Charm and bottom Heavy baryon mass spectrum from Lattice QCD with 2+1 flavors

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July-31-2007

Lattice 2007, U of Regensburg, Germany
Outline

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**Introduction**

- Singly and doubly charmed heavy baryons
- Singly and doubly bottom heavy baryons:

\[
\begin{align*}
\Lambda_H, \Sigma_H, \Sigma^*_H, \Xi_H, \Xi'_H, \Xi^*_H, \Omega_H, \Omega^*_H \\
\Xi_{HH}, \Xi^*_{HH}, \Omega_{HH}, \Omega^*_{HH}
\end{align*}
\]

- **Lattice QCD with 2+1 flavors**

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• **Lattices and Propagators**

• MILC coarse lattices
  – \(20^3 \times 64\), \(a \approx 0.12\) fm
  – 3 ensembles with four different time sources
    • \(m_l = 0.007\) \(m_s = 0.05\)
    • \(m_l = 0.01\) \(m_s = 0.05\)
    • \(m_l = 0.02\) \(m_s = 0.05\)

• Propagators
  – 9 different staggered light valence quarks
    • 0.005 ~ 0.02
  – 3 different staggered strange valence quarks
    • 0.024, 0.03, 0.0415
  – One valence clover heavy quark
    • \(k = 0.122\) (Tuned for charm quark)
      – 007 : 545 confs 010 : 591 confs 020 : 459 confs
    • \(k = 0.086\) (for bottom)
      – 007 : 554 confs 010 : 590 confs 020 : 452 confs
- **Formalism**
- **Operators** *(K.C. Bowler et al., PRD 54, 3619 (1996))*

\[ O_5 = \varepsilon_{abc} (\psi_1^{aT} C \gamma_5 \psi_2^b) \Psi_H^c, \quad O_\mu = \varepsilon_{abc} (\psi_1^{aT} C \gamma_\mu \psi_2^b) \Psi_H^c \]

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• Two point function with a Staggered light quark and a Wilson heavy quark
  • Conversion between a Naive propagator and a Staggered propagator!

\[
G_\psi (x; y) = \Omega(x) G_\phi \Omega(y)^+ \\
G_\phi = \hat{I}_4 G_\chi (x, y)
\]

\[
G_\psi (x, y) = \Omega(x) \Omega^+(y) G_\chi (x, y)
\]

where \( \Omega(x) = \prod_\mu (\gamma_\mu)^{x_\mu/a} \)

• Now, we can write the Heavy-Light correlator

\[
\sum_x e^{ip\cdot x} \langle W_{\Gamma_{sc}}^+ (x) W_{\Gamma_{sk}} (0) \rangle = \sum_x e^{ip\cdot x} Tr\left[ \Gamma_{sc} G_\psi (0; x) \Gamma_{sk}^+ G_H (x; 0) \right] \\
= \sum_x e^{ip\cdot x} \sum_{c, c'} [\text{tr} \{ \Gamma_{sc} \Omega^+(x) \Gamma_{sk}^+ G_H^{c'c} (x; 0) \} G_\chi^{cc'} (0; x)]
\]

where \( W_\Gamma = \overline{\Psi}_H (x) \Gamma \Psi(x) \)

• (M. Wingate et al. PRD67, 054505 (2003))
Two point function for the heavy baryon

\[
C_5(\vec{p}, t) = \sum_\vec{x} e^{-i\vec{p} \cdot \vec{x}} \left\langle O_5(\vec{x}, t) \bar{O}_5(\vec{0}, 0) \right\rangle \\
= \sum_\vec{x} e^{-i\vec{p} \cdot \vec{x}} \varepsilon_{abc} \varepsilon_{a'b'c'} \text{tr}[G_{1}^{aa'}(x, 0)C_{\gamma_5}G_{2}^{bb'}(x, 0)(C_{\gamma_5})^{+}]G_{H}^{cc'}(x, 0)
\]

\[
C_5(\vec{p}, t) = \sum_\vec{x} e^{-i\vec{p} \cdot \vec{x}} \varepsilon_{abc} \varepsilon_{a'b'c'} \text{tr}[\Omega^T(x)C_{\gamma_5}\Omega(x)(C_{\gamma_5})^{+}] \\
\times G_{1\chi}^{aa'}(x, 0)G_{2\chi}^{bb'}(x, 0)G_{H}^{cc'}(x, 0)
\]

\[
\text{tr}[\Omega^T(x)C_{\gamma_5}\Omega(x)(C_{\gamma_5})^{+}] = \text{tr}[(-1)^{x_1+x_3}(-1)^{x_1+x_3}]
\]

= 4

Finally,

\[
C_5(\vec{p}, t) = \sum_\vec{x} e^{-i\vec{p} \cdot \vec{x}} 4\varepsilon_{abc} \varepsilon_{a'b'c'} G_{1\chi}^{aa'}(x, 0)G_{2\chi}^{bb'}(x, 0)G_{H}^{cc'}(x, 0)
\]

\[
C_{\mu\nu}(\vec{p}, t) = \sum_{\vec{x}} e^{-i\vec{p} \cdot \vec{x}} 4(-1)^{x_\mu} \delta_{\mu\nu} \varepsilon_{abc} \varepsilon_{a'b'c'} G_{1\chi}^{aa'}(x, 0)G_{2\chi}^{bb'}(x, 0)G_{H}^{cc'}(x, 0)
\]
• Taste mixing?

\[ O_\mu = \varepsilon_{abc}(\psi_1^{aT} C \gamma_\mu \psi_2^b)\Psi_H^c \]

\[ D_\mu = (\psi_1^T(x) C \gamma_\mu \psi_2(x)) \]

\[ \psi^{\alpha'}(x) = \Omega^{\alpha'} a(x) \chi^a(x) \]

\[ q^{\alpha i,a}(y) = \frac{1}{8} \sum_{\xi} \Omega^{\alpha i}(\xi) \chi^a(y + \xi) \]

\[ x = y + \xi \]

\[ \chi^a(y + \xi) = 2\Omega^{+i\alpha}(\xi)q^{\alpha i,a}(y) \]

\[ \psi^{\alpha'}(x) = \Omega^{\alpha'} a(\xi) \chi^a(y + \xi) = \Omega^{\alpha'} a(\xi)2\Omega^{+i\alpha}(\xi)q^{\alpha i,a}(y) \]
• **Di-quark operator**

\[
D_\mu = (\psi_1^T (x) C \gamma_\mu \psi_2 (x))
\]

\[
D_{\mu}^{\text{conti}} (y) = \sum_i (\psi_1^T (x) C \gamma_\mu \psi_2 (x))
\]

\[
D_{\mu}^{\text{conti}} (y) = \sum_i 2 \Omega^{+ i a} (\xi) q^{\alpha i, a} (y) \Omega^{T a a'} (\xi) (C \gamma_\mu)^{\alpha' b'} (\xi) 2 \Omega^{+ j b} (\xi) q^{b, j, b} (y)
\]

\[
= \sum_i 4 \Omega^{+ i a} (\xi) q^{\alpha i, a} (y) (-1)^{\xi} (C \gamma_\mu)^{a b} \Omega^{+ j b} (\xi) q^{b, j, b} (y)
\]

\[
\sum_i \Omega^{+ i a} (\xi) (-1)^{\xi} \Omega^{+ j b} (\xi) = 4 (C \gamma_\mu)^{a b} \otimes (\gamma_\mu C^{-1})_{i j}
\]

\[
D_{\mu}^{\text{conti}} (y) = 16 q^{\alpha i, a} (y) (C \gamma_\mu)^{a b} \otimes (\gamma_\mu C^{-1})_{i j} q^{b, j, b} (y) (C \gamma_\mu)_{a b}
\]

\[
D_{5}^{\text{conti}} (y) = 16 q^{\alpha i, a} (y) (C \gamma_5)^{a b} \otimes (\gamma_5 C^{-1})_{i j} q^{b, j, b} (y) (C \gamma_5)_{a b}
\]

Overlap with $1^+$ and $0^+$ spin state with single taste

K. Nagata et al., arXiv:0707.3537

\(a, b\) : Copy index \hspace{1cm} \(i, j\) : Taste index

\(\alpha, \beta\) : Staggered spin index \hspace{1cm} \(\alpha', \beta'\) : Naive spin index
Two-point function of the di-quark operator

\[ C_{\mu \nu}^{\text{conti}}(y;0) = < D_{\mu}^{\text{conti}}(y) \overline{D}_{\nu}^{\text{conti}}(0) > \]

\[ = 16^2 \text{Tr}[G_1(y,0)(C_{\gamma \mu}) \otimes (C_{\gamma \mu})^+ G_2(y,0)(C_{\gamma \nu})^+ \otimes (C_{\gamma \nu})] \]

\[ \times (C_{\gamma \mu})_{ab} \otimes (C_{\gamma \mu})_{b'a'}^+ \delta_{bb'} \delta_{aa'} \]

\[ = 16^2 \text{Tr}[G_1(y,0)(C_{\gamma \mu}) \otimes (C_{\gamma \mu})^+ G_2(y,0)(C_{\gamma \nu})^+ \otimes (C_{\gamma \nu})] \]

\[ \times \text{Tr}[(C_{\gamma \mu})(C_{\gamma \nu})^+] \]

\[ \text{Tr}[(C_{\gamma \mu})(C_{\gamma \nu})^+] = \begin{cases} 0 & \mu \neq \nu \\ 4 & \mu = \nu \end{cases} \]
• **Data analysis**

• Fit model function

\[
P(t) = A e^{-mt} + A e^{-m(T-t)} + (-1)^t \tilde{A} e^{-\tilde{m}t} + (-1)^t \tilde{A} e^{-\tilde{m}(T-t)}
+ A^* e^{-m^*t} + A^* e^{-m^*(T-t)} + (-1)^t \tilde{A}^* e^{-\tilde{m}^*t} + (-1)^t \tilde{A}^* e^{-\tilde{m}^*(T-t)}
\]

• Correlated least squares fit

• Error estimation
  – 1000 bootstrap samples

• Linear chiral extrapolation
• **Results**
  
  • $1/2^+$ singly charmed heavy baryons
• $1/2^+$ singly charmed heavy baryons conti.

\[
M_{\text{phy}} = M_{\text{cal}} + \Delta \\
M_{\text{kin}} = \frac{|\vec{p}|^2 - [M_{\text{cal}}(\vec{p}) - M_{\text{cal}}(0)]^2}{2[M_{\text{cal}}(\vec{p}) - M_{\text{cal}}(0)]}
\]

Constant Mass Shift

= Average (\(M_{\text{exp}} - M_{\text{cal}}\))
- $1/2^+$ singly charmed heavy baryons: Other groups (Quenched calculations)

K.C. Bowler et al.,
PRD 54,3619 (1996)

R. Lewis et al.,
PRD 64,094509 (2001)
• $1/2^+$ singly bottom heavy baryons

Recent measurements from CDF and D0
• 1/2\(^+\) singly bottom heavy baryons conti.
• $1/2^+ \text{ singly bottom heavy baryons: Other groups (Quenched calculations)}$

K.C. Bowler et al., PRD 54, 3619 (1996)
N. Mathur et al., PRD 66, 014502 (2002)
- Doubly charmed heavy baryons (Preliminary)
- Doubly bottom heavy baryons (Preliminary)
Future study

- Fine lattice
  - $a \approx 0.09$, $m_l = 0.2m_s$, $m_l = 0.4m_s$
- Increase statistics
- More about error analysis
- Finite size effect
- Discretization errors
- Excited states $(3/2^+, 1/2^-, 3/2^-)$
Mass differences between bottom and charm hadrons