) \equiv \frac{8}{3M_\pi} \frac{a^3}{|\Lambda_0|} \sum_{x; x_0 \in \Lambda_0} \left[ \frac{\sum_y \langle j(0) \ T^{\text{hp}}_{00}(x_0) \ j(\frac{L_0}{2}, y) \rangle}{\sum_{y'} \langle j(0) \ j(\frac{L_0}{2}, y') \rangle} \right] - \langle T^{\text{hp}}_{00}(x_0) \rangle
$$

- $\Lambda_0 = \{ x_0^{\text{min}}, \ldots, \frac{L_0}{2} - x_0^{\text{min}} - a, \frac{L_0}{2} + x_0^{\text{min}}, \ldots, L_0 - x_0^{\text{min}} - a \}.$

- $\langle x \rangle^{(\pi)}_{g,\text{eff}} \xrightarrow{L_0, x_0^{\text{min}} \to \infty} \langle x \rangle^{(\pi)}_g$
Wilson action $\beta=6.0$ $\kappa=0.1515$

A plateau is seen at (surprisingly) early time $x_0$
Lattice Sum Rule and Normalization of $\langle x \rangle_g$

**Lattice sum rule**: [HM, '06]

$$\frac{3}{4} \left( E - \frac{1}{3} \sum_k \frac{\partial E}{\partial \log L_k} \right) = \langle \Phi | a^3 \sum_x Z_g(g_0) T_{00}^{g,\text{bare}} + Z_f(g_0) T_{00}^{f,\text{bare}} | \Phi \rangle |_{\Phi=\psi}$$

where

$$a^3 \sum_x T_{00}^{g,\text{bare}} (x_\circ) = \frac{2}{a g_0^2} \sum_x \text{Re} \text{Tr} \left[ \sum_k P_{0k}(x) - \sum_{k<l} \frac{1}{2} [P_{kl}(x) + P_{kl}(x + a \hat{a})] \right]$$

$$a^3 \sum_x T_{00}^{f,\text{bare}} (x_\circ) = \frac{3a^3}{4} \sum_x \bar{\psi}(x) [D_0 \gamma_0 - \frac{1}{2} aD_0^* D_0 - \frac{1}{3} (D_k \gamma_k - \frac{1}{2} aD_k^* D_k)] \psi(x)$$

- $Z_g(g_0) = \frac{1}{2} \frac{\partial \log (\beta_\sigma / \beta_\tau) (a_\sigma, a_\tau)}{\partial \log a_\tau}$, $Z_f(g_0) = - \frac{\partial \log (\kappa_\sigma / \kappa_\tau) (a_\sigma, a_\tau)}{\partial \log a_\tau}$

- this shows, in a particular regularization, that the momentum sum rule $\langle x \rangle_g + \langle x \rangle_f = 1$ holds for appropriate scheme-independent $Z_{f,g}(g_0)$. 

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Renormalization of $\langle x \rangle_g$

- since it is a flavour-singlet operator, $\bar{T}_\mu^g$ mixes with $\bar{T}^f_{\mu\nu}$
- if $\bar{T}_{00}^{f,g}(g_0)$ has been normalized as on previous slide,

$$
\begin{bmatrix}
\bar{T}_{00}^g(\mu) \\
\bar{T}_{00}^f(\mu)
\end{bmatrix}
= \begin{bmatrix}
Z_{gg} & 1 - Z_{ff} \\
1 - Z_{gg} & Z_{ff}
\end{bmatrix}
\begin{bmatrix}
\bar{T}_{00}^g(g_0) \\
\bar{T}_{00}^f(g_0)
\end{bmatrix},
$$

- NB. this form simplifies the calculation of the step-scaling matrix $\Sigma(\bar{g}^2(\mu), a_\mu) = Z(a_\mu, g_0)Z^{-1}(2a_\mu, g_0)$ in a finite-volume scheme
- $\Rightarrow$ there are only two independent anomalous dimensions
- $c_{gg,ff}(\bar{g} = 0) = \frac{N_f}{12\pi^2}, \frac{4}{9\pi^2}$ Gross, Wilczek; Georgi, Politzer '74
- the asymptotic glue momentum fraction is $16/[16 + 3N_f]$ for any hadron.
Result in the Quenched Approximation

- $Z_{gg} = 1$ due to the absence of quark loops $\Rightarrow$

\[
\langle x \rangle_g(\mu^2) = Z_g(g_0)\langle x \rangle_g^{\text{bare}} + [Z_f(g_0) - Z_{ff}(a_\mu, g_0)Z_f(g_0)] \langle x \rangle_f^{\text{bare}}
\]

- $Z_{ff}(a_\mu, g_0)Z_f(g_0) = 0.99(4)$ for the $\overline{\text{MS}}$-scheme at $\mu = 2\text{GeV}$
  [ZeRo Collab, Guagnelli et al. 03; 04]

- $\langle x \rangle^{\text{bare}} = 2 \times 0.3080(18)$ (disregarding disconnected diagrams) [ZeRo Collab, Guagnelli et al. 04]

- $Z_f(g_0) = 1 + O(g_0^2)$ itself is not known yet beyond treelevel; we take $1.0(2) \Rightarrow$ with 3000 configurations we obtain

\[
\langle x \rangle_g^{(\pi)}(\mu^2_{\overline{\text{MS}}} = 4\text{GeV}^2) = 0.37(8)_{\text{stat}}(12) Z_f \quad (M_\pi = 890\text{MeV})
\]

- **phenomenological estimates**: $\langle x \rangle_g \approx 0.38(5)$ assuming sea quarks account for $\sim 0.1$ of the momentum (A.D. Martin et al. ’92)
Outlook: gluonic observables on the lattice

- repeat the calculation for the proton, with dynamical quarks

- other observables: trace anomaly contribution to the mass, glue contribution to the nucleon spin, ... are of great interest (RHIC-spin program), but even harder

- techniques used here carry over to all these cases

- HYP-smeared $T_{\mu\nu}$ may prove useful in thermodynamic and transport coefficient calculations.
Recent Results on the Viscosities

Pressure

Viscosities

Cost formula with multi-level algorithm: 

\[ \#(PC - \text{days}) = 860 \left( \frac{0.051\, \text{fm}}{a} \right)^9 \left( \frac{L}{1.4\, \text{fm}} \right)^3 \left( \frac{1.65\, T_c}{T} \right)^6 \left( \frac{\Delta C(L_0/2)/C(L_0/2)}{4\%} \right)^2 \] (vs. \( a^{-14} \))

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Motivation: why calculate the viscosities?

- observation of flow at RHIC:
  - anisotropic flow incompatible with $\eta/s \gtrsim 0.2$ (D. Teaney ’03)
  - how will $\nu_2(p_T)$ look like at LHC?

- fundamental quantities of thermal QCD:
  - more difficult for a model to predict than the pressure and energy density, $\epsilon$ and $P$
  - thermal correlators ($=\text{the primary lattice data}$) are interesting for themselves (test of conformality, . . . )

- perturbative and AdS/CFT calculations:

$$
\eta/s, \, \zeta/s = \begin{cases} 
\frac{0.484}{\pi^2 \alpha_s^2 \log(0.608/\alpha_s)}, & N_f = 0 \, PT \\
\frac{1.25\alpha_s^2}{\pi^2 \log(4.06/\alpha_s)} & N = 4 \, SYM. 
\end{cases}
$$

Arnold, Moore, Yaffe ’03; Arnold, Dogan, Moore ’06;
Policastro, Son, Starinets ’01; Kovtun, Son, Starinets ’04
Results

Pressure

![Pressure Graph]

Karsch, *Hard Probes '06*

Viscosities

![Viscosities Graph]

$\eta/s$ [0704.1801]

$\eta/s$ prelim.

$\zeta/s$ prelim.

$N_T = 8 \quad N_f = 0$  

HM '07

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Linear response theory and Kubo formulas

- the shear and bulk viscosity are the linear coefficients in a gradient-expansion of the energy-momentum tensor

Jeon, Yaffe '96:

\[
\eta = \lim_{\omega \to 0} \frac{1}{\omega} \frac{1}{10} \int_{0}^{\infty} dt \int d^{3}x \ e^{i\omega t} \langle [\pi_{ij}(t, x), \pi_{ij}(0, 0)] \rangle_{eq}
\]

\[
\zeta = \lim_{\omega \to 0} \frac{1}{\omega} \int_{0}^{\infty} dt \int d^{3}x \ e^{i\omega t} \langle [P(t, x), P(0, 0)] \rangle_{eq}
\]

- \( P \) = pressure
- \( \pi_{ij} = T_{ij} - \frac{1}{3} \delta_{ij} T_{kk} \) the traceless part of the stress tensor
- in Euclidean Yang-Mills theory, \( T_{\mu\nu} = \overline{T}_{\mu\nu} + \frac{1}{4} \delta_{\mu\nu} \theta \),

\[
\overline{T}_{\mu\nu} = \frac{1}{4} \delta_{\mu\nu} F_{\rho\sigma}^{a} F_{\rho\sigma}^{a} - F_{\mu\alpha}^{a} F_{\nu\alpha}^{a},
\]

\[
\theta = \beta(g)/(2g) F_{\rho\sigma}^{a} F_{\rho\sigma}^{a}
\]
The shear viscosity from two-point functions

- Euclidean formalism, temperature $T = 1/L_0$:

$$C(x_0) = L_0^5 \int d^3x \langle \overline{T}_{12}(0) T_{12}(x_0, x) \rangle.$$ 

- the spectral function $\rho(\omega, 0)$ satisfies

$$C(x_0) = L_0^5 \int_0^\infty d\omega K(x_0, \omega) \rho(\omega),$$  
$$K(x_0, \omega) = \frac{\cosh \omega \left( \frac{1}{2} L_0 - x_0 \right)}{\sinh \frac{\omega L_0}{2}}.$$ 

- the shear viscosity is then given by

$$\eta(T) = \pi \left. \frac{d\rho}{d\omega} \right|_{\omega=0}.$$ 

- $\rho(\omega)/\omega \geq 0$
- $\rho(-\omega) = -\rho(\omega)$

Karsch, Wyld '86; Nakamura, Sakai '04
Lattice formulation

Wilson action: \( S_g = \frac{1}{g_0^2} \sum_x \sum_{\mu \neq \nu} \text{Re} \text{Tr} \left\{ 1 - P_{\mu\nu}(x) \right\} \)

This is the first calculation to . . .

1. use a two-level algorithm (HM, ’04)
2. implement treelevel improvement (discretization errors are \( O(g_0^2 a^2) \))
3. normalize \( \overline{T}_{\mu\nu} \) non-perturbatively
4. take into account the trace-anomaly of \( T_{\mu\nu} \)
5. enforce the perturbative large-\( \omega \) behavior of \( \rho(\omega) \).
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<th>$a$ [fm]</th>
<th>$T/a$</th>
<th>$L/a$</th>
<th>$T/T_c$</th>
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- sim. in red running on single-rack Blue Gene L at M.I.T.
The $\langle T_{12} T_{12} \rangle$ correlator

- Improvement in accuracy over previous calculations about a factor 20 thanks to the two-level algorithm
- 860 PC days invested into this plot.
The reconstructed spectral function

\[ \rho(\omega) \frac{K(x_0=1/2T,\omega)}{T^4} \]

\[ T=1.24T_c \]

\[ T=1.65T_c \]

- $\eta/T^3 = \frac{\pi}{2} \times \text{intercept}$
- black curve = normalized $\mathcal{N} = 4$ SYM spectral function.
A new way to extract $\rho(\omega)$

- **assumption:** $\rho(\omega)$ is smooth on the scale of $T$

- $\tilde{\rho}(\omega) \equiv \rho(\omega)/\tanh \frac{1}{2} \omega L_0$, \hspace{1cm} $\tilde{K}(x_0, \omega) \equiv K(x_0, \omega) \tanh \frac{1}{2} \omega L_0$

- $\tilde{\rho}(\omega) = m(\omega) \left[ 1 + \sum_\ell c_\ell a_\ell(\omega) \right]$, where $m(\omega) \overset{\omega \to \infty}{\sim} \tilde{\rho}_{\text{tree level}}(\omega)$

- $\{a_\ell(\omega)\}$ an appropriate basis of orthogonal functions

- $a_\ell(\omega) =$ eigenfunctions of the kernel $G(\omega, \omega') \equiv \int_0^{L_0} \frac{dx_0}{L_0} M(x_0, \omega) M(x_0, \omega')$, where $M(x_0, \omega) \equiv K(x_0, \omega) m(\omega)$
the new method gives the correct viscosity within $\sim 16\%$ for $N_T = 8$, assuming the correlator is known exactly

(Mathematica routine for $\rho_{SYM}$ provided by P. Kovtun)
Result for $\eta$ from $N_T = 8$

$$\frac{\eta}{s} = \begin{cases} 
0.134(33) & (T = 1.65 T_c) \\
0.102(56) & (T = 1.24 T_c).
\end{cases}$$

arXiv:0704.1801
The bulk viscosity from the \( \langle \theta \ \theta \rangle \) correlator

\[ \bar{C}(x_0) \]

- \( 1.65T_c, \ \text{LT}=5 \)
- \( 2.22T_c, \ \text{LT}=6 \)
- \( 3.22T_c, \ \text{LT}=6 \)

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The reconstructed spectral function for $\langle \theta\theta \rangle$

\[ \rho(\omega) K(x_0=1/2T,\omega)/T^4 \]

- $1.24T_c$ (blue)
- $1.65T_c$ (red)
- $2.22T_c$ (pink)
- $3.22T_c$ (cyan)

\[ \zeta/T^3 = \left( \frac{\pi}{18} \times \text{intercept} \right) \text{ is increasing for } T \to T_c \]

cf. (Kharzeev, Tuchin '07)
Summary: results from $N_f = 8$

Pressure

Viscosities

Karsch, *Hard Probes* '06

$N_f = 0$  HM '07

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Conclusions

- **shear viscosity of SU(3) gluodynamics:**
  \[ \frac{\eta}{s} = 0.08 - 0.18 \text{ for } 1.2T_c < T < 3.5T_c \]

  - no significant temperature dependence
  - expect similar elliptic flow phenomenon at LHC temperatures as at RHIC (caveat: \( N_f = 0 \), smoothness assumption)

- **bulk viscosity \( \zeta \):** much smaller than \( \eta \),
  \[ \frac{\zeta}{s} \approx 0.02 \text{ at } T = 1.65T_c, \]

  but apparently rising for \( T \to T_c \) (to be confirmed)

- **effort to reduce systematic uncertainties** is ongoing: cutoff and finite-volume effects, increase resolution \( N_\tau \)

- **the AdS/CFT prediction** of \( \frac{\eta}{s} = 1/4\pi \) and \( \zeta = 0 \) gives the right picture in this temperature range in spite of visible conformality breaking in the correlators.
Finite-volume effects larger than for $\epsilon, P$

planning to run at $LT = 6$. 

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Cutoff effects at $1.24T_c$ (after treelevel improvement)

- Production of $N_T = 10, 12$ and 16 is underway.

- The glue content of the pion