Charm spectroscopy on dynamical 2+1 flavor domain wall fermion lattices with a relativistic heavy quark action

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Outline

1. Introduction
   - Heavy quarks
   - Relativistic Heavy Quark action
   - What can we do in full QCD?

2. Methods
   - Quantities to match
   - How to determine the RHQ parameters and $a$?

3. Analysis and Results
   - Lattices
   - Box sources and effective mass plots
   - Other concerns
   - Results

4. Outlook and Summary

Min Li
Charm spectroscopy with RHQ action
Heavy quarks

Problems

- $ma \ll 1$ no longer true, error terms contain $(ma)^n$
- Superfine lattice needed by brutal force simulation

Possible solutions

- NRQCD – no continuum limit
- HQET – not possible for quarkonia
- Anisotropic lattices – $O(\alpha_s ma_s)$
- Relativistic Heavy Quark (RHQ) action
Relativistic Heavy Quark action

The lattice form

\[ S = \sum_{n',n} \bar{\psi}_{n'} (m_0 a + \gamma_0 D_0 + \zeta \vec{\gamma} \cdot \vec{D} - \frac{1}{2} r_t (D^0)^2 - \frac{1}{2} r_s (\bar{D})^2 \]

\[ + \sum_{i,j} \frac{i}{4} c_B \sigma_{ij} F_{ij} + \sum_{i} \frac{i}{2} c_E \sigma_{0i} F_{0i} \] \( n',n \psi_n \)


Discretization errors up to order \((a\Lambda_{QCD})^2\)

Any choice of \(r_s\) and \(r_t\), as long as they agree when \(ma \rightarrow 0\). We choose \(r_s = \zeta\) and \(r_t = 1\) for simplicity.

\(c_E = c_B = c_P\) can be imposed. [N. Christ et. al. hep-lat/0608005]

Only parameters \(m_0 a, c_P\) and \(\zeta\) need to be tuned.
Fit to experimental values

- Determine the 3 RHQ parameters, with lattice spacing from other methods (at least 3 quantities needed)
- Predict other quantities of interest (eg. exotic mesons).
- Determine the lattice spacing as well as the RHQ parameters (at least 4 quantities needed)
Quantities to match

- Spin-averaged \((\eta_c, J/\Psi, D_s, D_s^*)\)

\[
\begin{align*}
m_{sa}^{hh} &= \frac{1}{4}(m_{PS}^{hh} + 3m_V^{hh}) \\
m_{sa}^{hl} &= \frac{1}{4}(m_{PS}^{hl} + 3m_V^{hl})
\end{align*}
\]

- Hyperfine splitting

\[
\begin{align*}
m_{hs}^{hh} &= m_V^{hh} - m_{PS}^{hh} \\
m_{hs}^{hl} &= m_V^{hl} - m_{PS}^{hl}
\end{align*}
\]

- Dispersion relation (mass ratio)

\[
E^2 = m_1^2 + \frac{m_1}{m_2} p^2
\]

- Spin-orbit averaged and splitting \((\chi_{c0} \text{ and } \chi_{c1})\)

\[
\begin{align*}
m_{sos}^{hh} &= m_{AV}^{hh} - m_S^{hh} \\
m_{soa}^{hh} &= \frac{1}{4}(m_S^{hh} + 3m_{AV}^{hh})
\end{align*}
\]

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Charm spectroscopy with RHQ action
How to determine the RHQ parameters and $a$

- Make a good guess – from $16^3$ lattice prediction.
- Linear approximation in the appropriate region

\[
Y(a) = \begin{pmatrix}
m_1 a \\
m_2 a \\
m_3 a \\
m_4 a \\
1
\end{pmatrix} = J \cdot \begin{pmatrix}
m_0 a \\
c_P \\
\zeta
\end{pmatrix} + A
\]

Obtain parameters and $a$ by minimizing the $\chi^2$ defined as:

\[
\chi^2 = (J \cdot X + A - Y(a))^T W^{-1} (J \cdot X + A - Y(a))
\]

It is a quadratic function of

- $X = (m_0 a, c_P, \zeta)^T$ if $a$ is known,
- $X^{New} = (m_0 a, c_P, \zeta, a)^T$ if $a$ is unknown.
Determine the coefficients J and A

How to get J and A?

7 points in the 3d space
\[ \{m_0, a, c_P, \zeta\} = \{0.43, 2.45, 1.29\} \]
and box size \[ \{0.2, 0.2, 0.04\} \]

Run 7 parameter sets in the 3d parameter space, which will enable us to get the derivatives of the quantities w.r.t each parameter (J) and the constant (A).
Dynamical DWF lattices

- **IWASAKI $\beta=2.13$ lattices**

<table>
<thead>
<tr>
<th>volume</th>
<th>$L_s$</th>
<th>$(m_{sea}, m_s)$</th>
<th>Traj(step)</th>
<th># of configs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$24^3 \times 64$</td>
<td>16</td>
<td>(0.005, 0.04)</td>
<td>900-4500(40)</td>
<td>91x4</td>
</tr>
<tr>
<td>$24^3 \times 64$</td>
<td>16</td>
<td>(0.01, 0.04)</td>
<td>900-4500(40)</td>
<td>91</td>
</tr>
<tr>
<td>$24^3 \times 64$</td>
<td>16</td>
<td>(0.02, 0.04)</td>
<td>1885-3605(20)</td>
<td>87x4</td>
</tr>
<tr>
<td>$24^3 \times 64$</td>
<td>16</td>
<td>(0.03, 0.04)</td>
<td>1000-3060(20)</td>
<td>104</td>
</tr>
</tbody>
</table>

For the $m_{sea}$ 0.005 and 0.02 ensembles, we placed the sources at time 0 as well as 16, 32 and 48. For the others sources are only at time 0.
Box sources and effective mass plots

Box source sizes are 4 and 12. The disagreement of the plateaus for box size 4 and 12 suggests more statistics are needed for $m_{\text{sea}} = 0.01$ and 0.03. All the chiral limit results we present here are extrapolated from $m_{\text{sea}} 0.005$ and 0.02 only.
which source to choose?

- Both box size 4 and 12 sources are used to extract $m_{\eta_c}$ and $J/\psi$ with double state fit, time range $t=(4,24)$.
- According to the graph, effective mass($\chi_{c1}$) of box size 4 has a fake plateau when statistics is not enough. Box size 12 is more appropriate for extract $m_{\chi_{c0}}$ and $m_{\chi_{c1}}$, fitting time range $t=(9,15)$.
- Currently running on box size 8 for heavy strange, for most efficient single state fit.
Other concerns

- **Quark propagator precision**
  - Heavy propagator, relative error $< 10^{-4}$ for every $t$, up to $t = 32$.
  - Light propagator, DWF, CG stop condition $10^{-10}$

- **Mass ratio $m_1/m_2$**
  - 3 momentum used, $[0,0,0] [0,0,1] [0,1,1]$
  - using $\eta_c$ and $J/\Psi$ momentum dependence gives consistent results
  - final results are based on $\eta_c$ momentum dependence.
**RHQ parameters**

Determined RHQ parameters using quantities (1)(2)(3), $a^{-1}=1.62\text{GeV}$ is assumed from static potential.

<table>
<thead>
<tr>
<th></th>
<th>$m_{sea}$</th>
<th>$m_0a$</th>
<th>$c_P$</th>
<th>$\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>0.410(8)</td>
<td>2.356(16)</td>
<td>1.270(7)</td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.387(14)</td>
<td>2.305(26)</td>
<td>1.276(13)</td>
<td></td>
</tr>
<tr>
<td>0.02</td>
<td>0.371(9)</td>
<td>2.263(14)</td>
<td>1.272(9)</td>
<td></td>
</tr>
<tr>
<td>0.03</td>
<td>0.348(23)</td>
<td>2.130(28)</td>
<td>1.234(26)</td>
<td></td>
</tr>
<tr>
<td>$-m_{res}$</td>
<td>0.432(13)</td>
<td>2.406(26)</td>
<td>1.269(12)</td>
<td></td>
</tr>
</tbody>
</table>

Again, only $m_{sea} = 0.005$ and 0.02 are used to do the chiral extrapolation.
Comparing with the experiment values, there are about 1% difference.

It may suggest wrong lattice spacing.

only $m_{sea}$ 0.005 and 0.02 are used to extrapolate.
The lattice spacing determined from 0.005 and 0.02 ensemble. 
\( a^{-1} = 1.71(9) \text{ Gev} \). From \( \Omega \), \( a^{-1} = 1.73(2) \text{ Gev} \).
Some consistency check

At $m_{\text{sea}} = 0.005$....

<table>
<thead>
<tr>
<th>quantities</th>
<th>$m_0a$</th>
<th>$c_P$</th>
<th>$\zeta$</th>
<th>$a^{-1}\text{(Gev)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)-(3)</td>
<td>0.410(8)</td>
<td>2.356(16)</td>
<td>1.270(7)</td>
<td></td>
</tr>
<tr>
<td>(1)-(4)</td>
<td>0.411(8)</td>
<td>2.354(16)</td>
<td>1.268(7)</td>
<td>1.62</td>
</tr>
<tr>
<td>(1)-(5)</td>
<td>0.407(8)</td>
<td>2.345(16)</td>
<td>1.270(7)</td>
<td>fixed</td>
</tr>
<tr>
<td>(1)-(5)</td>
<td>0.32(9)</td>
<td>2.19(16)</td>
<td>1.25(2)</td>
<td>1.674(56)</td>
</tr>
<tr>
<td>(1)-(3)</td>
<td>0.319(8)</td>
<td>2.190(15)</td>
<td>1.252(7)</td>
<td></td>
</tr>
<tr>
<td>(1)-(4)</td>
<td>0.319(8)</td>
<td>2.188(16)</td>
<td>1.25(8)</td>
<td>1.674</td>
</tr>
<tr>
<td>(1)-(5)</td>
<td>0.318(8)</td>
<td>2.190(16)</td>
<td>1.254(8)</td>
<td>fixed</td>
</tr>
</tbody>
</table>
Some consistency check

At $m_{\text{sea}}=0.02$....

<table>
<thead>
<tr>
<th>quantities</th>
<th>$m_0 a$</th>
<th>$c_P$</th>
<th>$\zeta$</th>
<th>$a^{-1}$ (Gev)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)-(3)</td>
<td>0.371(9)</td>
<td>2.263(14)</td>
<td>1.272(9)</td>
<td></td>
</tr>
<tr>
<td>(1)-(4)</td>
<td>0.371(9)</td>
<td>2.265(14)</td>
<td>1.273(9)</td>
<td>1.62</td>
</tr>
<tr>
<td>(1)-(5)</td>
<td>0.372(8)</td>
<td>2.267(14)</td>
<td>1.273(9)</td>
<td>fixed</td>
</tr>
<tr>
<td>(1)-(5)</td>
<td>0.38(9)</td>
<td>2.28(16)</td>
<td>1.27(2)</td>
<td>1.614(54)</td>
</tr>
<tr>
<td>(1)-(3)</td>
<td>0.382(9)</td>
<td>2.282(14)</td>
<td>1.274(9)</td>
<td></td>
</tr>
<tr>
<td>(1)-(4)</td>
<td>0.382(9)</td>
<td>2.284(14)</td>
<td>1.275(9)</td>
<td>1.614</td>
</tr>
<tr>
<td>(1)-(3)</td>
<td>0.382(8)</td>
<td>2.284(13)</td>
<td>1.275(9)</td>
<td>fixed</td>
</tr>
</tbody>
</table>
Some consistency check

at different lattice spacing, we can predict the masses of $\chi_{c0}$ and $\chi_{c1}$. The following $\chi^2$ should be at a minimum when $a^{-1}$ is around 1.71, as suggested by the lattice spacing obtained.

$$\chi^2 = \sum_{i=0,1} \frac{(m_{\chi ci}^{pred} - m_{\chi ci}^{phys})^2}{\sigma^2(m_{\chi ci}^{pred})}$$
Very preliminary results on $D_s$ and $D_s^*$

$m_{\text{sea}} = 0.005$ only
The $D_s$ and $D_s^*$ sample effective mass on 0.005 ensemble, with 91 configs and input bare strange mass 0.036.
with 7 quantities: $\eta_c, J/\psi, m_1/m_2, \chi_{c0}, \chi_{c1}, D_s$ and $D_s^*$. ($m_{\text{sea}}=0.005$ only)

<table>
<thead>
<tr>
<th>quantities</th>
<th>$m_0a$</th>
<th>$c_P$</th>
<th>$\zeta$</th>
<th>$a^{-1}$</th>
<th>$\chi^2$/d.o.f</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_c, J/\psi, \frac{m_1}{m_2}$</td>
<td>0.410(8)</td>
<td>2.356(16)</td>
<td>1.270(7)</td>
<td>0.00/0</td>
<td></td>
</tr>
<tr>
<td>$D_s, D_s^*, \frac{m_1}{m_2}$</td>
<td>0.498(65)</td>
<td>2.55(17)</td>
<td>1.277(7)</td>
<td>0.00/0</td>
<td></td>
</tr>
<tr>
<td>$\eta_c, J/\psi, \frac{m_1}{m_2}, \chi_{c0}, \chi_{c1}$</td>
<td>0.407(8)</td>
<td>2.345(16)</td>
<td>1.270(7)</td>
<td>1.62</td>
<td>7.76/2</td>
</tr>
<tr>
<td>$\eta_c, J/\psi, \frac{m_1}{m_2}, D_s, D_s^*$</td>
<td>0.424(8)</td>
<td>2.326(16)</td>
<td>1.237(7)</td>
<td>fixed</td>
<td>142/2</td>
</tr>
<tr>
<td>All</td>
<td>0.417(8)</td>
<td>2.309(15)</td>
<td>1.238(7)</td>
<td>153/4</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>quantities</th>
<th>$m_0a$</th>
<th>$c_P$</th>
<th>$\zeta$</th>
<th>$a^{-1}$</th>
<th>$\chi^2$/d.o.f</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_c, J/\psi, \frac{m_1}{m_2}$</td>
<td>0.228(9)</td>
<td>2.029(15)</td>
<td>1.238(8)</td>
<td>0.00/0</td>
<td></td>
</tr>
<tr>
<td>$D_s, D_s^*, \frac{m_1}{m_2}$</td>
<td>0.177(60)</td>
<td>1.90(15)</td>
<td>1.238(10)</td>
<td>0.00/0</td>
<td></td>
</tr>
<tr>
<td>$\eta_c, J/\psi, \frac{m_1}{m_2}, \chi_{c0}, \chi_{c1}$</td>
<td>0.235(8)</td>
<td>2.038(15)</td>
<td>1.234(8)</td>
<td>1.73</td>
<td>7.46/2</td>
</tr>
<tr>
<td>$\eta_c, J/\psi, \frac{m_1}{m_2}, D_s, D_s^*$</td>
<td>0.228(9)</td>
<td>2.033(15)</td>
<td>1.241(8)</td>
<td>fixed</td>
<td>1.19/2</td>
</tr>
<tr>
<td>All</td>
<td>0.235(8)</td>
<td>2.046(14)</td>
<td>1.237(8)</td>
<td>9.64/4</td>
<td></td>
</tr>
</tbody>
</table>
## Results

<table>
<thead>
<tr>
<th>quantities</th>
<th>$m_0a$</th>
<th>$c_P$</th>
<th>$\zeta$</th>
<th>$a^{-1}$</th>
<th>$\chi^2$/d.o.f</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_c, J/\psi, \frac{m_1}{m_2}, \chi_{c0}, \chi_{c1}$</td>
<td>0.319(89)</td>
<td>2.189(159)</td>
<td>1.252(19)</td>
<td>1.674(59)</td>
<td>3.85/1</td>
</tr>
<tr>
<td>$\eta_c, J/\psi, \frac{m_1}{m_2}, D_s, D_s^*$</td>
<td>0.241(22)</td>
<td>2.053(32)</td>
<td>1.240(8)</td>
<td>1.722(10)</td>
<td>0.46/1</td>
</tr>
<tr>
<td>All</td>
<td>0.257(22)</td>
<td>2.076(34)</td>
<td>1.237(8)</td>
<td>1.716(12)</td>
<td>7.04/3</td>
</tr>
</tbody>
</table>
Next Step to do?

- More statistics on $m_{sea}=0.01$ and $m_{sea}=0.03$, make a good prediction on $a^{-1}$ with a few percent error.
- Finish charm strange system on different sea quarks
- Heavy light system
- Predictions of more states, including exotic ones
- Bottom physics?
Conclusions:

- \((m_0a,c_P,\zeta)\) is determined to match to physical quantities and extrapolated to chiral limit, makes it possible to do predictions on various meson states.
- Results are consistent when different quantities are used to determine the RHQ parameters and lattice spacing.
- Lattice spacing can be determined with certain accuracy provided more statistics.
- Heavy strange and heavy light systems may help to determine the RHQ parameters, lattice spacing and predict meson masses with more precision.