Sign problem in two-color two-flavor QCD with quark and isospin chemical potentials

Presenter: Kenji Fukushima — RIKEN BNL Research Center

Kenji Fukushima

We revisit the sign problem in two-color QCD at finite density. It is well-known that the Dirac determinant in two-color QCD takes a real value. In the presence of two degenerate fermion species (i.e. quark flavors), therefore, the Dirac determinant is non-negative so that the Monte-Carlo simulation on the lattice is possible. We have looked into the eigenvalue distribution of the Dirac operator given as the Wilson fermion to confirm a quartet structure on the complex eigenvalue plane. Because of the characteristic pattern of the eigenvalue distribution, we have shown that the Dirac determinant is "almost" non-negative even in case of one quark flavor. Minor exceptions would originate from the eigenvalue distribution widened by the Wilson term which may have an overlap on the origin in the complex eigenvalue plane. If the overlap takes place, the Dirac determinant could take a negative value. This should be, however, regarded as a finite-density extension of the parity broken (Aoki) phase, which is quite common in the formulation of the Wilson fermion. Thus, we would insist that the lattice simulation is feasible in two-color QCD even without two (or even-number) degenerate quark flavors.

The physically interesting application is two-color QCD with u and d flavor quarks with not only a quark chemical potential but also an isospin chemical potential, or equivalently, two independent chemical potentials for u and d quarks. In such a circumstance, the Cooper pairs in the superfluid phase in two-color QCD at high density feel a 'pressure' that tends to tear the pair apart. Then, inhomogeneous pairing is favored energetically rather than a homogeneous superfluid, which leads to a crystalline structure of the pairing gap energy.

In my talk, I will discuss the sign problem from the point of view of the eigenvalue spectrum in two-color QCD. I will then point out the feasibility to probe the structure of the crystalline superfluid phase in the lattice simulation.