2+1 flavor topological susceptibility from the asqtad action at 0.06 fm


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Outline

1. Full QCD Topological Susceptibility since 2001
   - An assignment from S. Dürr in 2001

2. MILC Asqtad Program: $\chi_{\text{topo}}$
   - First MILC $\chi_{\text{topo}}$ Result 2003
   - Rooted Staggered Chiral Perturbation Theory
   - Billeter, DeTar, and Osborn: $\chi_{\text{topo}}$ from RS$\chi$PT

3. 2007 Methods and Results
   - Methods
   - Results
   - Caveats

4. Conclusions
For small $m_q$

$$\chi_{\text{topo}} \sim \frac{f^2 m^2}{2N_f}$$

Where’s the $m \to 0$ suppression in the data?

“...roughly consistent with each other, but follow theoretical expectations to a limited degree”
Data used for 2001 comparison:

- CP-PACS: Iwasaki action + Wilson clover fermions \((16^3 \times 32, 24^3 \times 48)\)
- UKQCD: Wilson gauge + Wilson clover fermions \((16^3 \times 32)\)
- SESAM/T\(\chi\)L: Wilson gauge and Wilson fermions \((16^3 \times 32, 24^3 \times 40)\)
- PISA: Wilson gauge + staggered fermions \((16^4)\)

Advances since 2001:

- Highly Improved Actions — 2+1 flavor full QCD
- Staggered Chiral Perturbation Theory
- Bigger Computers
MILC Improved Action Program

- Asqtad Action: $O(\alpha_s a^2)$ improved gauge and fermion actions
- Topological Charge via Boulder $F_{\mu\nu} \tilde{F}^{\mu\nu}$ and HYP Smearing

By 2003 — 2 lattice spacings: $a = 0.13$ and $0.09$ fm

- $a \to 0$ extrapolation is essential
- Data compared to continuum $\chi$PT:

$$\chi_{\text{topo}} \sim \frac{f_\pi^2 m_\pi^2}{4(1 + m_{u,d}/2m_s)}$$

- 2 lattice spacings...
- Encouraging results

(Rooted) Staggered Chiral Perturbation Theory

Lee and Sharpe  
—1 flavor × 4 tastes

Aubin and Bernard  
—$n_f$ flavors × 4 tastes, and  
—$n_f$ flavors × (4 tastes)$^{1/4}$

How to handle taste breaking effects at finite $a$. 

![Graphs showing the relationship between $(m_x+m_y)r_1 \times (Z_m/Z_m^{\text{fine}})$ and $(f_{\pi} r_1)/\sqrt{2}$](image-url)
Billeter, DeTar, and Osborn: Anomaly couples to $\phi_{ol}$


- From the **Rooted Staggered Chiral Lagrangian**

\[
\mathcal{L} = \frac{f_\pi^2}{8} \text{Tr}(\partial_\mu U^\dagger \partial U) + \sum C_i \mathcal{O}_i \\
- \frac{\mu f_\pi^2}{4} \text{Tr}[\mathcal{M}(U^\dagger + U)] + \frac{m_o^2}{2} \phi_{ol}^2 + \ldots
\]

- $\chi_{\text{topo}} = \frac{f_\pi^2 m_{\pi, l}^2}{1 + m_{\pi, l}^2/2m_{ss,l} + 3m_{\pi, l}^2/2m_o^2}$

- $\chi_{m \rightarrow 0} \sim \frac{f_\pi^2 m_{\pi, l}^2}{8}$

- $\chi_{m \rightarrow \infty} \sim \frac{f_\pi^2 m_o^2}{12} = 0.06/r_0^4$

Pion taste multiplet masses $m_{\pi, l}^2$ at fixed $a$, versus quark mass $m_q$

**Lesson:**

Use $m_{\pi, l}^2$ not $m_{\pi}^2$, Goldstone
Topological Susceptibility Measurements

- **Boulder extended** $F_{\mu\nu} \tilde{F}^{\mu\nu}$ definition
  

- Use 3 **HYP** Smoothing sweeps to find $q(r) = F_{\mu\nu} \tilde{F}^{\mu\nu}$

- Use $\langle Q^2 \rangle / V = \int dr \langle q(r) q(0) \rangle$

- **Fit long distance behavior** of $\langle q(r) q(0) \rangle$ to analytical form to $\eta + \eta'$ scalar propagator.
Long distance topological charge density correlator fit

Defining $C(r) = \langle q(r)q(0) \rangle$

$$\langle Q^2 \rangle / V = \sum_{r \leq r_c} C_{\text{meas}}(r) + \sum_{r > r_c} C_{\text{fit}}(r)$$

$$C_{\text{fit}}(r) = D(m_{\eta}, r) + D(m_{\eta'}, r)$$

$$D(m, r) = \frac{m}{4\pi^2 r} K_1(mr)$$
New Results from $a = 0.06$ fm $48^3 \times 144$ lattices

- Simultaneously Fit data at different lattice spacings and masses to $f(m, a)$

$$\frac{1}{\chi_{\text{topo}} r_0^4} = f(m^2 \pi, I, a)$$

$$= A_0 + (A_1 + A_2 a^2 + A_3 a^4) / m^2 \pi, I$$

- Continuum = $f(m, 0)$

- Compare with L.O. $2+1 + \infty$ RS$\chi$PT
Autocorrelation of Q

\begin{figure}
\centering
\includegraphics[width=\textwidth]{autocorr.png}
\caption{Autocorrelation of Q}
\end{figure}
Conclusions

- 2+1 flavor $\chi_{\text{topo}}$ quite consistent with $S\chi$PT

- 0.06 fm data already very close to continuum for small $m_\pi, l$

- 1/4 root method is supported, since continuum formula for $\chi_{\text{topo}}$ is quite sensitive to $n_f$ and $m_f$