Preliminary non-perturbative results of static $B_s - \bar{B}_s$ matrix elements in tmQCD

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with

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Lattice 2007 – August 2$^{nd}$, 2007
the $B$-parameter is defined as the deviation of the four-fermion $\Delta B = 2$ m.e. from the VSA

$$B_{B_q} = \frac{3}{8} \frac{\langle \bar{B}_q | O_{LL}^{\Delta B=2} | B_q \rangle}{f_{B_q}^2 m_{B_q}^2}, \quad O_{LL}^{\Delta B=2} = [\bar{\psi}_b \gamma_\mu (1 - \gamma_5) \psi_q] [\bar{\psi}_b \gamma_\mu (1 - \gamma_5) \psi_q]$$

the LL-operator is perturbatively matched to HQET via

$$O_{LL}^{\Delta B=2} (m_b) = C_1(m_b, \mu) Q_1(\mu) + C_2(m_b, \mu) Q_2(\mu) + O \left( \frac{1}{m_b} \right)$$

$$Q_1 \equiv O_{stat}^{VV+AA} = (\bar{\psi}_h \gamma_\mu \psi_q) (\bar{\psi}_h \gamma_\mu \psi_q) + (\bar{\psi}_h \gamma_\mu \gamma_5 \psi_q) (\bar{\psi}_h \gamma_\mu \gamma_5 \psi_q)$$

$$Q_2 \equiv O_{stat}^{SS+PP} = (\bar{\psi}_h \psi_q) (\bar{\psi}_h \psi_q) + (\bar{\psi}_h \gamma_5 \psi_q) (\bar{\psi}_h \gamma_5 \psi_q)$$

better to work with RGI matrix elements through RG perturbative evolution

$$\begin{bmatrix} Q_1^{RGI} \\ Q_2^{RGI} \end{bmatrix} = U_{MS/NDR \rightarrow RGI}(\mu) \begin{bmatrix} Q_1(\mu) \\ Q_2(\mu) \end{bmatrix}$$

...in twisted-mass QCD

- In order to eliminate the mixing due to $\chi$-breaking, we change operator basis

\[
\begin{bmatrix}
Q_1^{\text{RGI}} \\
Q_2^{\text{RGI}}
\end{bmatrix} = \begin{bmatrix}
Q_1^{\text{RGI}} \\
Q_1^{\text{RGI}} + 4Q_2^{\text{RGI}}
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
1 & 4
\end{bmatrix} \begin{bmatrix}
Q_1^{\text{RGI}} \\
Q_2^{\text{RGI}}
\end{bmatrix} = R \begin{bmatrix}
Q_1^{\text{RGI}} \\
Q_2^{\text{RGI}}
\end{bmatrix}
\]

- And regularize the $s$-quark in tmQCD at full twist $\alpha = \pi/2$ ($\psi_s \rightarrow \exp\{-i\alpha\gamma_5\tau_3/2\}\psi_s$)

\[
\langle Q_1^{\text{RGI}} \rangle = -i \lim_{\alpha \rightarrow 0} \hat{Z}_{1,\text{RGI}}(g_0)\langle Q_1'(a) \rangle_{\text{tmQCD}}^{\alpha=\pi/2}
\]

\[
\langle Q_2^{\text{RGI}} \rangle = -i \lim_{\alpha \rightarrow 0} \hat{Z}_{2,\text{RGI}}(g_0)\langle Q_2'(a) \rangle_{\text{tmQCD}}^{\alpha=\pi/2}
\]

- I.e. parity-even operators are mapped onto their parity-odd companions

\[
Q_1' = O_{\text{VA+AV}} = (\bar{\psi}_h \gamma_\mu \psi_s) (\bar{\psi}_h \gamma_\mu \gamma_5 \psi_s) + (\bar{\psi}_h \gamma_\mu \gamma_5 \psi_s) (\bar{\psi}_h \gamma_\mu \psi_s)
\]

\[
Q_2' = O_{\text{VA+AV}} + 4O_{\text{PS+SP}} = (\bar{\psi}_h \gamma_\mu \psi_s) (\bar{\psi}_h \gamma_\mu \gamma_5 \psi_s) + (\bar{\psi}_h \gamma_\mu \gamma_5 \psi_s) (\bar{\psi}_h \gamma_\mu \psi_s) + 4[(\bar{\psi}_h \gamma_5 \psi_s) (\bar{\psi}_h \psi_s) + (\bar{\psi}_h \psi_s) (\bar{\psi}_h \gamma_5 \psi_s)]
\]
Schrödinger functional correlators

- Interpolating operators of the $B_s$-meson provided by pseudoscalar boundary bilinears

$$O_{sh} = \frac{\alpha^3}{2\pi} \sum_{yz} \bar{\xi}_s(y) \gamma_5 \zeta_h(z), \quad O'_{sh} = \frac{\alpha^3}{2\pi} \sum_{y'z'} \bar{\xi}'_{s}(y') \gamma_5 \zeta'_h(z')$$

- Local operators in the bulk. 2- and 3-point correlation functions

$$f_x(x_0) = -\frac{\alpha^3}{2} \sum_x \langle X_{hs}(x) O_{sh} \rangle$$
$$f'_x(x_0) = -\frac{\alpha^3}{2} \sum_x \langle O'_{sh} X_{sh}(x) \rangle$$
$$F_y(x_0) = \alpha^3 \sum_x \langle O'_{sh} Y_{hs\bar{s}}(x) O_{sh} \rangle$$

- The $B$-parameter is extracted from the ratio of correlators

$$R_i(x_0) = \frac{\frac{f'_{A}}{F_{A}}(x_0)}{\frac{Q'_{i}}{Q_{i}}(x_0)} \quad i = 1, 2$$

- Where the axial correlator is fully twisted

$$h_{A-iv}(x_0) = \frac{1}{\sqrt{2}} \left[ Z_{A,RGI}^{stat} f_{A}(x_0) - Z_{V,RGI}^{stat} f_{V}(x_0) \right], \quad h'_{A-iv}(x_0) = \frac{1}{\sqrt{2}} \left[ Z_{A,RGI}^{stat} f'_{A}(x_0) - Z_{V,RGI}^{stat} f'_{V}(x_0) \right]$$

Lattice 07 - 02.07.07

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$B_s - \bar{B}_s$ matrix elements in (quenched) tmQCD
bare matrix elements are $O(\alpha)$-improved

- proof along the lines of (Frezzotti, Martinelli, Papinutto, Rossi, hep-lat/0503034)

$\tilde{z}_{k}^{RGI}$ are not $O(\alpha)$-improved (talk by M. Papinutto)

- to obtain $O(\alpha)$-improved renormalization constants, the SF scheme has to be modified

Frezzotti, Rossi, hep-lat/0511034
<table>
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<tr>
<th>$\beta$</th>
<th>$T \times L^3$</th>
<th>$\kappa_{cr}$</th>
<th>$\kappa$</th>
<th>$\mu$</th>
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<tr>
<td>6.0</td>
<td>$32 \times 16^3$</td>
<td>0.135196</td>
<td>0.135181</td>
<td>0.028669</td>
</tr>
<tr>
<td>6.1</td>
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<td>6.2</td>
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<td>0.135785</td>
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</tbody>
</table>

- **static action:** HYP2 \{HYP link with $(\alpha_1, \alpha_2, \alpha_3) = (1.0, 1.0, 0.5)$\}
- **relativistic action:** tmQCD action with Sheikoleslami-Wohlert term
- **quenched approximation**

- $\kappa_{cr}$ from PCAC without mass-twisting
- $\kappa_s$ from (Rolf and Sint, JHEP 0212:007,2002)
- $(\kappa, \mu) = \text{Twist}(\kappa_s, 0)\vert_{\alpha=\pi/2}$
\( Z_{V}^{\text{stat}}(\text{HYP2}) \)

- \( Z_{V}^{\text{stat}} \) from
  - Della Morte, Shindler, Sommer, JHEP 0508, 051 (2005)

- new determination of the scale independent ratio \( Z_{V}^{\text{stat}} / Z_{A}^{\text{stat}} \) from an AWI

\[
\langle Q_{A}^{2h}(x_{0})P^{h2}(y) \rangle = 2\langle Q_{V}^{1h}(x_{0})P^{h2}(y) \rangle \left\{ \left[ Q_{A}^{1}(t_{2}) - Q_{A}^{1}(t_{1}) \right] - 2m \int_{R} d^{4}z P^{1}(z) \right\}
\]

\[
Q_{A}^{a}(x_{0}) = \int d^{3}x A_{0}^{a}(x), \quad Q_{A}^{kh}(x_{0}) = \int d^{3}x A_{0}^{kh}(x), \quad Q_{V}^{kh}(x_{0}) = \int d^{3}x V_{0}^{kh}(x)
\]
Analysis of the excited states: 2-point correlator

- Binding energy is the standard way to look at the excited states.

\[ aE_{\text{eff}}(x_0) = \frac{1}{2} \log \left\{ \frac{h_{\lambda-\nu}(x_0 - \alpha)}{h_{\lambda-\nu}(x_0 + \alpha)} \right\} \]

- Fundamental state isolated at \( x_0^{\text{min}} \)
- If \( x_0^{\text{min}} < T/2 \) then 
  \[ [x_0^{\text{min}}, T - x_0^{\text{min}}] \]
  
  ... is a good plateau region for \( R_j \)
- Present case is critical: small mass gap \( \Rightarrow x_0^{\text{min}} \approx T/2 \)
- Smearing based on boundary wave-functions useless without all-to-all propagators

\[ \mathcal{O}(\omega) = a^3 \sum_{y,z} \omega(y - z) \bar{\zeta}(y) \gamma_5 \zeta(z) \]
Analysis of the excited states: 3-point correlator

- $F_1(x_0)$
- $F_2(x_0)$
- $h(A_iV(x_0)h'(A_iV(T-x_0))$
Analysis of the excited states: ratio of correlators

\[ R_i(x_0) = \frac{3}{8} \frac{F_{Q_i}(x_0)}{[2h_{A-iV}(x_0)][2h'_{A-iV}(T-x_0)]} \quad i = 1, 2 \]

- we observe a strong cancellation of the excited state contaminations!
- is this an accident? no

\[ R_i(x_0) = B_i^{(0,0)} \frac{1 + \sum_{(n,m)\neq(0,0)} B_i^{(n,m)} f_{nm} g_{nm} e^{-(T-x_0)\Delta^{(a)}_{n_0}} e^{-x_0\Delta^{(a)}_{n_0}}}{1 + \sum_{(n,m)\neq(0,0)} f_{nm} g_{nm} e^{-(T-x_0)\Delta^{(a)}_{n_0}} e^{-x_0\Delta^{(a)}_{n_0}}} \]

\[ f_{nm} = \frac{\langle i_B | n, B \rangle \langle m, B | i_B \rangle}{\langle i_B | 0, B \rangle \langle 0, B | i_B \rangle}, \quad g_{nm} = \frac{\langle n_B | A_0 | 0, 0 \rangle \langle 0, 0 | A_0 | m, B \rangle}{\langle 0, B | A_0 | 0, 0 \rangle \langle 0, 0 | A_0 | 0, B \rangle} \]

\[ B_i^{(n,m)} = \frac{\langle n, B | Q_i' | m, B \rangle}{\frac{8}{3} \langle n, B | A_0 | 0, 0 \rangle \langle 0, 0 | A_0 | m, B \rangle} \]
Analysis of the excited states: VSA

\[ R_i(x_0) = \frac{3}{8} \frac{F_{Q'_i}(x_0)}{[2h_{\alpha-R}(x_0)][2h'_{\alpha-R}(T-x_0)]} \quad i = 1, 2 \]

- we observe a strong cancellation of the excited state contaminations!
- is this an accident? no

\[ R_i(x_0) = B_i^{(0,0)} \frac{1 + \sum_{(n,m)\neq(0,0)} B_i^{(n,m)} f_{nm} g_{nm} e^{-\left(T-x_0\right)\Delta_{n0}^{(e)} } e^{-x_0 \Delta_{n0}^{(e)}}}{1 + \sum_{(n,m)\neq(0,0)} f_{nm} g_{nm} e^{-\left(T-x_0\right)\Delta_{n0}^{(e)} } e^{-x_0 \Delta_{n0}^{(e)}}} \]

- in practice the only relevant term is \( z_{10} = B_i^{(1,0)} / B_i^{(0,0)} \)
- in the vacuum saturation approximation (VSA): \( B_i^{(n,m)} = 1 \)
- if the VSA depends smoothly upon the mass of the external state \( \Rightarrow z_{10} \approx 1 \)
- the VSA acts as an additional damping of the excited states
a stochastic two-state model

- to quantify how the shape of $R_i$ changes with $z_{10}$, we introduce a two-state model

$$
\epsilon(e, x_0, T) = e + \frac{1}{2} \log \left\{ \frac{1 - pe^{-\Delta(x_0 - 1)}}{1 - pe^{-\Delta(x_0 + 1)}} \right\}
$$

$$
\rho(z, x_0, T) = \frac{1 - zp \left[ e^{-\Delta(x_0)} + e^{-\Delta(T - x_0)} \right]}{1 - p \left[ e^{-\Delta(x_0)} + e^{-\Delta(T - x_0)} \right]}
$$

- with distribution probabilities

$$
P(\Delta) = \frac{1}{\sigma_{\Delta} \sqrt{2\pi}} \exp \left( -\frac{\Delta - \bar{\Delta}}{2\sigma^2_{\Delta}} \right), \quad (\bar{\Delta}, \sigma_{\Delta}) = (0.22, 0.03)
$$

$$
P(p; \bar{p}, \sigma_p) = \frac{1}{p \sigma_p \sqrt{2 \pi}} \exp \left\{ -\frac{(\ln p - \bar{p})^2}{2 \sigma^2_p} \right\}
$$

- bind. energy and $B$-parameter from averages

$$
\mathcal{E}(e, x_0, T) = \langle \epsilon(e, x_0, T) \rangle \simeq e + \frac{1}{2N} \sum_{i=1}^{N} \log \left\{ \frac{1 - p_i e^{-\Delta_i(x_0 - 1)}}{1 - p_i e^{-\Delta_i(x_0 + 1)}} \right\}
$$

$$
\mathcal{R}(z, x_0, T) = \langle \rho(z, x_0, T) \rangle \simeq \frac{1}{N} \sum_{i=1}^{N} \frac{1 - z p_i \left[ e^{-\Delta_i(x_0)} + e^{-\Delta_i(T - x_0)} \right]}{1 - p_i \left[ e^{-\Delta_i(x_0)} + e^{-\Delta_i(T - x_0)} \right]}
$$

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$B_s - \bar{B}_s$ matrix elements in (quenched) tmQCD
Before giving a continuum limit estimate we would like

a. have a better control of the excited state contaminations

b. simulate lattice spacing closer to the continuum

extensions to $N_f = 2$

concerning the renormalization, cf. the poster by C. Pena