Rigidity of center vortices

P. V. Buividovich$^\text{b}$, M.I. Polikarpov$^\text{b}$

$^\text{b}$BSU (Minsk), $^\text{b}$ITEP (Moscow)

Lattice 2007
Plan

1. Monopoles and Vortices
   - Monopoles
   - Vortices
   - Monopoles belong to Vortices

2. Long Range Forces on vortices
   - Monopole Trajectory
   - \( \langle \text{Tr} \, F_{\mu\nu}^2(x) \text{Tr} \, F_{\mu\nu}^2(y) \rangle \) correlator on the vortex
     \[ [P.V. Buividovich, M.I.P. (2007)] \]

3. Rigidity
   - Analytical example
   - Lattice definition of curvature
   - Numerical results

4. Discussion
Monopoles and Vortices

Monopoles

$SU(2) \rightarrow U(1)$ Monopole Current (Closed lines on 4D lattice)

Confinement $\Leftrightarrow$ Monopoles

[H. Shiba and T. Suzuki (1994)]
$SU(2) \rightarrow Z(2)$ Center Vortices (Closed surfaces on 4D lattice)

Confinement $\Leftrightarrow$ Center Vortices

[L. Del Debbio, M. Faber, J. Greensite, S. Olejnik (1997)]
Monopole Currents are lying on Center Vortices

Abelian Monopoles and Center Vortices are interrelated

- Removing monopoles (or removing center vortices) ⇒
- Removing center vortices (or removing monopoles) ⇒
- remove confinement and chiral symmetry breaking.

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Correlation of the directions of the monopole currents

The probability to have the same (opposite) direction at the distance \( l \):

\[
P_{\text{same}}^{\text{opposite}} = 1 \pm A_0 \exp^{-\mu_{s,o}l},
\]

\( \mu_s \approx \mu_o \approx 270 \text{ Mev} \) [V.G. Bornyakov, P.Yu. Boyko, M.I.P., V.I. Zakharov (2005)].

The glueball mass is

\( m(0^{++}) = 1.65 \pm 0.05 \text{ Gev} \) [Teper (1998)]
Calculation of $\langle Tr F_{\mu\nu}^2(x) Tr F_{\mu\nu}^2(y) \rangle$ correlator on the vortex is equivalent to calculation of the normalized correlator:

$$\rho \left[ Tr U_p, Tr U_{p'} \right] = \frac{\langle Tr U_p Tr U_{p'} \rangle - \langle Tr U_p \rangle^2}{\langle (Tr U_p)^2 \rangle - \langle Tr U_p \rangle^2}$$
\[ \langle \text{Tr} F_{\mu \nu}^2(x) \text{Tr} F_{\mu \nu}^2(y) \rangle \text{ correlator on the vortex [P.V. Buividovich, M.I.P. (2007)]} \]

\[ \rho[\text{Tr} U_p, \text{Tr} U_{p'}] \]

\[ a, \text{fm} \]

\[ d_2 = 1, d_4 = 1 \]
\[ d_2 > 6, d_4 = 1 \]

\[ \text{SU}(2) \text{ LGT, } L^4 = 24 \div 28 \]
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Thus we see that there is something nontrivial (long range correlations) on the center vortex world sheet. Fields on the world sheet (monopoles) can induce long range correlations and nontrivial action for vortices:

\[
\exp(-W[S]) = \int \mathcal{D}\phi \exp\left(-\int_S d^2\xi \sqrt{g} L[\phi]\right)
\]

\[
W = \sigma \times \text{Area} + \gamma \times (\text{internal curvature}) + \kappa \times (\text{extrinsic curvature}) + \ldots
\]
Thus we see that there is something nontrivial (long range correlations) on the center vortex world sheet. Fields on the world sheet (monopoles) can induce long range correlations and nontrivial action for vortices:

\[
\exp(-W[S]) = \int \mathcal{D}\phi \exp \left( - \int_{S} d^{2}\xi \sqrt{g} L[\phi] \right)
\]

\[
W = \sigma \times \text{Area} + \gamma \times (\text{internal curvature}) + \kappa \times (\text{extrinsic curvature}) + \ldots
\]
We calculated numerically coefficients $\sigma$, $\gamma$ and $\kappa$ for SU(2) lattice gauge theory.

$$ W [S] = \int_S d^2 \xi \sqrt{g} \left( \sigma_0 (a) a^{-2} + \gamma (a) R + \kappa (a) K \right) $$

$\sqrt{g} = \sqrt{\det g_{ab}}$, $g_{ab} = \frac{\partial X^{\mu}}{\partial \xi^a} \frac{\partial X^{\mu}}{\partial \xi^b}$ is the induced metric on the surface, $a = \Lambda_{UV}^{-1}$ is the lattice spacing. $R$ ($K$) is the internal (extrinsic) curvature.
Internal curvature in lattice units for hypercubic lattice is
[Ambjorn (1994)]:

\[ a^2 R_s = 4 - n_s, \]

where \( n_s \) is the number of neighbors of the site \( s \). Extrinsic curvature in lattice units is [Ambjorn (1994)]:

\[ a^2 K_s = \Delta x_s^\mu \Delta x_s^\mu \]

\[ a^2 K_s = 0 \quad a^2 K_s = 1 \]

\[ a^2 K_s = 2 \quad a^2 K_s = 3 \]
Numerical results

Bare string tension of center vortex

Extrinsic curvature coupling

Intrinsic curvature coupling
Extrapolation to continuum limit $a \to 0$

$$
\sigma_0 (0) = 0.192 \pm 0.006 \\
\kappa (0) = 0.066 \pm 0.003 \\
\gamma (0) = 0.08 \pm 0.02
$$

$$
W [S] = \int_S d^2 \xi \sqrt{g} \left( \sigma_0 (a) a^{-2} + \gamma (a) R + \kappa (a) K \right)
$$
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Rigidity of center vortices was first suggested by Engelhardt and Reinhardt in 1999 from phenomenological analysis.

P.V. Buividovich and M.I.P. arXiv:0705.3745 correlated the center vortex excess action with intrinsic and extrinsic vortex curvature (this talk).

The world sheet action with intrinsic and extrinsic curvature naturally arises after integration over fields (monopoles?) living on the vortex.

The sign of the existence of the fields living on the center vortex is long range correlations on the world sheet.
Self consistency check. Our result is

\[ \sigma = a^{-2} \sigma_0 \approx a^{-2} A + a^{-1} B, \]

\[ A \approx 0.192, \; B \approx 2.2. \]

If divergence of \( \sigma \) corresponds to self-energy of percolating one-dimensional object (monopole trajectory) on the world sheet [J. Ambjorn (1994)] then density of monopoles on the world sheet is \( \rho_{1D} = \frac{B}{\ln 4} \approx 1.5 \text{fm}^{-1} \) which corresponds to the monopole bare mass \( m_{bare} = a^{-1} \ln 4 \). The density of vortices is \( \rho_v \approx 24 \text{fm}^{-2} \), thus density of monopoles in four-dimensional space is \( \rho'_{1D} = 37.9 \text{fm}^{-3} \), which should be compared with density of Abelian monopoles \( \rho_m \approx 31 \text{fm}^{-3} \).