Thermodynamics of $N_f = 2$ QCD on anisotropic lattices
– Progress Report –

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1 Introduction

[Purpose 1] Determination of $T_c$ in full QCD

- For $N_f = 2 + 1$ KS-type fermion, continuum extrapolations were performed by several groups. But values of $T_c$ are not consistent, yet.

  \[
  T_c(a \to 0) = 192(7)(4) \text{ [MeV]} \quad \text{RBC-Bielefeld, 2006}
  \]

  \[
  T_c(a \to 0) = 151(3)(3) - 176(3)(4) \text{ [MeV]} \quad \text{Wuppertal, 2006}
  \]

  \[
  T_c(a \to 0) = 169(12)(4) \text{ [MeV]} \quad \text{MILC, 2005}
  \]

  \[\rightarrow\] Crosscheck with different type of fermion may be useful to determine $T_c$.

- For Wilson-type fermion, continuum limit has not been obtained, except one group($N_f = 2$, plaq+NP).

  \[
  T_c(a \to 0) = 166(3) - 173(3) \text{ [MeV]} \quad \text{Y. Nakamura et al, 2006}
  \]

  \[\rightarrow\] More works are needed. So, we perform simulations with Wilson-type fermion.

- (For chirally improved fermion, no continuum extrapolation has been taken.)
[Purpose 2] Calculation of light hadron spectrum around $T_c$

- Light hadron spectrum at finite $T$ are needed for experimental analysis.

- Determination of the pattern of QCD phase transition

  ◊ Pattern 1 : conventional $\sigma$ model type
  $\pi$ is a chiral partner of $\sigma$

  ◊ Pattern 2 : vector manifestation type
  $\pi$ is a chiral partner of $\rho$
  ($m_\rho \to 0$ as $m_{quark} \to 0$ at $T_c$)

$\rightarrow$ We want to clarify which pattern is realized in QCD.
2 Simulation setup

- $N_f = 2$ RG improved gauge + Clover fermion(TD) action
  
  ◊ Improved actions allow us to take continuum extrapolations from coarse lattices.

- Anisotropic lattices with $\xi \equiv a_s/a_t = 2$
  
  ◊ Anisotropic lattices are effective in reducing discretization errors of thermodynamic quantities and sampling data in the temporal direction.

Simulations with the same action on “isotropic” lattices are performed by WHOT-QCD collab.
[Lattice action]

- Bare anisotropies $\gamma_G$, $\gamma_F$ are tuned non-perturbatively so that $\xi \equiv a_s/a_t = 2$ \text{CP-PACS, 2003.}

\[
S = S_G + S_F
\]

\[
S_G = c_0 \beta \left\{ \frac{1}{\gamma_G} \sum_x \sum_{i,j} P_{ij}(x) + \gamma_G \sum_x \sum_i P_{i4}(x) \right\}
\]

\[
+ c_1 \text{(rectangular terms)}, \quad c_0 = 3.648, \quad c_1 = -0.331, \quad \beta \equiv 6/g_0^2
\]

\[
S_F = \sum_{x,y} \overline{\psi(x)} K(x, y) \psi(y)
\]

where

\[
K(x, y) \equiv \delta_{x,y} - \kappa_s \sum_i \left( (r - \gamma_i) U_i(x) \delta_{x+\hat{i},y} + (r + \gamma_i) U^\dagger_i(x - \hat{i}) \delta_{x-\hat{i},y} \right)
\]

\[
- \kappa_s \gamma_F \left( (1 - \gamma_4) U_4(x) \delta_{x+\hat{4},y} + (1 + \gamma_4) U^\dagger_4(x - \hat{4}) \delta_{x-\hat{4},y} \right)
\]

\[
+ (C_{SW} \text{ terms}), \quad r = 1/\xi, \quad 1/\kappa \equiv 1/\kappa_s - 2(\gamma_F + 3r - 4)
\]

\[
P_{\mu\nu}(x) \equiv 1 - \frac{1}{N_c} \text{Re } tr \left\{ U_\mu(x) U_\nu(x + \hat{\mu}) U^\dagger_\mu(x + \hat{\nu}) U^\dagger_\nu(x) \right\}, \quad U_\mu(x) = e^{ig_0 A_\mu(x)}
\]
Algorithm : HMC
Machines : BlueGene @KEK, clusters @Univ. of Tsukuba, Nagoya Univ.

- We use three kinds of $N_t$ toward continuum extrapolations
- Aspect ratio of our main runs is two ($LT = 2$)
- Additional runs with larger spatial volumes ($LT = 3$) are performed for finite size checks

<table>
<thead>
<tr>
<th>Size($N_s^3 \times N_t$)</th>
<th>$\beta$</th>
<th>$\kappa(m_{PS}/m_V)$ (%)</th>
<th>$N_{traj}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8^3 \times 8$</td>
<td>1.95-2.10</td>
<td>0.110-0.1275 (0.98-0.3)</td>
<td>1000-9000</td>
</tr>
<tr>
<td>$10^3 \times 10$</td>
<td>1.95-2.10</td>
<td>0.117-0.1275 (0.88-0.3)</td>
<td>1000-6000</td>
</tr>
<tr>
<td>$12^3 \times 12$</td>
<td>2.00-2.10</td>
<td>0.117-0.1274 (0.88-0.3)</td>
<td>1000-7000</td>
</tr>
<tr>
<td>$12^3 \times 8$</td>
<td>2.00</td>
<td>0.117-0.1265 (0.88-0.5)</td>
<td>1000-4000</td>
</tr>
</tbody>
</table>
3 Result 1 (critical temperature)

[Pseudo-order parameter : Polyakov loop]

- Since \( N_f = 2 \) QCD has a crossover at finite temperature, there is no order parameter.
  \[ \rightarrow \] We use \( Z(3) \)-rotated Polyakov loop \( L \) as a pseudo-order parameter. (\( L \) is an order parameter in \( m_q \to \infty \) limit)

\[
L \equiv \frac{1}{N_c} \sum_x \text{Tr} \prod_{t=1}^{N_t} U_4(x, t), \quad U_\mu(x) = e^{ig_0 A_\mu(x)}
\]

\[ U_4(x) \]
[Phase structure on $8^3 \times 8$ lattices]

- $\langle L \rangle = 0$ in the confined phase, and $\langle L \rangle \neq 0$ in the deconfined phase.
[Susceptibility of Polyakov loop on $8^3 \times 8$ lattices]

- Pseudo-critical point is determined by peak locations of Polyakov loop susceptibility: $\chi_L \equiv \langle L^2 \rangle - \langle L \rangle^2$

\begin{align*}
\chi_L
\end{align*}
Sectional view of $\langle L \rangle$ and $\chi_L$ with $\beta = 2.00$ on $8^3 \times 8$ lattices

- Pseudo-critical point is determined by the peak location of $\chi_L$, which is obtained by Gaussian fit around it.
[Finite size check]

- We compare results on $N_s^3 \times N_t = 8^3 \times 8$ with those on $12^3 \times 8$. → No spatial size dependence is observed, which is consistent with crossover.

- (As in the case of KS fermion, we must check finite size scaling at the physical point to prove QCD has a crossover. But we have not done it yet.)

\[
m_{\text{quark}}^q(N_s) - m_{\text{quark}}^q(N_s \to \infty) \propto N_s^{-1/\nu}
\]

$1/\nu = 3$ for 1st and $1/\nu \sim 1.3$ for 2nd order($O(4)$) phase transition.
[Chiral extrapolation of \( T_{pc} \) to obtain “\( T_c \) at \( m_{quark} = 0 \)”]

- Since simulations are performed with relatively heavy quark masses, extrapolations to the physical point (chiral extrapolations) are needed.

\[
\beta_c = \beta_{pc} + B (m_{VWI} (\kappa pc))^{1/\beta\delta}, \quad 1/\beta\delta = 0.537 \text{ for } O(4)
\]

\[
T_c = 1/N_{t}a_t(\beta_c)
\]
• Our value of $T_c$ is around 160 MeV, which is slightly smaller than other group results. Anisotropic lattices may reduce the discretization error of $T_c$, effectively.

• We need more statistics for the continuum extrapolation.
4 Result 2(spectrum around $T_{pc}$)

We perform spectroscopy at $T \sim T_{pc}$, and $T = 0$ for comparison.

[Difficulty to obtain hadron spectrum at $T \neq 0$]

- It is hard to extract masses, due to small number of data in the temporal direction. (finite $T = 1/N_t a_t$ is realized by small temporal extent.)

  ◇ Anisotropic lattices ($a_s > a_t$) have more data than isotropic lattices, but no plateau is observed in effective masses with the point source.
[Meson wavefunctions at $T \neq 0$]
Before going on to spectroscopy at finite $T$ with the smeared source, we checked wavefunctions.

- Wavefunctions are exponentially localized in space, even around $T_{pc}$.
  $\rightarrow$ Mesons seem to be bound states even around $T_{pc}$.

$$
\phi(r) \equiv \frac{\sum_{x} \langle \overline{q}(x, t) \Gamma q(x + r, t) (\overline{q}(x, t) \Gamma q(x, t)) \rangle^{\dagger}}{\sum_{x} \langle \overline{q}(x, t) \Gamma q(x, t) (\overline{q}(x, t) \Gamma q(x, t)) \rangle^{\dagger}}, \quad \Gamma = \gamma_{5}, \gamma_{i}, i = 1, 2, 3
$$
Hadron spectrum at $T \sim T_{pc}$ – Preliminary –

- We smear meson operators by use of wavefunctions.

$$\overline{q}(x, t) \Gamma q(x, t) \rightarrow \sum_{\mathbf{r}} \phi_{\Gamma}(\mathbf{r}) \overline{q}(x + \mathbf{r}, t) \Gamma q(x, t)$$

- We can see plateau in the smeared source and point sink propagators (though the plateau window is small).

- We fit the plateau data to extract masses.
[Quark mass dependence of meson masses at $T \sim T_{pc}$] – Preliminary –

Only statistical errors are estimated (systematic errors are not included)

- We do not see clear mass shifts at $T \sim T_{pc}$.
- In vector manifestation scenario, $m_V \to 0$ as $m_{quark} \to 0$ at $T = T_c$.
  But, our data imply $m_V \neq 0$ in the chiral limit.

◊ $O(a)$ explicit chiral symmetry breaking by Wilson-type fermion
  may be distorting the results.
  → Continuum extrapolation($a \to 0$) is needed for conclusion.

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5 Summary

We performed $N_f = 2$ lattice QCD simulations at finite temperature.

- $T_c \sim 160[\text{MeV}]$ on $N_t = 8, 10, 12$ lattices with $\xi \equiv a_s/a_t = 2$.
  $\rightarrow$ More statistics are needed for continuum extrapolations.

- Spectroscopy at finite $T$ is ongoing.
  
  ◇ Meson wavefunctions imply that mesons are bound states even around $T_{pc}$.

  ◇ No significant mass shifts are observed around $T_{pc}$ at $N_t = 8$
  $\rightarrow$ Measurements on $N_t = 10, 12$ and continuum extrapolations are needed for definite conclusions.
Appendix