The spatial string tension and dimensional reduction in QCD.

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For RIKEN-BNL-Columbia-Bielefeld collaboration

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The spatial string tension and dimensional reduction in QCD.
Lattice sizes.
- $16^34$ and $24^36$.

Action.
- Gauge
  - Improved $1 \times 2$ gluon action.
- Fermion
  - p4
  - fat3
  - $N_F = 2 + 1$, with $m_s \approx m_s^{\text{phys}}$

LCP.
- $m_\pi \approx 220\text{MeV}$
- $m_K \approx 500\text{MeV}$

RHMC algorithm.

Statistics $O(10^4)$ for each data set.
Spatial Wilson loops analysed.

\[ aV(R) = \lim_{Z \to \infty} \log \frac{W(R, Z)}{W(R, Z + 1)} \]

- 30 smearings.
- \( Z = 3 \) sufficient for stability.

Figure: \( \frac{T}{T_c} = 1.66, 24^36. \)
To parameterise known cut-off effects.

\[
\frac{1}{[R]} = 4\pi \int_{-\pi}^{\pi} \frac{d^3 k}{(2\pi)^3} e^{-k.R} D_0(k)
\]

\(D_0\): Free lattice gluon propagator.

\[
aV(R) = -\frac{\hat{\alpha}}{[R]} + \hat{c}_0 + \hat{s}_s[R]
\]
The spatial string tension and dimensional reduction in QCD.

Introduction
Spatial string tension
Dimensional reduction
Conclusions

Lattice setup
Finding the potential
Fitting the potential

The spatial string tension and dimensional reduction in QCD.
Example: \( \frac{T}{T_c} = 1.66, \ R_{\text{max}} = 8 \)

Correlation in the potential increases with distance.

- Control by truncating fits.
- Need to be sensitive to \( \alpha \).
  - \( \hat{\alpha} \approx 0 \) for \( R_{\text{min}} \geq 4 \)
  - Take \( R_{\text{min}} = 3 \)

Typical fit range.

- \( 0.2 \text{fm} \rightarrow 0.8 \text{fm} \).
From QCD to EQCD

\[ S_{\text{QCD}} = \int_0^\frac{1}{T} dt \int d^3x \left( \frac{1}{g^2} F^{\mu\nu} F_{\mu\nu} + \bar{\psi} \gamma. D\psi \right) \]

- Masses of modes.
  - Bosonic: \(2\pi n T\).
  - Fermionic: \(2\pi (n + 1) T\).
- Only zero mode of the bosonic field remains massless.
  - Integrate out massive modes.
    - Effective theory for length scales \(>> \frac{1}{T}\).

\[ S_{\text{EQCD}} = \int d^3x \frac{1}{g_E^2} \text{Tr} F_{ij}(x) F_{ij}(x) + \text{Tr}[D_i, A_0(x)]^2 + m_E^2 \text{Tr} A_0(x)^2 \]

\[ g_E^2 = g^2(T) T \quad \quad m_E^2 \approx g^2 T^2 \]
From EQCD to MQCD

\[
S_{\text{EQCD}} = \int d^3 x \frac{1}{g_E^2} \text{Tr} F_{ij}(x) F_{ij}(x) + \text{Tr}[D_i, A_0(x)]^2 + m_E^2 \text{Tr} A_0(x)^2
\]

- Integrate out \(A_0\).
  - Pure gauge theory.
  - Effective theory for length scales \(\gg \frac{1}{g_T}\).

\[
S_{\text{MQCD}} = \int d^3 x \frac{1}{g_M^2} \text{Tr} F_{ij}(x) F_{ij}(x)
\]
\[ g_M^2 \text{ sets a scale.} \]

\[ \sqrt{\sigma_s} = c g_M^2 \]

\[ g_M^2 = g^2(T) T \]

\[ \frac{T}{\sqrt{\sigma_s}} = \frac{1}{c} g^{-2}(T) \]

\[ g^{-2}(T) = 2 \beta_0 \ln \left( \frac{T}{\Lambda_{\sigma}} \right) + \frac{\beta_1}{\beta_0} \ln \left( 2 \ln \left( \frac{T}{\Lambda_{\sigma}} \right) \right) \]
Results

- Fit to $\Lambda_\sigma/T_c$ and $c$ for $T/T_c \geq 2$. 

![Graph showing the spatial string tension and dimensional reduction in QCD.](image)
Results

\[ \frac{T}{\sqrt{\sigma_s}} = \frac{1}{c} 2\beta_0 \ln \left( \frac{T}{\Lambda_{\sigma}} \right) + \frac{\beta_1}{\beta_0} \ln \left( 2 \ln \left( \frac{T}{\Lambda_{\sigma}} \right) \right) \]

<table>
<thead>
<tr>
<th>( N_T = 8 ), Pure gauge</th>
<th>( \Lambda_{\sigma} )</th>
<th>( T_c )</th>
<th>( c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_T = 6 ), ( N_F = 2 + 1 )</td>
<td>0.099(87)</td>
<td>0.575(14)</td>
<td></td>
</tr>
<tr>
<td>( N_T = 4 ), ( N_F = 2 + 1 )</td>
<td>0.100(23)</td>
<td>0.607(39)</td>
<td></td>
</tr>
</tbody>
</table>

**Table:** Fits parameters, \( T > 2T_c \)

Compare with other measurements.

\[ c_{3D}^{\text{gauge}} = 0.553(1)^1 \]

2-loop prediction for $g_E^2(T)$ exists\textsuperscript{2}.

Connection between $g_M^2$ and $g_E^2$ known to 2-loop.

$$\frac{\sqrt{\sigma_s}}{g_M^2} = 0.553(1)$$

We need.

- $\Lambda_{\overline{MS}}$.
- $T_c$ at our lattice spacings.

\textsuperscript{2}M. Laine and Y Schröder JHEP 0503:067,2005.

Determination of $\Lambda_{\overline{MS}}$

- Obtained from measurement of $\alpha_V(7.5 \text{GeV}) = 0.2082(40)$
  - Matching
    - Potential calculated in $\overline{V}$ scheme.
    - Potential calculated in $\overline{MS}$ scheme to two-loops.
  - $\Lambda_{\overline{MS}}$ found through 2-loop $\beta$-function.
  - Ambiguity in matching schemes.

$$\Lambda_{\overline{MS}} = 0.322(19) \text{ GeV} \quad \Lambda_{\overline{MS}} = 0.285(15) \text{ GeV}$$

Matching at same scale. Matching $\alpha_V(7.5 \text{GeV})$ with $\alpha_{\overline{MS}}(e^{-\frac{5}{6}}7.5 \text{GeV})$

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Results, $N_T = 4$

- Band represents $0.260 < \Lambda_{\overline{MS}} < 0.341$
Results, $N_T = 6$

- Band represents $0.260 < \Lambda_{\overline{MS}} < 0.341$
Spatial string tension, $\sigma_s$, found for.

- $N_F = 2 + 1$, $N_T = 4, 6$.
- $1 \leq \frac{T}{T_c} \leq 4$.

2 loop reduction formulae provides a good description.

- Large ambiguity is $\Lambda_{\overline{MS}}$. 