Neutral mesons and disconnected diagrams in Twisted Mass QCD

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Topics

TMQCD has disconnected contributions for neutral mesons. Here $N_f = 2$ degenerate sea quarks with valence=sea. update of hep-lat/0701012 & Plenary C. Urbach

- Evaluation of disconnected diagrams (variance reduction)
- TMQCD: order($a^2$) effects in pions
- Flavour singlet pseudoscalar meson ($\eta_2$)
- Vector mesons, decay,..
- Flavour singlet scalar meson ($f_0$)
Disconnected diags - variance reduction

TMQCD has a degenerate pair of $u, d$ quarks with Wilson-Dirac matrix:

$$M_{u,d} = M_W \pm i\mu \gamma_5.$$ 

Hence $1/M_u - 1/M_d = -2i\mu(1/M_d)\gamma_5(1/M_u)$.

This can be evaluated efficiently using the ‘one-end-trick’ with a stochastic volume source $\xi$. One solves to obtain $\phi = (1/M_u)\xi$. Then the required disconnected loop is given by

$$\sum X (1/M_u - 1/M_d) = -2i\mu \sum \langle \phi^* X \gamma_5 \phi \rangle_r$$

signal/noise: Conventional $1/\sqrt{V}$  Here $V/\sqrt{V^2} = 1$
Disconnected diags - variance reduction II

Example: $\beta = 3.9$, $\mu = 0.004$, $X = \gamma_5$

$M(\pi) \approx 300$ MeV, $a = 0.086$ fm, $La = 2.1$ fm, for $\eta_2$ correlator.

$\sigma = 18$ from inherent gauge-time variation

$\sigma = 69.5$ from 24 samples of volume source (conventional)

$\sigma = 16.6$ from above method (12 samples).

So 12 inversions give the disconnected correlator from all $t$ to all $t'$ with no significant increase in errors from the stochastic evaluation.

For cases where this method cannot be used

eg $\pi^0$ with $\bar{\psi}\gamma_5\tau_3\psi \rightarrow \bar{\psi}I\psi$

- we use hopping parameter variance reduction instead.
Neutral pions

\[ M(\pi^+) \approx 300 \text{ MeV}, \ a = 0.086 \text{ fm}, \ La = 2.1 \text{ fm} \]

\[ \bar{\psi} I \psi \quad \bar{\psi} \gamma_4 \gamma_5 \tau_3 \psi \]

Disconnected pieces are relatively large - and reduce charge splitting. \( \pi^0 \) lighter than \( \pi^+ \). \( \Delta m a = 0.028(7) \) from \( 4 \times 4 \) fit.
Pion order($a^2$) effects

We expect $r_0^2(m(\pi^0)^2 - m(\pi^+)^2) = c(a/r_0)^2$

Flavour splitting decreases as $a^2$.
Sign and behaviour consistent with Chiral PT and nature of phase transition where $m(\pi^0) = 0$.
Eg. $m(\pi^+) = 200$ MeV with less than 20% splitting if $r_0/a > 8.2$. 
Flavour singlet PS meson: $\eta_2$

With $N_f = 2$ degenerate quarks, the flavour singlet pseudoscalar meson (called $\eta_2$) is related to the experimental $\eta'(958)$ and is expected to have mass around 800 MeV. hep-lat/0006020

Here we summarise results with light pions and $a < 0.1$ fm ($r_0/a > 4.5$).

$\eta_2$ mass is consistent with a constant behaviour in the chiral limit with $m(\eta_2) \approx 0.88$ GeV ($r_0 m(\eta_2) = 2$).
Flavour singlet PS meson: errors

Measure all $t$ and $t'$; 888 configurations; stochastic error small:
Yet disconnected correlator is noisy- why?

Problem is that signal comes from only a small part of the data sample
For instance (at $\mu = 0.004$) with $|t' - t| = 10$, 2.1% of the data contribute
26% of the signal.
Message: even more configurations are needed to get errors down further.
Vector mesons

We compare the LL correlators (including disconnected parts for neutral meson) for vector mesons:

Ratio is consistent with constant (ie no mass splitting) implying $Z_A/Z_V \rightarrow \sqrt{1.5}$ (consistent with Dimopoulos talk).

Note disconnected contribution is negligible.
Vector mesons - masses

From fits (from connected neutral contribution only here) we obtain masses (in lattice units)

<table>
<thead>
<tr>
<th>Meson</th>
<th>$\mu = 0.004$</th>
<th>$\mu = 0.004$</th>
<th>$\mu = 0.0085$</th>
<th>$\mu = 0.003$</th>
<th>$\mu = 0.006$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>32</td>
<td>24</td>
<td>24</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>$\rho^+$</td>
<td>.416(12)</td>
<td>.404(22)</td>
<td>.428(8)</td>
<td>.337(20)</td>
<td>.337(12)</td>
</tr>
<tr>
<td>$\rho^0$</td>
<td>.400(25)</td>
<td>.395(17)</td>
<td>.419(17)</td>
<td>.372(29)</td>
<td>.346(12)</td>
</tr>
</tbody>
</table>

So consistent with no flavour splitting.
Vector mesons- decays

Transition \( \rho^0(0) \rightarrow \pi^+(1)\pi^-(1) \), \( q(\pi) = \pm 2\pi/L \).

\[ M(\pi^+) \approx 300 \text{ MeV}, \quad a = 0.086 \text{ fm}, \quad La = 2.1 \text{ fm} \]

Curve from 2 state model with energy gap \( \Delta ma = 2aE(\pi) - am(\rho) \).

Value of coupling \( x \) is consistent with empirical \( \rho \) decay width.

One can estimate effect of this mixing transition on \( \rho \) mass: obtaining a downward shift of \( ma \) of .02 (eg from .41 to .39).

This shift is comparable to our statistical error in determining the \( \rho \) mass.
Flavour singlet scalar mesons

scalar glueball, $\bar{u}u + \bar{d}d$, $\bar{s}s$, $\pi\pi$ in an S-wave, ... all contribute.

A first look in TMQCD:

$M(\pi^+) \approx 300$ MeV, $a = 0.086$ fm, $La = 2.1$ fm

$6 \times 6$ correlator matrix ($P, S, A_4$), 2 state fit ($\pi$ and $f_0$ mesons):

$m_a = 0.103(5) (\pi^0)$  $m'_a = 0.227(28) (f_0$ energy consistent with $2m(\pi^0)$).

This is not unexpected - but emphasises the problems of studying scalar mesons with light quarks in dynamical LGT.
Summary

- Evaluation of disconnected diagrams (variance reduction)
  TMQCD allows efficient evaluation of Disconnected Contributions

- TMQCD: order($a^2$) effects in pions $\pi$ charge splitting goes to zero like $a^2$, sign consistent with phase transition.

- Flavour singlet pseudoscalar meson ($\eta_2$) $\eta_2$ has mass around 800 MeV in chiral limit.

- Vector mesons, decay,.. Negligible charge splitting; decay accessible.

- Flavour singlet scalar meson ($f_0$) $f_0$ meson has $\pi \pi$ contribution which obscures further study.