Higgs mechanism in five dimensional gauge theories

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Outline

Gauge theories in extra dimensions
\( SU(N) \) on \( \mathbb{R}^4 \times S^1 / \mathbb{Z}_2 \)

What perturbation theory tells us
- Kaluza-Klein decomposition and mass eigenvalues
- the Coleman Weinberg potential at 'infinite cutoff'

Lattice simulations

Perturbation theory at finite cutoff
- the Coleman Weinberg potential revisited
- connection to lattice results

Conclusions and outlook
Gauge theories in extra dimensions

- **explain origin of the Higgs**: some of the extra dimensional components of the gauge field play the role of a fundamental scalar in 4d.

- Higgs potential is generated by quantum corrections. [Coleman, Weinberg, 1973; Hosotani, 1983]

- finiteness of the Higgs mass to all orders without SUSY.

- compact extra dimensions of size $R$, here: 4 + 1 d on $S^1/\mathbb{Z}_2$

- **triviality**, interactions only at finite cutoff.

Parameter space described by dimensionless quantities

$$N_5 = \pi R \Lambda, \quad \beta = \frac{2N}{g_5^2 \Lambda}.$$  

\begin{center}
\begin{tikzpicture}
\fill[red] (0,0) circle (1pt) node[above left] {trivial point};
\fill[blue] (0,1) circle (1pt) node[below] {trivial point: $g_4^2 = \frac{N}{\beta N_5} \to 0$};
\fill[blue] (1,0) circle (1pt) node[above right] {Coulomb phase};
\fill[blue] (0,0) circle (1pt) node[below] {confined};
\fill[blue] (0,1) circle (1pt) node[above right] {Coulomb?};
\fill[blue] (1,0) circle (1pt) node[above] {trivial point?};
\draw[->] (0,0) -- (1,1); \\
\end{tikzpicture}
\end{center}
$SU(N)$ on $\mathbb{R}^4 \times S^1/\mathbb{Z}_2$

identify gauge fields related by the reflection $x_5 \to -x_5$

- two orbifold fixed points at $x_5 = 0$, $x_5 = \pi R \Rightarrow 4d$ boundaries
- identification up to gauge group conjugation by $g$: Dirichlet b.c.

\[
\begin{align*}
&gA_\mu g^{-1} = A_\mu \\
gA_5g^{-1} = -A_5
\end{align*}
\] \Rightarrow \quad g^2 \in \text{center of } SU(N)

breaking of gauge symmetry

- only even field components have zero modes

$SU(2) \to U(1) : (g = -i\sigma^3) \Rightarrow$ even fields:
$A_5^{1,2}$ "Higgs" and $A_5^3$ "Z"

$SU(3) \to SU(2) \times U(1) : (g = \text{diag}(1,1,-1)) \Rightarrow$ even fields:
$A_5^{4,5,6,7}$ "Higgs doublet" and $A_\mu^{1,2,3,8}$ "$W^\pm$, Z, photon"
What perturbation theory tells us
Kaluza-Klein decomposition and mass eigenvalues

- Expand fields in KK basis

\[
E(x,x_5) = \frac{1}{\sqrt{2\pi R}} E^{(0)}(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} E^{(n)}(x) \cos(nx_5/R) \quad \text{even fields}
\]

\[
O(x,x_5) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} O^{(n)}(x) \sin(nx_5/R) \quad \text{odd fields.}
\]

- \(A_5\) is a scalar from 4 d point of view \(\Rightarrow\) can acquire a vev \(\langle A_5 \rangle \neq 0\).

We define

\[
\alpha = g_5 \langle A_5 \rangle R.
\]
Lagrangian and mass eigenvalues

\[ \mathcal{L} = -\frac{1}{2g_5^2} \text{tr}\{F_{\mu\nu}F_{\mu\nu}\} - \frac{2}{2g_5^2} \text{tr}\{F_{\mu5}F_{\mu5}\} - \frac{1}{g_5^2 \xi} \text{tr}\{(\bar{D}_M A_M)\}^2 \]

where \( \bar{D}_M F = \partial_M F + [\langle A_M \rangle, F] \).

\( \Rightarrow \) mass operator \( \bar{D}_5 \bar{D}_5 \) for \( A_\mu \) and \( A_5 \) with the eigenvalues:

for \( SU(2) \) [Kubo, Lim, Yamashita, 2002]

- zero modes: \( A_\mu^{3,0} \) ('Z boson') \( m_Z R = \alpha \)
  \( A_5^{1,0} \) ('scalar') \( m_{A_5} R = \alpha, 0 \)

- higher modes: \( (m_n R)^2 = \frac{n^2}{R^2}, \frac{(n\pm \alpha)^2}{R^2} \)
What perturbation theory tells us
Coleman Weinberg potential at 'infinite cutoff'

One loop potential in 4 d for scalar particle of mass $M$ [Coleman, Weinberg, 1973]

$$\int D\phi e^{-S_E} \approx e^{-V} \equiv \det [\partial_\mu^2 + M^2]^{-\frac{1}{2}} \implies V = -\frac{1}{2} \sum_n \int \frac{dt}{t} \text{tr}\{e^{-t(m_n^2 + p^2)}\}$$

We sum over all KK mass states in $V$. After a Poisson resummation, we obtain for $SU(2)$

$$V = -\frac{9}{64\pi^6 R^4} \sum_{m=1}^{\infty} \frac{\cos(2\pi m\alpha)}{m^5} \quad \text{Minimum at } \alpha = 0 \mod \mathbb{Z}!$$

- gauge particle mass: $m_Z = 0$
- remnant $U(1)$ symmetry not broken!
- scalar mass: $(m_H R)^2 = \frac{N}{N_5 \beta} \frac{d^2 V}{d\alpha^2} \bigg|_{\alpha=\alpha_{\text{min}}} = g_4^2 \frac{9\zeta(3)}{16\pi^4} \bigg|_{\alpha=0} \to 0$

at trivial point where $g_4 \to 0$.

Is this true everywhere in the parameter space?  No!

$$\Rightarrow$$ lattice simulations [Knechtli, Irges, 2006]
Lattice simulations

- Simulation of pure $SU(2) \rightarrow U(1)$ on $\frac{T}{a} \times \frac{L}{a}^3 \times N_5$ lattices. Periodic boundary conditions in 4 d, Dirichlet b.c. in $d = 5$, Wilson plaquette action.

- Compact geometry $N_5 \ll L/a, T/a$, here $T/a = 96, L/a = 12, N_5 = 6$

- massive Z contrary to CW result $\Rightarrow$ SSB! Higgs phase.

- Higgs significantly heavier than suggested by 1-loop pt

- phase transition [Creutz, 1979] at $\beta_c = 1.607$, masses only for $\beta > \beta_c$. 
describe lattice action by an effective lagrangean [Symanzik, 1981]

\[-\mathcal{L} = \frac{1}{2g_5^2} \text{tr}\{F_{MN}F_{MN}\} + \sum_{p_i} c^{(p_i)}(N_5, \beta) \ a^{p_i-4} \ \mathcal{O}^{(p_i)} + \ldots\]

with \(\mathcal{O}^{(p_i)}\) operators of dimension \(p_i > 4\).

\[c \ \mathcal{O}^{(6)} = \sum_{M,N} \frac{c}{2} \text{tr}\{F_{MN}(D_M^2 + D_N^2)F_{MN}\}, \quad c \equiv c^{(6)}(N_5, \beta) = \frac{1}{12} + \ldots\]

\[c_0 \ \mathcal{O}^{(5)} = \frac{\pi ac_0}{4} F_{5\mu} F_{5\mu} [\delta(x_5) + \delta(x_5 - \pi R)], \quad c_0 \equiv c^{(5)}(N_5, \beta)\]
Perturbation theory at finite cutoff
Coleman Weinberg potential revisited

\[ c \mathcal{O}^{(6)} = \sum_{M,N} \frac{c}{2} \text{tr} \{ F_{MN} (D_M^2 + D_N^2) F_{MN} \}, \quad c \equiv c^{(6)}(N_5, \beta) = \frac{1}{12} + \ldots \]

\[ c_0 \mathcal{O}^{(5)} = \frac{\pi a c_0}{4} \hat{F}_{5\mu} \hat{F}_{5\mu} [\delta(x_5) + \delta(x_5 - \pi R)], \quad c_0 \equiv c^{(5)}(N_5, \beta) \]

modified mass operator

\[ \bar{D}_5 \bar{D}_5 + \frac{a^2}{12} (\bar{D}_5 \bar{D}_5)^2 \quad \text{boundary correction for gauge particle} \]

new eigenvalues (truncated at \( O(a^2), O(\frac{1}{n}) \))

zero mode \((m_{ZR})^2 = \alpha^2 + \frac{c_0 \alpha^2}{2} \frac{\pi}{N_5} + c \alpha^2 \frac{\pi^2}{N_5^2} \)

\[(m_{nR})^2 = n^2, \quad n > 0\]

\[ = (n \pm \alpha)^2 + \frac{c_0 \alpha^2}{2} \frac{\pi}{N_5} + c(n \pm \alpha)^2 \frac{\pi^2}{N_5^2}, \quad n \geq 0 \]
Perturbation theory at finite cutoff
Coleman Weinberg potential revisited

$V_{\text{eff}}(c, c_0) = 13.00, c_0 = 0.0121$
$c = 0, c_0 = 0.0121$
$c = 13.00, c_0 = 0$
$c = c_0 = 0$

◮ $S^1(c_0 = 0)$, SSB occurs for $c > 1.72$, sharp transition of $\alpha_{\text{min}} \rightarrow \frac{1}{2}$
◮ orbifold: periodicity of potential is lost, SSB for large enough $c$, $\alpha_{\text{min}}$ varies continuously
◮ No SSB induced by boundary term alone
Comparison to lattice results

gauge boson masses

1. We compute $m_Z$, $m_Z^*$ and $\langle \text{tr} \{\phi \phi^\dagger\} \rangle$ from $L/a = 12$, $T/a = 96$, $N_5 = 6$ lattice.

2. determine $\alpha_{\text{lat}}(\beta)$ by

$$\alpha_{\text{lat}}(\beta) = \sqrt{\frac{\langle \text{tr} \{\phi \phi^\dagger\} \rangle N_5^2}{2\pi}}$$

- compare to corrected KK masses at $c = 13.0$, $c_0 = 0.0121$, (ground state, 1st excited state)
- minimum of CW potential for these coefficients $\alpha_{\text{min}} = 0.225$
Comparison to lattice results

ratio of Higgs to Z mass

\[
\rho_{HZ^0} = \frac{m_H}{M_Z}
\]

1. determine \( \alpha_{\text{lat}}(\beta) \) from simulation
2. tune \( c, c_0 \) such that CW potential such that 
   \( \alpha_{\text{min}} = \alpha_{\text{lat}}(\beta) \)
3. compute the Higgs mass from the potential

\[
(m_H R)^2 = \frac{N}{N_5 \beta} R^4 \frac{d^2 V}{d\alpha^2} \bigg|_{\alpha_{\text{min}}}
\]

\( \rho_{HZ^0} > 1 \) can be reached on the lattice.
Conclusions

- Lattice simulations of $SU(2)$ on the orbifold. $SU(2) \xrightarrow{Z_2} U(1)$ with a massive gauge particle.
- Contradiction to 1-loop perturbative result can be resolved by taking into account an explicit cutoff in perturbation theory.
- We get good qualitative agreement between the new CW result and the simulation data.
Outlook

- SU(3): CW calculation at finite cutoff shows that experimental value of the Weinberg angle \(\cos \theta_W \approx 0.877\) can be reached.
- SU(3): \(\rho_{HZ^0} > 1.25\) is possible for small \(N_5\) ⇒ anisotropic lattices
- localization \([\text{Dvali, Shifman}]\) for large \(N_5\) close to phase transition.