Group Theoretical Construction of Nucleon Operators using All-to-All Propagators

Keisuke Jimmy Juge
University of the Pacific
(LHP Collaboration)
Introduction

Physically large lattices with light dynamical quarks are being generated by many collaborations.

As quark masses become lighter, many single particle states start to mix with multi-particle states.

All-to-All quark propagators may become essential in studying these multi-particle states.

Try the Noise-Dilution method (TrinLat) to construct group-theory motivated baryon operators (LHPC) for single particle states.
Group-Theoretical Construction of Operators


Project out irreps by combining elemental operators (of “length” 0, 1, 2, 3)

\[ \Phi^{ABC}_{\alpha \beta \gamma; ijk}(t) = \sum \varepsilon_{abc} \left( (\tilde{D}^{(p)}_i \tilde{\psi})^{A}_{a \alpha}(\vec{x}, t) \times (\tilde{D}^{(p)}_j \tilde{\psi})^{B}_{b \beta}(\vec{x}, t) (\tilde{D}^{(p)}_k \tilde{\psi})^{C}_{c \gamma}(\vec{x}, t) \right) \]

Combine with appropriate coefficients to form ops
All-to-All Quark Propagators

Noise-Dilution Method


- Stochastic noise $\mathbb{Z}(4)$ noise source $\eta$
  (one for each quark)

- Dilute

\[ \eta = \eta^{(0)} + \eta^{(1)} + \cdots + \eta^{(N_{\text{dil})}} \]

- Invert Dirac operator to get solutions $\varphi^{(i)}$

Propagator $\sum_{i} \varphi^{(i)}(x, t) \otimes \eta^{\dagger(i)}(x_0, t_0)$

- Construct baryon operators with $\varphi^{(i)}$ and $\eta^{(i)}$
Constructing Baryon Operators (Basak et al.)

\[ C^{(N)}_{ij}(t) = c^{(i)}_{\mu \nu \tau} \bar{c}^{(j)}_{\mu \nu \tau} \]

\[
\left\{ \begin{align*}
\bar{G}^{(uud)}_{(\mu|\bar{\mu})(\nu|\bar{\nu})(\tau|\bar{\tau})} + \bar{G}^{(uud)}_{(\tau|\bar{\nu})(\nu|\bar{\nu})(\mu|\bar{\mu})} \\
+ \bar{G}^{(uud)}_{(\mu|\bar{\nu})(\nu|\bar{\mu})(\tau|\bar{\mu})} - \bar{G}^{(uud)}_{(\mu|\bar{\nu})(\nu|\bar{\nu})(\tau|\bar{\mu})} \\
- \bar{G}^{(uud)}_{(\mu|\bar{\nu})(\nu|\bar{\mu})(\tau|\bar{\mu})} - \bar{G}^{(uud)}_{(\nu|\bar{\nu})(\tau|\bar{\mu})(\mu|\bar{\mu})} \\
- \bar{G}^{(uud)}_{(\nu|\mu)(\tau|\bar{\nu})(\mu|\bar{\tau})} + \bar{G}^{(uud)}_{(\tau|\bar{\nu})(\nu|\bar{\nu})(\mu|\bar{\mu})} \end{align*} \right\}
\]

\( \bar{G} \)'s are the colour contracted 3 quark propagators.
All-to-all simplifications

Source ($\bar{\mu}$'s) and sink ($\mu$'s) indices could not be separated in the previous formula

All-to-all allows us to “separate” the source and sinks

$$C^{(N)}_{IJ}(t) = \sum_{\tilde{i}} \sum_{\tilde{j}} \sum_{\tilde{k}} c_{\mu \nu \tau} B^{\tilde{i}\tilde{j}\tilde{k}}_{[ABC] \mu \nu \tau}(\vec{x}, t) \times$$

$$c^{(J)}_{\mu \nu \tau} \left\{ 2B^{\tilde{i}\tilde{j}\tilde{k}}_{[ABC] \mu \nu \tau} + 2B^{\tilde{k}\tilde{i}\tilde{j}}_{[CBA] \mu \nu \tau} - B^{\tilde{k}\tilde{i}\tilde{j}}_{[ACB] \mu \nu \tau} - B^{\tilde{k}\tilde{i}\tilde{j}}_{[CAB] \mu \nu \tau} \right\} (\vec{x}_0, t_0)$$
Colour contracted objects

$$B_{i,j,k}^{\mu\nu\tau}[012](\vec{x}, t) = \epsilon_{abc} \psi_{\mu}^{(i)a}(\vec{x}, t) \psi_{\nu}^{(j)b}(\vec{x}, t) \psi_{\tau}^{(k)c}(\vec{x}, t)$$

$$B_{i,j,k}^{\mu\nu\tau}[012](\vec{x}, t) = \epsilon_{abc} \eta_{\mu}^{(i)a\dagger}(\vec{x}, t) \eta_{\nu}^{(j)b\dagger}(\vec{x}, t) \eta_{\tau}^{(k)c\dagger}(\vec{x}, t)$$

The quarks may be displaced ...

$$U_y(\vec{x}, t) U_y(\vec{x} + a\hat{e}_y, t) U_y(\vec{x} + 2a\hat{e}_y, t) \psi(\vec{x} + 3a\hat{e}_y, t)$$

Combine these objects (which are labeled by 3 displacement and 3 spin indices) with the right coefficients

There are many pieces that are in common amongst different operators $\Rightarrow$ only compute them once!
Effective Masses

Comparison:
Point-to-All and
Time-diluted all-to-all

$12^3 \times 48$ Lattice

$M(PS) \sim 700$ MeV

20 configurations

1.7x more inversions

Single-Site operator
(Nucleon G1g)
Time + Spin Dilution

# of quark inversions

48 (time) \times 4 \text{ (spin)} \times 3 \text{ (quarks)} = 576 \text{ inv}

to be compared with

7 \text{ (displacements)} \times 4 \text{ (spin)} \times 3 \text{ (color)} = 84 \text{ inv}
Dilution Comparison

$12^3 \times 48$ Lattice, 20 configs, DDL-type operator

**Time** Dilution
144 inversions ($x1.3^2$)

**Time+Spin** Dilution
576 inversions ($x2.6^2$)
Diagonalization

Noise vectors introduce fluctuations in the temporal direction (noise on different timeslices are independent) making diagonalization unstable ...

Effective masses bounce around more than point-to-all data

Fixed coefficient

- Fix $t_0$ to 1 and $t$ to 3

$$C(t) \, v = \lambda \, C(t_0) \, v$$

Use $v$ to rotate to the optimal basis
Lowest Three States

Time-dilution 20 configs

point-to-all
Lowest Three Nucleon States

Time+Spin-dilution

20 configs

point-to-all

Graphs showing energy E vs. time t for different states.
Optimized Ground State

Time+Spin-dilution point-to-all

20 configs
Adding Colour Dilution

- Time+Spin dilution
- Time+Spin+Colour dilution
  (Single-Site 2 operator)

- 1 configuration
- Average of 100 cfgs using point-to-all shown in black
- Indication that colour dilution will further reduce errors
Effect of # sources used

Sources on different timeslices almost independent?

... it pays off to sample the whole lattice
Hybrid Method

Combining with low-lying eigenvectors \( \vec{v} \)

One would like to continue to work with `sparse source vectors`

Can use the same trick as in Foley et al.

- Store dot products of eigenvectors and noise sources that are needed to orthogonalize \( \eta^{(i)} \)
- The original noise sources continue to ensure the number of non-zero contractions are small ...

\[
\epsilon_{abc} \eta^{(i) a}(t_0) \eta^{(j) b}(t_0) \sum_{\alpha} v^{(\alpha) c}(t_0) \langle v^{(\alpha)}, \eta^{(k)} \rangle
\]

work in progress ...
Conclusions

- The noise dilution method of estimating all-to-all propagators is effective for three-quark (baryon) operators as well.

- Focus on constructing the 3-quark operators.
  - many common elemental ops across channels
  - avoid computing zeros from the sparse noise source

- Excited states are accessible (diagonalization works).

- The “hybrid” method, ie combining eigenvectors, is being added (will need this at lighter quark masses).

- Simulation of multi-particle states underway...