Transverse Momentum Distributions of Quarks from the Lattice using Extended Gauge Link Operators

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presenting work in collaboration with LHPC and

Philipp Hägler, John Negele, Dru Renner, Andreas Schäfer, Meinulf Göckeler
fast nucleon: quarks (and gluons) look like “partons”, carrying

- a momentum fraction $x$ of the nucleon momentum $P$
- an *intrinsic transverse momentum* $k_T$
Motivation: Parton Picture

How are the quarks distributed with respect to $k_T$?

e.g. $f_1(k_T)$?

fast nucleon: quarks (and gluons) look like “partons”, carrying
  - a momentum fraction $x$ of the nucleon momentum $P$
  - an intrinsic transverse momentum $k_T$
example: **Semi Inclusive Deep Inelastic Scattering experiment**
$k_T$ dependence and Factorization

example: Semi Inclusive Deep Inelastic Scattering experiment

\[ \text{factorization} \implies \text{hard process} + \text{soft blobs (non-perturbative)} \]

\[ \rightarrow \text{Transverse Momentum dependent Parton Distribution Functions} \]

[Collins, Soper, Sterman PLB 83, NPB 85]
[Ji, Ma, Yuan PRD (2005)], [Mulders, Tangerman NPB (1996)]
$\k_T$ dependence and Factorization

\[
\Phi_{\Gamma}(k, P, S) \equiv \frac{1}{2(2\pi)^4} \int d^4 \ell \, e^{i k \cdot \ell} \, \langle P, S | \, q(0) \Gamma U q(\ell) \, | P, S \rangle
\]
\[ \langle P, S \mid \bar{q}(0) \Gamma U(0 \rightarrow \ell) q(\ell) \mid P, S \rangle \] is gauge invariant.

\[ U(0 \rightarrow \ell) \equiv \mathcal{P} \exp \left( -ig \int_0^\ell d\xi^\mu A^\mu(\xi) \right) \]
along path from 0 to \( \ell \)

- factorization in SIDIS: path runs to infinity and back
- here (up to now): straight path

\[ \rightarrow \] probability interpretation of distribution \( f(k_T) \)  [BBHM00]
Gauge Link Operator $\mathcal{U}$

$$\langle P, S | \bar{q}(0) \Gamma \mathcal{U}(0 \rightarrow \ell) q(\ell) | P, S \rangle$$

is gauge invariant.

**Continuum**

$$\mathcal{U}(0 \rightarrow \ell) \equiv \mathcal{P} \exp \left( -ig \int_0^\ell d\xi^\mu A_\mu(\xi) \right)$$

along path from 0 to $\ell$

- factorization in SIDIS: path runs to infinity and back
- **here** (up to now): straight path

$\rightarrow$ probability interpretation of distribution $f(k_T)$ [BBHM00]
Evaluated Quark-Quark Separations

quark separations in the $x,y$-plane:

263 paths in total:
- straight paths in $x, y, z,\ -x, -y, -z$ direction
- step-like paths in other directions

lattice operator
Extracting Nucleon Structure from the Lattice

Ingredients

- Gauge configs.
- Quark propagators
- Nucleon sequential propagators

Output: 3-point correlator $C_{3pt}$

[We calculate isovector quantities $(u - d) \Rightarrow$ no disconnected contributions.]
Parametrization and Ratios

parametrization of nucleon matrix element

\[ \langle P, S | \bar{q}(0) \Gamma U(0 \rightarrow \ell) q(\ell) | P, S \rangle \equiv \bar{U}(P, S) M_{\Gamma}(P) U(P, S) \]

Transfer matrix formalism ⇒ look at ratios!

ratio

\[ R_{\Gamma}(\tau) \equiv \frac{C_{3pt}(\tau|\bar{P}, t_{\text{sink}})}{C_{2pt}(t_{\text{sink}}, \bar{P})} \]

\[ 0 \ll \tau \ll t_{\text{sink}} \approx \frac{1}{2E(P)} \frac{\text{Tr} (\bar{P} + m_N) \Gamma^{3pt} (\bar{P} + m_N) M_{\Gamma}(P)}{\text{Tr} (\bar{P} + m_N) \Gamma^{2pt}} \]

⇒ Physics extracted from the plateau region!
Downloaded components:

gauge configurations 84 MILC lattices
from NERSC archive [Orginos, Toussaint 1999],
staggered ASQTAD action, 2+1 flavors,
$L^3 \times T = 20^3 \times 64$,
$a \approx 0.124$ fm,

HYP smeared and chopped: only time slices 0..31 are used

Propagators and sequential propagators from LHPC

hybrid action: Domain Wall valence fermions ($L_s = 16$)

$m_\pi \approx 596$ MeV,
source-sink-separation: 10

2 nucleon momenta available:

\[ \vec{P} = (0, 0, 0), \]

\[ \vec{P} = \frac{2\pi}{L} (-1, 0, 0) \approx 500 \text{ MeV} / c \]

nucleon spin projection operator

\[ \Gamma^{3\text{pt}} = \frac{1}{2} (1 + \gamma_4)(1 + i\gamma_5\gamma_3) \]
Sample Plateaus

ratio $\text{Re } R_{\Gamma}(\tau)$

$\vec{P} = (0, 0, 0)$

$\Gamma = \gamma^0$

separation $\vec{\ell} = (5, 0, 0)$

$\Rightarrow$ 5 links in x direction

ratio $-\text{Re } R_{\Gamma}(\tau)$

$\vec{P} = (0, 0, 0)$

$\Gamma = \gamma^3\gamma^5$

$\vec{\ell} = (6, 3, 0), \ |\vec{\ell}| = 6.7$

source at $t = 0$  

sink at $t = 10$

extraction at $t = 4, 5, 6$
survey: \( \text{Re} R_\Gamma \) for \( \vec{P} = (0, 0, 0) \), \( \Gamma = \gamma^0 \)

results for all 263 different link paths

⇒ Obviously, only quark separation length \( |\vec{l}| \) matters (for \( \vec{P} = 0 \)
Preliminary Results

Re $R_\Gamma$ for $\vec{P} = (0, 0, 0)$, $\Gamma = \gamma^0$

averaged over equivalent link paths in $x,y$ - plane.

fit with Gaussian: $C \exp\left(-|\vec{\ell}|^2/\sigma^2\right)$

<table>
<thead>
<tr>
<th>$C$</th>
<th>$\sigma$</th>
<th>$2/\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.826 ± 0.005</td>
<td>$(5.64 \pm 0.12)a = 0.70$ fm</td>
<td>$(563 \pm 12)$ MeV/c</td>
</tr>
</tbody>
</table>
Re $R_\Gamma$ for $\vec{P} = (0, 0, 0)$, $\Gamma = \gamma^0$

Averaged over equivalent link paths in $x,y$-plane.

Fit with two Gaussians:

$$ C_1 \exp\left(-|\vec{\ell}|^2/\sigma_1^2\right) + C_2 \exp\left(-|\vec{\ell}|^2/\sigma_2^2\right) $$

<table>
<thead>
<tr>
<th>$C_1$</th>
<th>$\sigma_1$</th>
<th>$2/\sigma_1$</th>
<th>$C_2$</th>
<th>$\sigma_2$</th>
<th>$2/\sigma_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.49</td>
<td>7.3$a$</td>
<td>$(433 \pm 15)$ MeV/c</td>
<td>0.37</td>
<td>3.4$a$</td>
<td>$(945 \pm 41)$ MeV/c</td>
</tr>
</tbody>
</table>
Preliminary Results: Fourier Transformed

\[ R_{\gamma^0} \xrightarrow{\text{Fourier transformation}} f_{1\text{lat}}(\vec{k}_T) \]

\[
\int dx \int dk^- \left( \int \frac{d^4 \ell}{2(2\pi)^4} e^{ik\cdot\ell} \langle P, S | \bar{q}(0) \gamma^+ U q(\ell) | P, S' \rangle \right)
\]

\[ = f_1(\vec{k}_T) \quad \text{see e.g., [Mulders, Tangerman NPB (1996)]} \]

\[ = \ldots \]

\[ = \int \frac{d^2 \vec{l}_T}{(2\pi)^2} e^{-i\vec{k}_T\cdot\vec{l}_T} R_{\gamma^0}(|\vec{l}_T|, \vec{P} = 0) \]
Preliminary Results: Fourier Transformed

\[ R_{\gamma^0} \xrightarrow{\text{Fourier transformation}} f^\text{lat}_1(\vec{k}_T) \]

\[
f^\text{lat}_1(|\vec{k}_T|) \quad \text{GeV}^{-2}
\]

\[
\begin{array}{cccccccc}
0.25 & 0.5 & 0.75 & 1 & 1.25 & 1.5 & |\vec{k}_T| & \text{GeV}
\end{array}
\]

Transverse
Momentum dependent
Parton
Distribution
Function!

\[
\sqrt{\langle \vec{k}_T^2 \rangle} = (533 \pm 11) \text{ MeV}
\]

Renormalization (e.g., to \( \overline{\text{MS}} \)) not done, yet!

comparison: phenomenology \( \rightarrow \) e.g., [Anselmino et al. PRD (2005)]

usual Ansatz:

\[ f_1(x, \vec{k}_T) \propto f_1(x) f_1(\vec{k}_T), \quad f_1(\vec{k}_T) \propto \exp\left( \frac{\vec{k}_T^2}{\langle \vec{k}_T^2 \rangle} \right), \]

\[
\sqrt{\langle \vec{k}_T^2 \rangle} \approx 500 \text{ MeV describes data.}
\]
Preliminary Results

$-\text{Re } R_\Gamma$ for $\vec{P} = (0, 0, 0), \Gamma = \gamma^3 \gamma^5$

averaged over equivalent link paths in $x,y$ - plane.

fit with Gaussian: $C \exp\left(-|\vec{\ell}|^2/\sigma^2\right)$

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<tr>
<td>$0.90 \pm 0.04$</td>
<td>$(6.58 \pm 0.12)a = 0.82$ fm</td>
<td>$(484 \pm 9)$ MeV/c</td>
</tr>
</tbody>
</table>
\[ \frac{1}{2} \text{Re} \left\{ R_\Gamma(\ell) + R_\Gamma(-\ell) \right\} \quad \text{for } \vec{P} = (-1, 0, 0), \ \Gamma = \gamma^0 \]

The graph shows link paths in the \(x\)-direction.

The fit with Gaussian:

\[ C \exp\left( -|\vec{\ell}|^2 / \sigma^2 \right) \]

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<th>( \sigma )</th>
<th>( 2/\sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.58 \pm 0.07</td>
<td>(5.4 \pm 0.5)a = 0.67 \text{ fm}</td>
<td>(666 \pm 52) \text{ MeV/c}</td>
</tr>
</tbody>
</table>
first conclusions

- Extraction of non-local correlators with good statistics possible.
- Seems like we see approximately Gaussian distribution of quark momenta, as expected.
- Width of the Gaussian is in the expected range.

things to do:

- details of parametrization
- more configurations, more Dirac- and link-structures.
- renormalization of the non-local operators
- Can we mimic links “to infinity and back” on the lattice? Relation to phenomenological TMPDFs.
first conclusions

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