Effects of the disconnected flavor singlet corrections on the hyperfine splitting in charmonium

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[Lattice 2007, Regensburg]
Motivation

Problem in charmonium hyperfine splitting:

\[ M(\psi(1S) - \eta_c(1S)) \]

- fine (◊): 106^{+2}_{-2} \text{ MeV}
- coarse (○): 102^{+1}_{-1} \text{ MeV}
- med coarse (+): 101^{+4}_{-4} \text{ MeV}
- extra coarse (×): 95.2^{+1.0}_{-1.2} \text{ MeV}
Diagram contributions to the full propagator

- Connected and disconnected (singlet) diagrams:

\[
\Gamma \quad \Gamma
\]

\[\sim \frac{1}{p^2 + m_c^2}\]

\[
\Gamma \quad \lambda \quad \Gamma
\]

\[\sim \frac{1}{p^2 + m_c^2} \frac{\lambda}{p^2 + m_c^2}\]
Lattice method for disconnected diagrams

The disconnected part of the correlator is calculated as:

\[ D(t) = \langle L(0)L^*(t) \rangle , \quad L(t) = \text{Tr}(\Gamma M^{-1}) \]

Previous works explore the ratio:

\[ \frac{D(t)}{C(t)} = \frac{\langle m^f \rangle}{\langle m^c \rangle} - 1 = \frac{A_f}{A_c} e^{(m_c - m_f)t} - 1. \]

Considering that the available lattices are quenched with respect to the charm quark, an appropriate fitting form would be

\[ \frac{D(t)}{C(t)} = (m_c - m_f)t + \frac{m_c - m_f}{m_c} \]

if correlators are normalized appropriately.
Previous results (2004)

- QCD-TARO: P. de Forcrand et al., JHEP 0408 (2004), hep-lat/0404016
  - $12^3 \times 32$, $a^{-1} \approx 1.2$ GeV, 3200 configs, $N_f = 2$ staggered with $am = 0.1$.
  - Valence quarks - Fermilab action. $k \in [0.093, 0.113]$.
  - Disc. diagrams estimated stochastically with Z2 noise and Gaussian smearing functions with width – expected charmonium radius. Random sources 50 - 300 (light - heavy mass).

  - $16^3 \times 32$, $a^{-1} \approx 1.8$ GeV, 778 configs, $N_f = 2$ clover fermions $k = 0.135$.
  - Valence quarks - Fermilab action. $k = 0.113, 0.119, 0.125$.
  - Disc. diagrams – with Z2 noise, 100 random sources, local and fuzzied operators.

CONCLUSIONS: Disconnected diagrams – very noisy. Signal worse for heavy valence quarks. For the $\eta_c$ at $k_c$, $m_f - m_c \approx \pm 20$ MeV is very roughly estimated.
**Our calculation**

We use 505 Asqtad 2+1 flavor lattices with $V = 40^3 \times 96$ and $a^{-1} \approx 0.09$ fm. The valence quarks are clover type with tuned $k_c = 0.127$.

**Improvements for the stochastic estimation of traces: Unbiased subtraction**

- We employ the unbiased subtraction technique (C. Thron et al., Phys. Rev. D57, 1642 (1998))

$$Var[M^{-1}_{ij}] = \frac{1}{L} \{ [M^{-1}_{ij}]^2 C_2^2 + \sum_{k \neq j} [M^{-1}_{ik}]^2 \}$$

- Unbiased subtraction reduces the off diagonal elements:

$$\text{Tr} \Gamma M^{-1} = \text{Tr}(\Gamma M^{-1} - \Gamma \sum_i Q_i)$$

where $Q_i$ are traceless operators with off diagonal elements similar to these of $M^{-1}$.

- Choosing $Q_i$ to be the terms in the hopping parameter expansion to $O(3)$:

$$Q_0 = I \quad Q_1 = kD \quad Q_2 = k^2 D^2 \quad Q_3 = k^3 D^3$$

If $\text{Tr}Q_i \neq 0$ the trace is explicitly computed and added above. With 12 Z2 spin and color diluted sources (144 inversions per lattice) the gauge fluctuations dominate the error.
The subtraction terms for the pseudoscalar ($\eta_c$)

\[
\begin{align*}
\text{Tr}(\gamma_5 M^{-1}) &= \text{Tr}(\gamma_5 M^{-1} - \gamma_5 - k\gamma_5 D - k^2\gamma_5 D^2 - k^3\gamma_5 D^3) \\
&\quad + k^2\text{Tr}(\gamma_5 D^2) + k^3\text{Tr}(\gamma_5 D^3)
\end{align*}
\]

\[
\begin{align*}
\text{Tr}(\gamma_5 D^2) &= \text{Tr}(\gamma_5 \sum_{\mu<\nu} \sum_{\rho<\eta} \sigma_{\mu\nu}\sigma_{\rho\eta} F_{\mu\nu} F_{\rho\eta})
\end{align*}
\]

\[
\begin{align*}
\text{Tr}(\gamma_5 D^3) &= \text{Tr}(\gamma_5 \sum_{\mu<\nu} \sum_{\rho<\eta} \sum_{\alpha<\beta} \sigma_{\mu\nu}\sigma_{\rho\eta}\sigma_{\alpha\beta} F_{\mu\nu} F_{\rho\eta} F_{\alpha\beta})
\end{align*}
\]

Improvement: using the point-to-point $\eta_c$ propagator

- Calculating the disconnected point-to-point propagator improves statistics. It has from one to three orders of magnitude smaller relative errors than the time-slice-to-time-slice disconnected propagator in the region where we have a signal.

- Data points at non-integer distances are included - beneficial since the charmonium signal disappears quickly at larger distances.
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Asymptotic behavior of the disconnected propagator

- At large distances the dominant behavior of the connected propagator is:

\[ C(r) \sim A e^{-mc r \frac{3}{r^2}}. \]

- The disconnected propagator asymptotically will be:

\[ D(r) \sim -\frac{d}{dm_c^2}C(r) \sim B e^{-mc r \frac{1}{r^2}}. \]

- Their ratio:

\[ \frac{D(r)}{C(r)} \approx \frac{B}{A} r \]

where

\[ \frac{B}{A} = mc - m_f. \]
Naive ratio $D(r)/C(r)$

- Lattice artifacts due to rotational symmetry breaking are visible.
- Ratio changes sign.
- Contributions of heavy excited states and light modes.
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Extracting the $\eta_c$ signal from $D(r)$

$D(r)$ is a sum of ground $\eta_c$ state, excited states and light states which dominate at large distances (and flip the sign of $D(r)$).
From studies of the connected propagator we have an idea of roughly how much the excited states contribute to the $\eta_c$ signal: at $r > 5$ less than 20%. For $r < 5$ both excited states contribution and the lattice artifacts are significant in the $D(r)$.

The light states are dominant for $r > 4$ but we estimate that the charmonium signal is still about 30% of the whole signal at $r = 5$.

We want a fitting form which includes the contributions of

- Light states: $\frac{L}{r^2} e^{-mr}$
- Disconnected $\eta_c$ signal and excited state: $\frac{B}{r^2} (e^{-m_cr} + e^{-m^*_cr})$
- “Mixed” excited state: $\frac{cB}{r^2} (e^{-m_cr} - e^{-m^*_cr})$

\begin{itemize}
  \item $m_c$ (ground)
  \item $m_c$ (excited)
  \item $m^*_c$ (mixed excited)
\end{itemize}
Fitting results

\[ D_{\text{fit}}(r) = \frac{B}{r^2} \left( e^{-m_c r} + e^{-m_c r^*} \right) + \frac{cB}{r^3} \left( e^{-m_c r} - e^{-m_c r^*} \right) + \frac{L}{r^2} e^{-m_l r} \]

- The light mass \( m_l = 0.43(1) \) is determined from a single exponential fit from \( r = 7 - 12 \). In the above fit it is fixed to that value.
- The connected \( \eta_c \) and \( \eta_c^* \) masses, \( m_c = 1.1598(7) \) and \( m_c^* = 1.51(5) \), are known from fits to the connected propagator \( C(t) \). They are used as constants in the fit as well.
- The constant \( c \approx 7 \) comes from various assumptions in our model. The fit is not very sensitive to its exact value.
- Results for fitting range \( r = 5 - 11 \):

\[ \frac{B_{\text{fit}}}{A_{\text{fit}}} = m_c - m_f \in [-4, -1] \text{MeV} \]

- Our fit favors disconnected diagram contribution which slightly increases the \( \eta_c \) mass. This is the opposite of the perturbative expectation of \( \sim 2.4 \text{ MeV decrease} \).
- If the OZI rule for the \( J/\Psi \) holds \( \Rightarrow \) slight decrease of the hyperfine splitting.
Summary

- We have calculated the disconnected pseudoscalar propagator with very high precision using both unbiased subtraction and point-to-point data.

- We estimate that the disconnected diagram contribution to $\eta_c$ mass will increase it by 1-4 MeV. The effect on the hyperfine splitting is possibly in the direction of slightly decreasing the splitting.

- **Future plans:**
  - Developing a fitting procedure which takes into account the lattice artifacts at small distances.
  - Studying the vector as well (data available).
  - Including calculations at other lattice spacings.