Modified Coulomb potential of QED in a strong magnetic field

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Modified Coulomb potential of QED in LLLA

- ***Based on***
  N.S. and A. Sodeiri Jalili, Sharif University of Technology, Tehran-Iran
  arXiv:0705.4384
  To appear in PRD (2007)

- ***The purpose of the talk***
  Analytical (perturbative) determination of static Coulomb potential in a strong magnetic field in two different regimes in a certain LLLA

- ***What we have found (preliminary analytic results)***
  A novel dependence on the angle between the external B field and the particle – antiparticle axis

- ***Question***
  What are the consequences of this angle-dependence?

Strong QED

In a strong magnetic field, QED

has, in addition to the

familiar weak coupling phase,

a nonperturbative strong coupling phase

characterized by

spontaneous chiral symmetry breaking

Motivation

- **High Energy Physics**
  - Novel interpretation of multiple correlated and narrow peak structures in electron positron spectra in heavy ion experiments
  - The electron-positron peaks are due to the decay of a bound state formed in this new phase induced by a strong and rapidly varying EM field present in the neighborhood of colliding heavy ions

Motivation

In this talk:

Bound state formation in constant but strong B fields

- **Astrophysics of compact stars**
  - Neutron stars
  - Radio pulsars
  - Soft gamma ray repeaters

10^8–10^{17} G

Earth's magnet field 1G

Physical Consequences

- Optical effects
- Photon splitting
- *etc.*

Theory of Magnetic Catalysis

Idea

- For nonzero B, as in non-relativistic QM, Landau levels can be built

- For strong enough magnetic fields the levels are well separated and Lowest Landau Level (LLL) approximation is justified

Hence: In the LLLA, an effective quantum field theory replaces the full quantum field theory

Properties

1. **Dynamical mass generation**

- Start with a chirally invariant theory in nonzero B
- Solve the DSE in the ladder approximation (w/out dyn. fermions)

$$m_{dyn.} = C\sqrt{eB}\exp\left(-\frac{\pi}{2}\left(\frac{\pi}{2\alpha}\right)^{1/2}\right) \quad \Rightarrow \quad m_{dyn.}^2 \ll |eB|$$

Fermion dynamical mass

Properties

2. **Dimensional Reduction from D to D - 2**

- Dynamics of 4-dim QED in a strong magnetic field is equivalent with dynamics of 2-dim Schwinger model

Example

In 2 dim. Schwinger model, photons are massive

In 4 dim. QED in LLLA, photons are also massive

Finite photon mass $M_\gamma^2 = \frac{2\alpha|eB|N_f}{\pi}$

Fermion propagator in LLLA

$$\mathcal{S}_F(x,y) = S_{\|}(\mathbf{x}_{\|} - \mathbf{y}_{\|}) P(\mathbf{x}_{\perp}, \mathbf{y}_{\perp})$$

- the longitudinal part

$$S_{\|}(\mathbf{x}_{\|} - \mathbf{y}_{\|}) = \int \frac{d^2k_{\|}}{(2\pi)^2} e^{i\mathbf{k}_{\|}\cdot(\mathbf{x}-\mathbf{y})^{\|}} \frac{i\mathcal{O}}{\gamma^{\|}\cdot\mathbf{k}_{\|} - m}, \qquad \mathcal{O} \equiv \frac{1}{2}\left(1 - i\gamma^1\gamma^2 \text{sign}(eB)\right)$$

- the transverse part

$$P(\mathbf{x}_{\perp}, \mathbf{y}_{\perp}) = \frac{|eB|}{2\pi} \exp\left(\frac{ieB}{2}\epsilon^{ab}x^a y^b - \frac{|eB|}{4}(\mathbf{x}_{\perp} - \mathbf{y}_{\perp})^2\right), \qquad a,b = 1,2$$

Photon propagator in LLLA

$$i\widetilde{\mathcal{D}}_{\mu\nu}(q) = \frac{g^{\parallel}_{\mu\nu}}{q^2 + \mathbf{q}_{\parallel}^2 \Pi\left(\mathbf{q}_{\perp}^2, \mathbf{q}_{\parallel}^2\right)}$$

$$\Pi(\mathbf{q}_{\perp}^2, \mathbf{q}_{\parallel}^2) = \frac{2\alpha|eB|N_f}{\mathbf{q}_{\parallel}^2} e^{-\frac{\mathbf{q}_{\perp}^2}{2|eB|}} \left(\frac{1}{2\sqrt{y(y-1)}} \ln\left(\frac{\sqrt{1-y}+\sqrt{-y}}{\sqrt{1-y}-\sqrt{-y}} \right) - 1 \right), \qquad y \equiv \frac{\mathbf{q}_{\parallel}^2}{4m_{dyn.}^2}$$

Expanding $\Pi(\mathbf{q}_{\perp}^2, \mathbf{q}_{\parallel}^2)$ for $\mathbf{y} \ll 1$ and $\mathbf{y} \gg 1$

(a) $\Pi(\mathbf{q}_{\perp}^2, \mathbf{q}_{\parallel}^2) \simeq +\frac{\alpha|eB|N_f}{3\pi m_{dyn.}^2} e^{-\frac{\mathbf{q}_{\perp}^2}{2|eB|}}$ for $|\mathbf{q}_{\parallel}^2| \ll m_{dyn.}^2 \ll |eB|$

(b) $\Pi(\mathbf{q}_{\perp}^2, \mathbf{q}_{\parallel}^2) \simeq -\frac{2\alpha|eB|N_f}{\pi\,\mathbf{q}_{\parallel}^2} e^{-\frac{\mathbf{q}_{\perp}^2}{2|eB|}}$ for $m_{dyn.}^2 \ll |\mathbf{q}_{\parallel}^2| \ll |eB|$

For regime (b) $\Pi(\mathbf{q}_{\perp}^2, \mathbf{q}_{\parallel}^2)$ has a pole at $\mathbf{q}_{\parallel}^2 = 0$ ►▷► Finite photon mass $M_{\gamma}^2 = \frac{2\alpha|eB|N_f}{\pi}$

Static Coulomb potential in LLLA

Our Results

N.S. and A. Sodeiri Jalili

0705.4384(hep-th)

to appear in PRD (2007)

We have derived the static Coulomb potential of charged particles in two different regimes in LLLA

- First regime ► $|\mathbf{q}_{\parallel}^2| \ll m_{dyn.}^2 \ll |eB|$
- Second regime ► $m_{dyn.}^2 \ll |\mathbf{q}_{\parallel}^2| \ll |eB|$

using two perturbative methods

1. V.E.V. of Wilson loop
2. Born approximation

1^{st} Method: Wilson Loop and Coulomb potential

Idea

- Create a particle-antiparticle pair at x=0 and adiabatically separate them to a distance R
- Held this configuration for an infinite time T
- Finally, bring the pair back together and let them annihilate

The potential between charged particle from the V.E.V. of a Wilson loop

$$V(R) = -\lim_{T\to\infty} \frac{1}{T} \ln\langle W_C[A]\rangle \qquad \text{with} \qquad W_C[A] \equiv \exp\left(ie \oint_C A_\mu(x) dx^\mu \right)$$

Perturbative expansion

$$V(R) = \lim_{T\to\infty} \frac{e^2}{2T} \oint_C \oint_C dx_\mu dy_\nu \, D_{\mu\nu}(x,y) + \mathcal{O}(e^3) \qquad \text{with} \qquad D_{\mu\nu}(x,y) \equiv \langle A_\mu(x) A_\nu(y) \rangle$$

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2^{nd} Method: Born approximation

Idea: Use the relation between the scattering amplitude and the potential

$$\langle p'|i\mathcal{M}|p\rangle = -iV(\mathbf{q})(2\pi)\delta(E_{p'} - E_p), \qquad \mathbf{q} = \mathbf{p}' - \mathbf{p}$$

Example:

- For QED (without radiative corrections)

$$i\mathcal{M} \sim -\frac{ie^2}{|\mathbf{p} - \mathbf{p}'|^2} \quad \Rightarrow \quad V(\mathbf{q}) = -\frac{e^2}{|\mathbf{q}|^2} \quad \Rightarrow \quad V(R) = -\frac{\alpha}{R}$$

- QED potential with radiative corrections

$$V(\mathbf{x}) = -e^2 \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\mathbf{q}\cdot\mathbf{x}}}{\mathbf{q}^2\left(1 - \Pi(\mathbf{q}^2)\right)}$$

Uehling potential

$$V(R) = -\frac{\alpha}{R}\left(1 + \frac{\alpha}{4\sqrt{\pi}}\frac{e^{-2mR}}{(mR)^{3/2}} + \cdots\right)$$

Static Coulomb potential in LLLA

The only ingredient in calculating the static Coulomb potential in LLLA is

the photon propagator in two different regions in the regime of LLL dominance

Results

Static Coulomb potential in the first regime ► $|\mathbf{q}_{\|}^2| \ll m_{dyn.}^2 \ll |eB|$

Photon propagator in coordinate space

$$\widetilde{\mathcal{D}}_{\mu\nu}(R,\theta,T) = \frac{\delta_{\mu\nu}^{\|}}{4\pi^2 a_1}\Bigg[\left(1+\frac{\gamma}{a_1}-\frac{4a_2+\gamma R^2\sin^2\theta}{2\beta a_1^2}+\frac{3a_2R^2\sin^2\theta}{2\beta^2 a_1^3}\right)$$
$$+\frac{4\gamma^2}{a_1^2}\left(1-\frac{3R^2\sin^2\theta}{2\beta a_1}+\frac{3R^4\sin^4\theta}{8\beta^2 a_1^2}\right)$$
$$-\frac{12\gamma a_2}{\beta a_1^3}\left(2-\frac{2}{\beta a_1}\left(2R^2\sin^2\theta+\frac{a_2}{\gamma}\right)+\frac{5}{4\beta^2 a_1^2}\left(R^4\sin^4\theta+\frac{4a_2R^2\sin^2\theta}{\gamma}\right)-\frac{15a_2R^4\sin^4\theta}{8\gamma\beta^3a_1^3}\right)$$
$$-\frac{\gamma}{|eB|\beta a_1^2}\left(1-\frac{3}{\beta a_1}\left(\frac{R^2\sin^2\theta}{2}+\frac{a_2}{\gamma}\right)+\frac{6}{\beta^2a_1^2}\left(\frac{R^4\sin^4\theta}{16}+\frac{a_2R^2\sin^2\theta}{\gamma}\right)-\frac{15a_2R^4\sin^4\theta}{8\gamma\beta^3a_1^3}\right)\Bigg]$$

with

$$\beta^{-1} \equiv 4\left(1+\frac{\alpha|eB|}{3\pi m_{dyn.}^2}\right) \qquad \text{and} \qquad \gamma(\alpha) \equiv \frac{2\alpha}{3\pi m_{dyn.}^2},$$

and

$$a_1(R,\theta,T) \equiv T^2 + R^2 f^2(\alpha,\theta), \qquad \text{with} \qquad f^2(\alpha,\theta) \equiv 1+\frac{\gamma|eB|}{2}\sin^2\theta,$$
$$a_2(R,\theta,T) \equiv \beta\gamma\left(T^2+R^2\cos^2\theta\right)$$

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Results

Static Coulomb potential in the first regime ► $|\mathbf{q}_{\parallel}^2| \ll m_{dyn.}^2 \ll |eB|$

$$V_1(R,\theta) = -2e^2 \int_0^\infty dT\, \widetilde{\mathcal{D}}_{00}(R,\theta,T)$$

$$V_1(R,\theta) = -\frac{\alpha}{R}\left(\mathcal{A}_1(\alpha,\theta) - \frac{\gamma\mathcal{A}_2(\alpha,\theta)}{R^2} + \frac{\gamma^2\mathcal{A}_3(\alpha,\theta)}{R^4}\right)$$

$$\mathcal{A}_1(\alpha,\theta) \equiv \frac{1}{f},$$

$$\mathcal{A}_2(\alpha,\theta) \equiv -\frac{1}{4f^3}\left(1 - \frac{3\cos^2\theta}{f^2} - \frac{3\sin^2\theta}{8\beta f^2} + \frac{15\sin^2\theta\cos^2\theta}{8\beta f^4}\right),$$

$$\mathcal{A}_3(\alpha,\theta) \equiv +\frac{9}{16f^5}\left(1 - \frac{5\sin^2\theta}{4\beta f^2} + \frac{35\sin^4\theta}{128\beta^2 f^4}\right)$$
$$-\frac{15\cos^2\theta}{8f^7}\left(3 - \frac{7}{4\beta f^2}\left(2\beta\cos^2\theta + 3\sin^2\theta\right) + \frac{63\sin^2\theta}{8\beta^2 f^4}\left(\beta\cos^2\theta + \frac{3\sin^2\theta}{16}\right) - \frac{693\cos^2\theta\sin^4\theta}{256\beta^2 f^6}\right)$$
$$-\frac{3}{16|eB|f^5\gamma\beta}\left(1 - \frac{5}{4\beta f^2}\left(4\beta\cos^2\theta + \sin^2\theta\right) + \frac{35\sin^2\theta}{4\beta^2 f^4}\left(\beta\cos^2\theta + \frac{\sin^2\theta}{32}\right) - \frac{315\cos^2\theta\sin^4\theta}{128\beta^2 f^6}\right)$$

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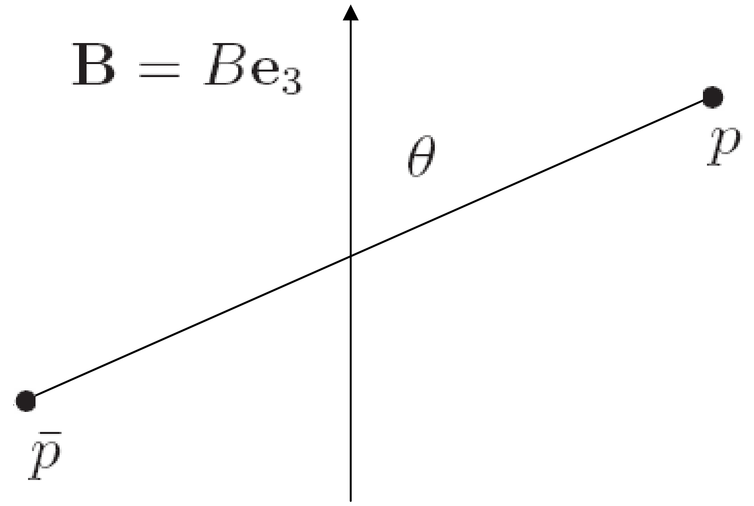
Novelty



A new dependence on the angle between the external magnetic field and the particle-antiparticle axis

Results

Static Coulomb potential in the second regime ► $m^2_{dyn.} \ll |\mathbf{q}^2_{\parallel}| \ll |eB|$

Photon propagator in coordinate space

$$\widetilde{\mathcal{D}}_{\mu\nu}(R,\theta,T) = \frac{\delta^{\parallel}_{\mu\nu}}{4\pi^2\left(1-\frac{\alpha}{\pi}\right)} \frac{\zeta}{\sqrt{T^2+R^2g^2(\theta)}} K_1\left(\zeta\sqrt{T^2+R^2g^2(\theta)}\right)$$

where

$$\zeta \equiv \sqrt{\frac{2\alpha|eB|}{\pi}} \qquad \text{and} \qquad g^2(\theta) \equiv \cos^2\theta + \frac{\sin^2\theta}{1-\frac{\alpha}{\pi}}$$

Here, for one fermion flavor ζ is the photon mass M_γ

Results

Static Coulomb potential in the second regime ► $m^2_{dyn.} \ll |\mathbf{q}^2_{\|}| \ll |eB|$

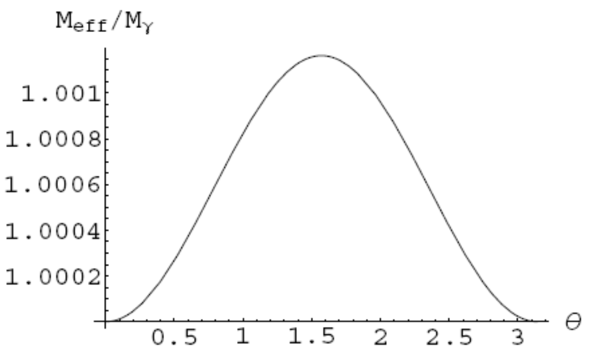
Using $$V(R,\theta) = -2e^2 \int_0^\infty dT\, \widetilde{\mathcal{D}}_{00}(R,\theta,T)$$

A Yukawa like potential can be derived

$$V_2(R,\theta) = -\frac{\alpha}{(1-\frac{\alpha}{\pi})g(\theta)R} e^{-M_\gamma g(\theta) R}$$

where the effective photon mass

$$M_{\text{eff.}} \equiv M_\gamma \sqrt{\frac{1-\frac{\alpha}{\pi}\cos^2\theta}{1-\frac{\alpha}{\pi}}}$$



depends on the angle between the direction of the magnetic field and particle-antiparticle axis

This result is in contrast to the previous results

by

A.E. Shabad and V.V. Usov

astro-ph/0607499

0704.2162 (astro-ph)

Yukawa like potential

$$V(\mathbf{x}) = -\frac{\alpha e^{M_\gamma R}}{R}$$

with the photon mass

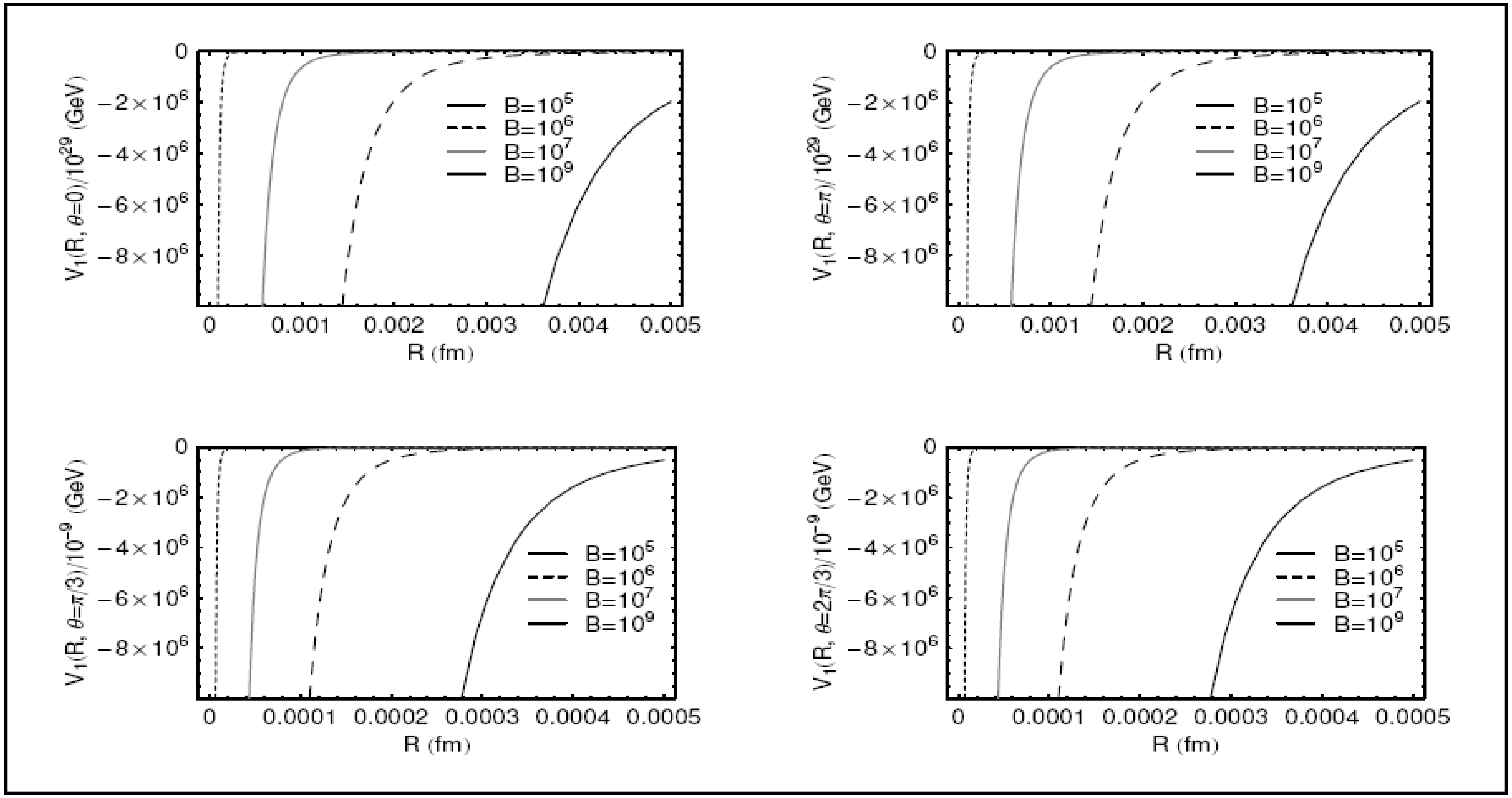
$$M_\gamma = \sqrt{\frac{2\alpha|eB|N_f}{\pi}}$$

Results

Modified Coulomb potential in the first regime

$$V_1(R,\theta) = -\frac{\alpha}{R}\left(\mathcal{A}_1(\alpha,\theta) - \frac{\gamma\mathcal{A}_2(\alpha,\theta)}{R^2} + \frac{\gamma^2\mathcal{A}_3(\alpha,\theta)}{R^4}\right)$$

Potential $V_1(R,\theta)$ in $q_{\parallel}^2 \ll m_d^2 \ll |eB|$ Regime for different θ and B



No qualitative changes by varying the angle θ

Results

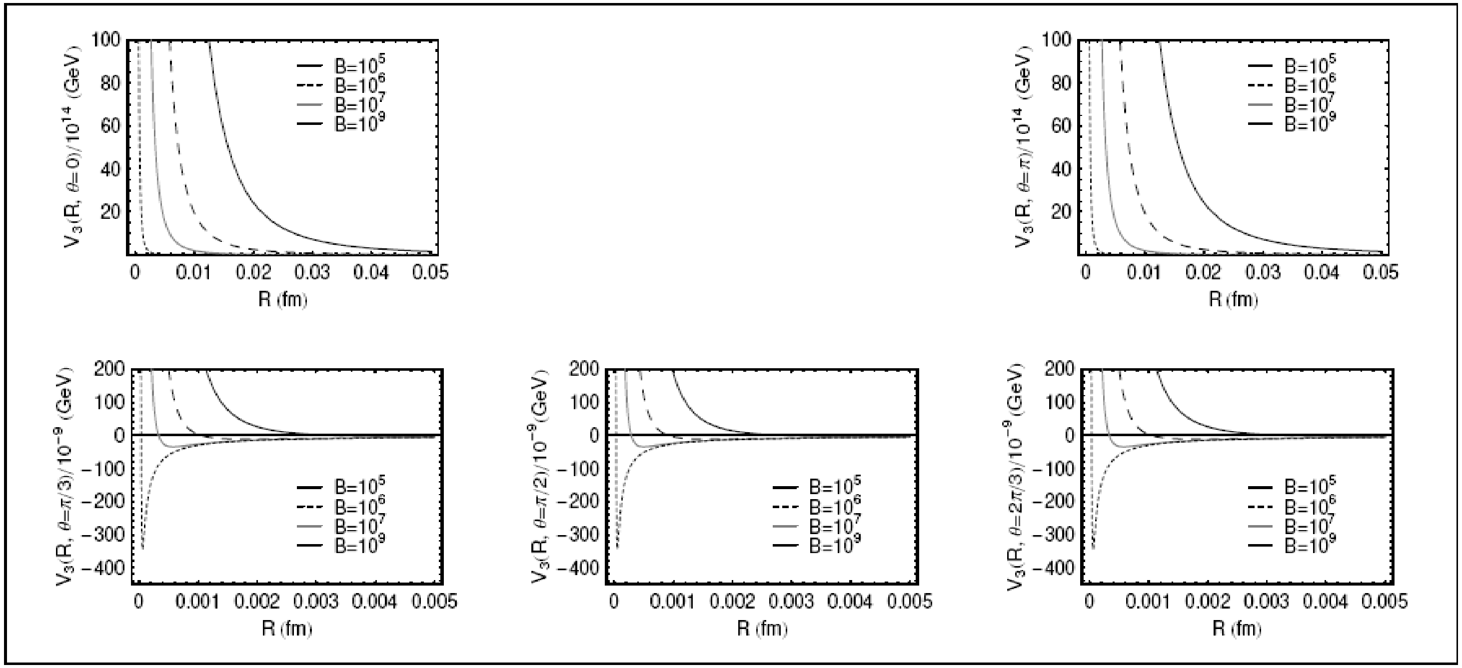
Modified Coulomb potential in the first regime

$$V_1(R,\theta) = -\frac{\alpha}{R}\left(\mathcal{A}_1(\alpha,\theta) - \frac{\gamma\mathcal{A}_2(\alpha,\theta)}{R^2} + \frac{\gamma^2\mathcal{A}_3(\alpha,\theta)}{R^4}\right)$$

For strong enough magnetic fields $H \geq 10^{24}$ G the coefficient $\mathcal{A}_3$ is small and can be neglected

Results

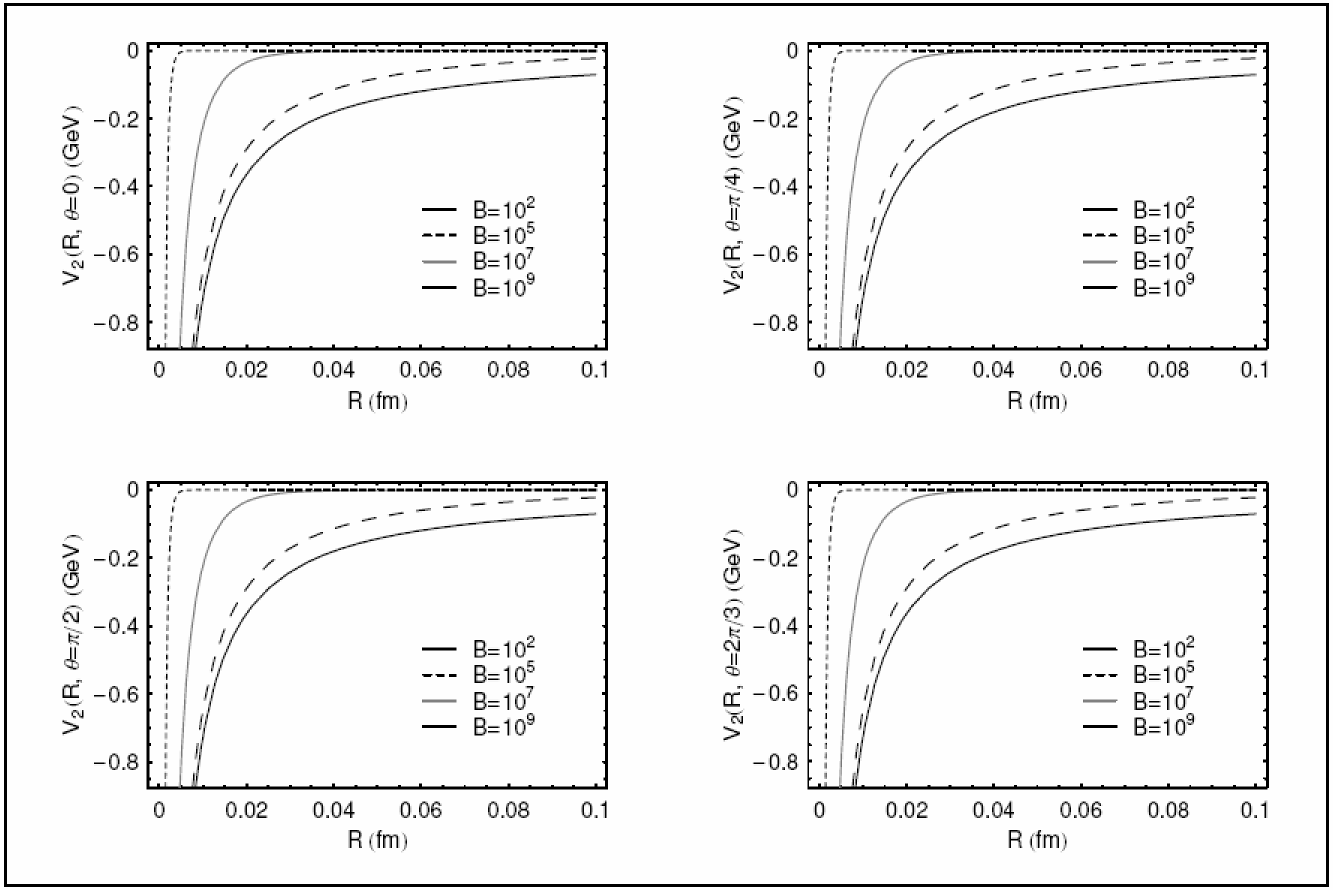
Potential $V_3(R,\theta)$ in $q_{\|}^2 << m_d^2 << |eB|$ Regime for different θ and B



- For $\theta = 0, \pi$ the potential is repulsive, whereas it is attractive for $\theta \in]0, \pi[$ and has a minimum
- The location of these minima $R_{min} \propto 1/\sqrt{B}$
- Hence: For srong enough magnetic fields bound states can be formed

Modified Coulomb potential in the second regime

$$V_2(R,\theta) = -\frac{\alpha}{(1-\frac{\alpha}{\pi})g(\theta)R}e^{-M_\gamma g(\theta)R}$$



No qualitative changes by varying the angle θ

Summary

- The modified Coulomb potential is calculated perturbatively in two different regimes in the LLLA

- In contrast to the previous results in the literature, our result depends on the angle between the external B field and the particle-antiparticle axis

- In the first regime in the LLLA, for strong enough magnetic field a qualitative change occurs by varying the angle
  - Whereas for $\theta = 0, \pi$ the potential is repulsive, it exhibits a minimum for angles $\theta \in]0, \pi[$
  - Therefore bound states can be formed in strong magnetic field for $H \geq 10^{25}$ G

Outlook

- In our calculation, the coupling constant was a bare one. It would be interesting to find the RG improved potential in the LLLA

- Using the corresponding RG equation, it is also possible to determine the beta-function of QED in the LLLA

- Potential of noncommutative U(1) gauge theory

  [0707.1885 (hep-th) by R.C. Helling and J. You]

  Duality of QED in LLLA and NC-U(1), *Miransky et al. (2004-05)*

- It is also necessary to determine the static potential using the alternative nonperturbative (lattice) methods