The chiral critical point of $N_f = 3$ QCD: 1. step towards the continuum

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- Introduction
- QCD phase diagram at $\mu = 0$
- The curvature of $T_c(\mu)$
- Conclusions (qualitative)

with Ph. de Forcrand (ETH/CERN)
The order of the transition for $N_f = 2 + 1, \mu = 0$: qualitative picture

Finite density, $\mu \neq 0$:

- $N_f = 3$: critical quark mass $m_c$

Fodor, Katz

Bielefeld; Columbia; de Forcrand, O.P.
The order of the transition for $N_f = 2 + 1, \mu = 0$: qualitative picture

Finite density, $\mu \neq 0$: 

X phys. point

- $N_f = 3$: critical quark mass $m_c$

Fodor, Katz

Bielefeld; Columbia; de Forcrand, O.P.

Finite density, $\mu \neq 0$: 

$\mu$  

$T$

$m_{u,d} \neq 0; m_s = \infty$
Gap between phys.point, critical line important:

\[ N_f = 3: \quad \frac{m_c(\mu)}{m_c(\mu = 0)} = 1 + c_1 \left( \frac{\mu}{\pi T} \right)^2 + \ldots \]

\[ c_1 \sim O(1) \Rightarrow m_c(\mu) \text{ slowly varying} \Rightarrow \mu_c(m) \text{ rapidly varying!} \]

When is the critical point at “small” \( \mu \)?

If \( c_1 = +1 \), then for \( \mu_B^c \lesssim 400 \text{ MeV} \)

need \[ 1 < \frac{m}{m_c(\mu = 0)} \lesssim 1.05 \]

\[ \text{larger gap} \Rightarrow \text{larger } \mu_B^c \]
Observable to measure criticality: Binder cumulant

\[ B_4(m, \mu) = \frac{\langle (\delta \bar{\psi} \psi)^4 \rangle}{\langle (\delta \bar{\psi} \psi)^2 \rangle^2}, \quad \delta \bar{\psi} \psi = \bar{\psi} \psi - \langle \bar{\psi} \psi \rangle \]

\( V \to \infty, \) step function

\[ B_4(m, \mu) \to 3 \quad \text{crossover} \]

\[ B_4(m_c, \mu_c) \to \text{critical value} \]

\[ = 1.604 \quad \text{for 3d Ising} \]

\[ B_4(m, \mu) \to 1 \quad \text{first order} \]
$N_f = 2 + 1$: phase-diagram at $N_t = 4, \mu = 0$

de Forcrand, O.P. 06

$N_t = 4$

standard staggered, $L = 8 - 16$, RHMC

Setting the scale (arrow):

<table>
<thead>
<tr>
<th>$(am_{u,d}, am_s)$</th>
<th>$m_\pi/m_\rho$</th>
<th>$m_K/m_\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.005, 0.25)</td>
<td>0.148(2)</td>
<td>0.626(9)</td>
</tr>
<tr>
<td>physical</td>
<td>0.18</td>
<td>0.6456</td>
</tr>
</tbody>
</table>
First step towards the continuum: $N_f = 3$ on $N_t = 6$

![Diagram showing the transition from $N_f = 2$ to $N_f = 3$ with critical lines and quark masses]

Critical line moving towards smaller quark masses......

cf. improved actions on $N_t = 4$: Bielefeld

\[
\frac{m^c}{T_c}\bigg|_{\text{standard}} \approx 4.4 \frac{m^c}{T_c}\bigg|_{p4}
\]

\[
\frac{m^c}{T_c}\bigg|_{N_t=4} \approx 5 \frac{m^c}{T_c}\bigg|_{N_t=6}
\]
Relation to continuum scales:

$T = 0$ simulation at critical parameters $\beta^c, am^c$ on $18^3 \times 24$

| $N_t$ | $m^c_{\pi}/T^c$ | $m_{\pi}/T_c|_{phys}$ |
|------|-----------------|-------------------------|
| 4    | 1.680(4)        | $\sim 0.37$             |
| 6    | 0.954(12)       | $\sim 0.75$             |
| $\infty$ | $\frac{m^c_{\pi}}{T^c}(N_t) = \frac{m^c_{\pi}}{T^c}\bigg|_{cont} + \frac{const}{N_t^2} + \ldots$ |

- first order region shrinks for $a \to 0$
- gap between phys. point and crit. surface widens
- expect large effect on critical endpoint
Finite density: imaginary $\mu$ + analytic continuation

fermion determinant positive $\Rightarrow$ no sign problem

idea: for small $\mu/T$, fit full simulation results of imag. $\mu$ by Taylor series

$$\langle O \rangle = \sum_{n}^{N} c_{n} \left( \frac{\mu_{i}}{\pi T} \right)^{2n} \Rightarrow \mu_{i} \longrightarrow i\mu_{i}$$

- ensemble has two parameters ($\mu$, $T$) $\Rightarrow$ uncorrelated data
- some control of systematic error
- limited to $\mu_{B} \leq 500$ MeV
Critical temperature $T_c(\mu)$

$T = 0.166 - 0.139 \mu_B^2 - 0.053 \mu_B^4$

$T_c(\mu, m) = 1 + 1.88 \left( \frac{\mu}{\pi T_c} \right)^2 + \ldots$

$\frac{T_c(\mu, m)}{T_c(\mu = 0, m_c(0))} = 1 + 2.111(17) \frac{m - m_c(0)}{\pi T_c} - 0.667(6) \left( \frac{\mu}{\pi T_c(0, m)} \right)^2 + \ldots$

$N_f = 3, N_t = 4,$ imag. $\mu = i\mu_i$

\[ \beta_c(\mu, m) = b_0(m) + b_1(m)(a\mu)^2 + b_2(m)(a\mu)^4 + \ldots \]
Alternative: direct calculation of derivatives ⇒ check systematic errors from fitting Taylor series

method to save computer time:

finite difference

\[
\frac{d\langle O \rangle}{d(a\mu)^2} = \lim_{(a\mu)^2 \to 0} \frac{\langle O((a\mu)^2) \rangle - \langle O(0) \rangle}{(a\mu)^2}
\]

evaluate by “infinitesimal reweighting” in imag. direction

\[
\langle O((a\mu_i)^2) \rangle = \left\langle O((a\mu_i)^2) \frac{\det M(\mu_i)}{\det M(0)} \right\rangle_{\mu_i=0}
\]

- imag. \( \mu \): no sign problem ⇒ better signal
- reweighting: eliminates fluctuation in \( \Delta O \), exact for \( a\mu \to 0 \)
result \[ \frac{d\beta_c}{d(a\mu)^2} \]

\[ N_t = 4: \]

quantitative agreement with earlier calculation

\[ N_t = 6: \]

preliminary, no indication for growing curvature \[ \frac{dT_c}{d\mu^2} \]
Critical surface: last year’s result

\[ am_c(\mu) = am_c(0) + c_1'(a\mu)^2 + \ldots \]

Continuum conversion: \( a(T(\mu)) > a(T(0)) \)

\[ \frac{m_c(\mu)}{m_c(\mu = 0)} = 1 - 0.7(4) \left( \frac{\mu}{\pi T} \right)^2 + \ldots \]
Check derivative directly

see de Forcrand, Tue 16:30

$$c'_1 = \left. \frac{d(am_c)}{d(a\mu)^2} \right|_{\mu=0} = \frac{\partial B_4}{\partial (a\mu)^2} \left( \frac{\partial B_4}{\partial am} \right)^{-1}$$

quantitative agreement with earlier result, smaller errors
Conclusions

• critical line moves towards smaller quark masses for $a \to 0$
  ⇒ physical point at $\mu = 0$ more deeply in crossover region

• curvature of $T_c(\mu)$ not (yet) increasing towards continuum
  ⇒ inconsistent with freeze-out curve?

• unconventional scenario confirmed with derivative method for $N_f = 3, N_t = 4$