$B \rightarrow D^* l \nu$ with $2+1$ flavors

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Outline

I. Brief Motivation for calculating $|V_{cb}|$

II. Overview of the lattice calculation and latest improvements

III. Error budget

IV. Final result
Constraining the Unitarity Triangle

\[ \sin 2\beta \]

\[ |V_{ub}/V_{cb}| \]

\[ \gamma \]

\[ \Delta m_d \]

\[ \Delta m_s \text{ & } \Delta m_d \]

\[ \{ = 1.24 \pm 0.04 \pm 0.06 \} \]

\[ \text{sol. w/ } \cos 2\beta < 0 \]

\[ \text{(excl. at CL > 0.95)} \]
Importance of $|V_{cb}|$

$|V_{cb}|$ is needed to constrain the apex of the unitarity triangle from kaon mixing (along with $B_K$). Given that

$$A = \frac{|V_{cb}|}{\lambda^2}$$

(1)

has $\approx 2\%$ error, we see that this contributes a $9\%$ error to $\epsilon_K$ because it appears in the formula below to the fourth power.

$$|\epsilon_K| = C_e B_K A^2 \bar{\eta}\{-\eta_1 S_0(x_c)(1 - \lambda^2/2) + \eta_3 S_0(x_c, x_t) + \eta_2 S_0(x_t) A^2 \lambda^2 (1 - \bar{\rho})\}$$

Given the recent RBC/UKQCD $\sim 5\%$ $B_K$ result, if we can get $|V_{cb}|$ errors down it puts pressure on the continuum perturbation theory community since the two-loop calculation of the Wilson coefficients has $\sim 7\%$ errors.
Methods for extracting $|V_{cb}|$

- Inclusive $b \rightarrow c\ell\nu$ can be calculated perturbatively, but is ultimately limited by the breakdown of local quark-hadron duality. Difficult to estimate systematics.

- Exclusive $B \rightarrow D\ell\nu$ has an $\sim 10\%$ error in the region of phase space accessible to the lattice.

- Exclusive $B \rightarrow D^*\ell\nu$ is experimentally cleaner and can be calculated on the lattice to the needed precision, as this talk will demonstrate.
Overview of the calculation

- We use the Fermilab method to treat heavy quarks, as in the original quenched calculation of Hashimoto et al, hep-ph/0110253.

- Now using the MILC 2+1 flavor lattices, so the calculation is unquenched, with improved staggered (asqtad) light fermions in valence and sea.

- Many MILC lattice ensembles were used. This work uses three lattice spacings ($a \approx 0.15$ fm, $a \approx 0.12$ fm, $a \approx 0.09$ fm).

- New double ratio is constructed which gives the answer more directly.

\[ \text{In this talk I will assume the validity of the 4th root trick.} \]
The Fermilab method uses the SW (clover) action

\[ S = \sum_{x,f} \bar{\psi}_x^f \psi_x^f - \sum_{x,y,f} \kappa_f \bar{\psi}_x^f M_{xy} \psi_y^f + \frac{i}{2} c_{SW} \sum_{x,f} \kappa_f \bar{\psi}_x^f \sigma_{\mu\nu} F_{\mu\nu} \psi_x^f \]

with

\[ am_{0f} = \frac{1}{u_0} \left( \frac{1}{2\kappa_f} - \frac{1}{2\kappa_{crit}} \right) \quad (2) \]

The bare mass, \( am_0 \) and clover coupling \( c_{SW} \) are adjusted so that the leading corrections in the heavy quark expansion are included. Errors scale as \( \alpha_s \bar{\Lambda}/m_Q \), even though \( m_Q \) is much greater than the lattice scale.
Obtaining $V_{cb}$ from $\bar{B} \rightarrow D^* l \bar{\nu}_l$

\[
\frac{d\Gamma}{dw} = \frac{G_F^2}{4\pi^3} m_{D^*}^3 (m_B - m_{D^*})^2 \sqrt{w^2 - 1} G(w) \\
\times |V_{cb}|^2 |\mathcal{F}_{B\rightarrow D^*}(w)|^2
\]

where $G(w)$ is a kinematic factor and $\mathcal{F}_{B\rightarrow D^*}$ is a nonperturbative matrix element. $w = v' \cdot v$ is the velocity transfer from initial ($v$) to final state ($v'$).
Calculating $B \to D^*$

\[ \mathcal{F}_{B \to D^*}(1) = h_{A_1}(1), \]  \hfill (4)

\[ \langle D^*(v)|A^\mu|\bar{B}(v)\rangle = i\sqrt{2m_B2m_{D^*}}\epsilon^{\mu\nu}h_{A_1}(1). \]  \hfill (5)

$h_{A_1}(1)$ is constrained by heavy quark symmetry:

\[ h_{A_1}(1) = \eta_A \left[ 1 - \frac{l_V}{(2m_c)^2} + \frac{2l_A}{2m_c2m_b} - \frac{l_P}{(2m_b)^2} \right] \]  \hfill (6)
Hashimoto et al, hep-ph/0110253 proposed three double ratios, one for each of the $1/m_Q^2$ coefficients on the previous slide. Fits to the three ratios using the HQET dependence on heavy quark masses yielded the $1/m_Q^2$ (and most of the $1/m_Q^3$) coefficients.

Advantages of the double ratios:

- Statistical errors cancel in the ratio
- Most of the axial current renormalization cancels with the vector current renormalization. The remainder can be computed perturbatively.
- As shown by Kronfeld (hep-lat/0002008), heavy quark symmetry constrains the discretization errors in the double ratios, so that for this quantity the leading corrections are of the order $\alpha_s(\Lambda/m_Q)^2$ and $\Lambda/m_Q^3$.
- All errors in double ratios $\mathcal{R}$ scaled as $\mathcal{R} - 1$ rather than as $\mathcal{R}$, since when $m_c = m_b$ the ratios were one by construction. This was especially important since Hashimoto et al were working in the quenched approximation.
New Method

\[ \frac{\langle D^* | \bar{c} \gamma_j \gamma_5 b | B \rangle \langle B | \bar{b} \gamma_j \gamma_5 c | D^* \rangle}{\langle D^* | \bar{c} \gamma_4 c | D^* \rangle \langle B | \bar{b} \gamma_4 b | B \rangle} = |h_{A_1}(1)|^2. \] (7)

- Statistical errors cancel in the ratio
- Most of the axial current renormalization cancels with the vector current renormalization. The remainder can be computed perturbatively.
- This ratio gives (the lattice approximation of) \( h_{A_1} \) directly to all orders in HQET
- The ratio can then be calculated at the tuned \( m_{b,c} \), so that many kappa values are not needed.
Numerical data

Extracting correlation functions that contain staggered quarks presents a special challenge because of the contributions of wrong parity excited states which introduce oscillations into the usual plateau fits. The average,

$$C_{avg}^{X \rightarrow Y}(0, t, T) \equiv \frac{1}{2} C^{X \rightarrow Y}(0, t, T) + \frac{1}{4} C^{X \rightarrow Y}(0, t, T + 1)$$

$$+ \frac{1}{4} C^{X \rightarrow Y}(0, t + 1, T + 1),$$

is equivalent to a smearing which suppresses the oscillating states, and has been applied to all of the data for the double ratios.
Approach to chiral fits

For the chiral fits we form two ratios:

\[ R_{\text{sea}} = \frac{h_{A_1}(m_x^{\text{fid}}, m_L, m_S, a)}{h_{A_1}(m_x^{\text{fid}}, m_L^{\text{fid}}, m_S^{\text{fid}}, a)}, \quad R_{\text{val}} = \frac{h_{A_1}(m_x, m_L, m_S, a)}{h_{A_1}(m_x^{\text{fid}}, m_L, m_S, a)}. \]

\[ m_x = m_x^{\text{fid}} \approx 0.5 m_{\text{physical strange}}, \]
\[ m_L = m_L^{\text{fid}} \approx 0.5 m_{\text{physical strange}}, \]
\[ m_S = m_S^{\text{fid}} \approx m_{\text{physical strange}}. \]

This has the advantage that correlators are even better behaved in what are now quadruple ratios. Another advantage is that heavy quark discretization errors largely cancel in these ratios, so that we can disentangle the heavy quark discretization effects and those of the staggered chiral logs.
$R_{\text{val}}$ plateau
Staggered ChPT formula

\[ h_{A_1}^{2+1} = 1 + X_A + \frac{g_\pi^2}{48\pi^2 f^2} \left[ \frac{1}{16} \sum_B (2F_{\pi B} + F_{KB}) - \frac{1}{2} F_{\pi I} + \frac{1}{6} F_{\eta I} 
\right.

\[ + a^2 \delta'_V \left( \frac{m_{SV}^2 - m_{\pi V}^2}{(m_{\eta V}^2 - m_{\pi V}^2)(m_{\pi V}^2 - m_{\eta V}'^2)} F_{\pi V} 
\right.

\[ + \frac{m_{\eta V}^2 - m_{SV}^2}{(m_{\eta V}^2 - m_{\eta V}'^2)(m_{\eta V}'^2 - m_{\pi V}^2)} F_{\eta V} 
\]

\[ + \frac{m_{SV}^2 - m_{\eta V}^2}{(m_{\eta V}^2 - m_{\eta V}'^2)(m_{\pi V}^2 - m_{\eta V}'^2)} F_{\eta V}' \right) + (V \rightarrow A) \right] , \]

where \( a \) is the lattice spacing, \( \delta'_V \), \( g_\pi \) and \( X_A \) are constants, and \( F \) is a complicated function involving logs. (Laiho and Van de Water, hep-lat/0512007)
Valence chiral extrapolation

![Graph showing valence chiral extrapolation](image-url)
Sea chiral extrapolation

![Graph showing $R_{seal}$ versus $m_{\pi,seal}^2$ (GeV$^2$)](image)

- Medium coarse (0.15 fm)
- Coarse (0.12 fm)
- Fine (0.09 fm)
- Extrapolated value
Chiral Extrapolation

The graph shows the extrapolation of $h_{A_1}(1)$ as a function of $m_{\pi}^2$ (GeV$^2$) for different lattice spacings: medium coarse (0.15 fm), coarse (0.12 fm), and fine (0.09 fm). The extrapolated value is indicated by a cross symbol.

- Phase Transition
- Finite-Volume Effects
## Total error budget

<table>
<thead>
<tr>
<th>Uncertainty</th>
<th>$h_{A_1}(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>statistical</td>
<td>1.2%</td>
</tr>
<tr>
<td>$g_\pi$</td>
<td>0.6%</td>
</tr>
<tr>
<td>NLO vs partial NNLO ChPT fits</td>
<td>0.9%</td>
</tr>
<tr>
<td>discretization errors</td>
<td>1.3%</td>
</tr>
<tr>
<td>kappa tuning</td>
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</tr>
<tr>
<td>perturbation theory</td>
<td>0.4%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>2.3%</strong></td>
</tr>
</tbody>
</table>
Discretization errors

\begin{figure}
\centering
\includegraphics[width=\textwidth]{discretization_errors}
\caption{Discretization errors for different values of $a^2$.}
\end{figure}

\begin{align*}
\text{Lat '07, August 2, 2007 – p.21/24}
\end{align*}
Kappa tuning error
Most of the current renormalization cancels in the double ratio, leaving only a small perturbative correction. In the present calculation this was 0.4%.

As a conservative estimate we take this as the uncertainty in the perturbative error.
Final Result

\[ h_{A_1}(1) = 0.924(11)(19) \]

where the first is error is statistical and the second systematic.

This is consistent with the earlier quenched result of \([0.913^{+0.024}_{-0.017}-0.030]\).

Plan for the future:

We plan to extend runs to “super fine” MILC lattices (0.06 fm) in order to reduce discretization errors and heavy quark mass tuning errors, our largest uncertainties.