Staggered Top-Higgs models and Higgs mass bounds

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what is the goal?

- Should we take the Lagrangian and “just calculate”?
- **QCD**: non-perturbative, lattice essential for continuum questions + quantities
- **Electroweak**: perturbative? lattice role?
- **Obstacle**: “theory not UV complete” (continuum) or “theory needs finite cutoff” (lattice) - the same thing?
- Interpret cutoff as herald of new physics?
- Loss of predictivity?
This talk: explore some relevant model, build up

SM-ultralite: Higgs + “Top”

already interesting: unstable vacuum, Higgs lower bound

use large N and lattice to show theory TRIVIAL: no instability

use lattice to calculate (limited) Higgs lower bound vs. cutoff

how much survives in SM? interpretation? low cutoff?

SM-lite: Higgs + Top (+ gluon) (next talk, Daniel Nogradi) overlap fermion (poster, Chris Schroeder)
Couple single Higgs to N “Top” quarks

1-loop beta function

\[ \frac{d\lambda}{d\mu} = \frac{1}{16\pi^2} \left( 3\lambda^2 - 8N\lambda y^2 - 48Ny^4 \right) \]

looks like possible vacuum instability if Higgs too light

Interpretation: new physics occurs at \( \lambda(\mu = \Lambda) = 0 \)

Figure: SM 2-loop running
Casas, Espinosa, Quiros PLB 342, 171 (1995)
instability and Higgs bound

if Higgs mass known determines location of instability

Electroweak precision prefers light Higgs

\[ m_{\text{Higgs}} = (76^{+33}_{-24}) \text{ GeV} \]

could SM be valid up to Planck scale?

could we conclude new physics possible at 1 TeV?

is this trustworthy at low cutoff?

\[ m_t = 175 \text{ GeV} \]
large \( N \)

Start with bare Lagrangian

\[
\mathcal{L} = \frac{1}{2} m_0^2 \phi_0^2 + \frac{1}{24} \lambda_0 \phi_0^4 + \frac{1}{2} \left( \partial_\mu \phi_0 \right)^2 + \bar{\psi}_0 \left( \gamma_\mu \partial_\mu + y_0 \phi_0 \right) \psi_0
\]

re-write in renormalized form

\[
= \frac{1}{2} m_0^2 Z_\phi \phi^2 + \frac{1}{24} \lambda_0 Z_\phi^2 \phi^4 + \frac{1}{2} Z_\phi \left( \partial_\mu \phi \right)^2 + Z_\psi \bar{\psi}^a \left( \gamma_\mu \partial_\mu + y_0 \sqrt{Z_\phi} \phi \right) \psi^a
\]

\[
= \frac{1}{2} \left( m^2 + \delta m^2 \right) \phi^2 + \frac{1}{24} \left( \lambda + \delta \lambda \right) \phi^4 + \frac{1}{2} \left( 1 + \delta z_\phi \right) \left( \partial_\mu \phi \right)^2 + \left( 1 + \delta z_\psi \right) \bar{\psi}^a \gamma_\mu \partial_\mu \psi^a + \bar{\psi}^a (y + \delta y) \phi \psi^a
\]

large \( N \): Top loops only

\[
m_0^2 Z_\phi = m^2 + \delta m^2, \quad \lambda_0 Z_\phi^2 = \lambda + \delta \lambda, \quad \sqrt{Z_\phi} y_0 = y, \quad \delta y = \delta z_\psi = 0
\]
(a) fix bare parameters, use some finite cutoff (Pauli-Villars, hard-mom.)
(b) choose renormalization conditions e.g.

\[ G_{\phi\phi,0}^{-1}(p^2 \to 0) = \frac{p^2 + m_H^2}{Z_\phi}, \quad m_t = yv, \quad m_H^2 = \lambda v^2 / 3 \quad \text{tadpole vanishes} \]

(c) solve exactly

1) large cutoff \( m_T, m_H, v \ll \Lambda \) couplings vanish logarithmically

\[ y^2 \to \left[ \frac{N_F}{8\pi^2} \ln \frac{\Lambda^2}{m_T^2} \right]^{-1}, \quad \lambda \to 12 \left[ \frac{N_F}{8\pi^2} \ln \frac{\Lambda^2}{m_T^2} \right]^{-1}, \quad \frac{m_H}{m_T} = \sqrt{\frac{\lambda}{3y^2}} \to 2 \]

2) small cutoff \( m_T, m_H, v \sim \Lambda \) couplings flow to bare values

\[ y \to y_0, \quad \lambda \to \lambda_0 \geq 0 \quad \text{well-defined functional integral} \]

**NO INSTABILITY**
large $N$

Higgs coupling RG

arbitrary choice $\lambda_0 = 0.1$ determine renormalized couplings

exact RG flow breaks from continuum RG near cutoff

stability of vacuum guaranteed by $\lambda_0 \geq 0$
can calculate Higgs effective potential using lattice simulations
single Higgs, 2 staggered “Top” HMC algorithm, no sign problem
vacuum absolutely stable

check simulations:
lattice renormalized pert. theory
perfect agreement
infinite-volume convergence of lattice PT to continuum PT below cutoff
Higgs bound via simulations

explore phase diagram
bare couplings: $m_0^2, \lambda_0, y_0$
tune to find the critical surface
fixed cutoff and Top mass:
one degree of freedom left

lightest Higgs in limit $\lambda_0 \to 0$
(heaviest Higgs as $\lambda_0 \to \infty$)
expect cutoff effects small enough if
$
\max(m_{\text{Higgs}a}, m_{\text{Top}a}) \leq 0.5
$
Higgs bound via simulations

Higgs lower bound for e.g.

\[ m_{\text{Top}}/v = 1.2, \quad m_{\text{Top}} = 295 \text{ GeV} \]

cutoff effects not constant along curve
difficult to make cutoff large and keep finite-volume effects under control
demonstration of principle
predictivity?

but what can we conclude with finite cutoff?
comparison of Higgs lower bound
in large N

higher-derivative Top propagator

\[
\frac{1}{k + m} \rightarrow \frac{1}{k(1 + k^2/\Lambda^2) + m}
\]

loss of universality bad at low cutoff
bounds merge as cutoff is increased

“new physics”: space-time is a lattice? continuum connection?
what can we conclude?

- simple Higgs + “Top” model not unstable
- Higgs lower bound follows from triviality
- **low cutoff**: absence of universality severe, prediction?
- do not know if all/any features survive in SM
- next step: **SM-lite** Higgs + overlap Top (+ gluon)
- Weak gauge couplings, new qualitative behavior
- future(?): include **continuum** new physics, SM as EFT e.g. higher-dimensional operators, new degrees of freedom