Long-distance color-dependent quark potentials in the Coulomb gauge QCD

-- Color-Coulomb instantaneous potentials between two quarks --

Takuya Saito (IIC, Kochi Univ.),
Atsushi Nakamura (RIIIE, Hiroshima Univ.)

Yoshiyuki Nakagawa (RCNP, Osaka Univ.)
Hiroshi Toki (RCNP, Osaka Univ.)

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In progress
Contents

1. QCD color potentials
2. Confinement: Coulomb gauge QCD
3. Color decomposed Polyakov loop correlators
4. Numerical Results
5. Divergence of color-non-singlet channels
6. Summay
Color-singlet 1

- Attraction. Strongest force between two quarks. Important for color confinement and/or color-singlet hadron bound state.
- Linearly rising potentials in the hadron phase.
- There are much many lattice studies. See a ref. Bali, PR343(2001).

- However, the gauge invariant Wilson loop or the gauge invariant Polyakov loop can not distinguish the color-singlet and color-octet contributions.

\[ 3 \otimes \bar{3} = 1 \oplus 8 \]
\[ C = \langle \lambda_i \lambda_j \rangle = -\frac{4}{3} \]
\[ V_1 \sim \frac{C}{R} \text{ at short distances} \]

Graph showing \( V_{qq}^1 \) at long distances.
Color anti-symmetric 3*

  
  (See a Review: Anselmino, et. al, RMP65(1995)1199. )

- Multi-quark state (reported by LEPS group), in which highly correlated di-quark? Jaffe and Wilczek, PRL95(2003)232003

- Hybrid, exotic particles?

- Di-quark condensate in the finite chemical potential.
  
  - Linearly rising potentials in the hadron phase? Its strength?
  
  - No lattice calculation for the color anti-triplet (di-quark) potential.

\[ 3 \otimes 3 = 6 \oplus \overline{3} \]

\[ C = \langle \lambda_i \lambda_j \rangle = -\frac{2}{3} \]

\[ V_3 \sim \frac{C}{R} \] at short distances

\[ V_{qq}^{3*} \]

at long distances

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Our aim in this study

Main aim

We’d like to clarify the long-distance behavior of color-dependent forces between two quarks in the lattice QCD simulation. Finally, we hope that this result may improve a model study of hadrons.

Today’s topic

- Color forces depend on the color (Casimir) factor at large distances?
- Volume dependence due to the divergence of color-flux in the color non-singlet channels?
In our study, we use the Coulomb gauge QCD, which has several good properties:

- Coulomb gauge is a physical gauge.
- Transverse mode of gluon propagators.
- No indefinite metric; negative spectral function will not appear.
- Instantaneous potential that can be defined in the Coulomb gauge is important to study the quark bound state, hadrons. (This is somewhat the analogy of QED.) This gives us the basic properties of color interactions among quarks.
- Coulomb gauge confinement scenario has been studied extensively now.
Coulomb gauge QCD

Hamiltonian in the Coulomb gauge QCD

\[ H = \frac{1}{2} \int d^3 x \left( E_i^2 + B_i^2 \right) + \frac{1}{2} \int d^3 x d^3 y \left( \rho(x) D(x, y) \rho(y) \right) \]

Faddeev-Popov term in the Coulomb gauge QCD

\[ D(\bar{x}, \bar{y}) = \int d^3 z \left[ \frac{1}{M(\bar{x}, \bar{y})} (-\partial_z^2) \frac{1}{M(\bar{x}, \bar{y})} \right] \quad M = (-\partial^2 + gA \times \partial) \]

Time-time component of the gluon propagators.

\[ g^2 \left\langle A_0(x) A_0(y) \right\rangle = V(x - y) + P(x - y) \]

Vacuum polarization (retarded) part

\[ V(x - y) = g^2 \left\langle D(\bar{x}, \bar{y}) \right\rangle \delta(x_4 - y_4) \]

Instantaneous part
Link-Link correlator

Correlators with partial length to the temporal direction. (Greensite, Olejnik, PRD67,094503(2003),PRD69,074506(2004).)

\[
G(R,T) = \frac{1}{3} \left\langle Tr \left[ L(R,T) L^\dagger (0,T) \right] \right\rangle, \ R = |\vec{x}|
\]

\[
V(R,T) = \log \frac{G(R,T)}{G(R,T+1)}
\]

\[
V(R,0) = -\log [G(R,1)] = -\log \left[ U_0 (R,1) U_0 (0,1) \right]
\]

Assume \(V(R,0)\) is instantaneous potential.

- Calculation is very easy.
- We can expect that instantaneous potential has clear signal.
- Good for study of the internal quark potential with PLC in the hadron phase.
Quark-antiquark potential

Color-average

\[ 3 \otimes \overline{3} = 1 \oplus 8 \]

\[ \langle \text{Tr}_L(R)\text{Tr}_L^+(0) \rangle = 1 \cdot \exp(-N_tV_1(R)) + (N^2 - 1) \cdot \exp(-N_tV_8(R)) \]

Color-singlet (attractive)

\[ e^{-V_1(R)} = \frac{1}{3} \langle \text{Tr}_L(R)L^+(0) \rangle \quad C = -\frac{4}{3} \]

Color-octet (repulsive)

\[ e^{-V_8(R)} = \frac{8}{9} \langle \text{Tr}_L(R)\text{Tr}_L^+(0) \rangle - \frac{3}{8} \langle \text{Tr}_L(R)L^+(0) \rangle \quad C = +\frac{1}{6} \]
Quark-quark potential

Color average

\[ 3 \otimes 3 = 6 \oplus \bar{3} \]

\[ \langle \text{Tr}_L(R)\text{Tr}_L(0) \rangle = \frac{1}{2} N(N+1) \cdot \exp(-N_t V_6(R)) + \frac{1}{2} N(N-1) \cdot \exp(-N_t V_6^*(R)) \]

Color-symmetric sextet (repulsive)

\[ e^{-V_6(R)} = \frac{3}{4} \langle \text{Tr}_L(R)\text{Tr}_L(0) \rangle + \frac{3}{4} \langle \text{Tr}_L(R)\text{L}(0) \rangle \quad C = +\frac{1}{3} \]

Color-antisymmetric anti-triplet (attractive)

\[ e^{-V_3^*(R)} = \frac{3}{2} \langle \text{Tr}_L(R)\text{Tr}_L(0) \rangle - \frac{3}{2} \langle \text{Tr}_L(R)\text{L}(0) \rangle \quad C = -\frac{2}{3} \]
Simulation parameters

- Wilson plaquette gauge action (simplest one) and quenched calculation.
- Hyper-cubic lattice with $N=12,18,24,32$.
- Gauge configuration number is $300 \sim 700$.
- Gauge couplings, $\beta = 5.9-6.0$, lattice cutoff $a \sim 0.1$ fm.
- **Coulomb gauge fixing** (and temporal gauge fixing) by numerical iterative method, a la Mandulor-Oglive.
- Precision of gauge fixing is up to $O(10^6)$.
- We use only the Polyakov line expectation values on the real-axis relating $Z(3)$ symmetry.
- **Computer facilities are SX-5 and SX-8 supercomputers at RCNP.**
Color-dependent instantaneous forces

\[ \beta = 6.0 \ a \sim 0.1 \text{ fm} \]

\[ C_1 = -\frac{4}{3}, \ \text{singlet} \]
\[ C_3 = -\frac{2}{3}, \ \text{anti-triplet} \]
\[ C_8 = \frac{1}{6}, \ \text{octet} \]
\[ C_6 = \frac{1}{3}, \ \text{sextet} \]
Ratios of the color forces between singlet and anti-triplet channels

- Ratio of the effective string tensions between singlet and ant-triplet.
- Volume dependence seems to be small.
Divergence part of color-Coulomb instantaneous potential

- Divergence parts

\[ V_c^{IS} = 4\pi (T_1^a T_2^b) \int_0^\infty dp \frac{1}{p^2} \]  
(from terms depending on R)

\[ \Sigma_c^{IS} = 4\pi (T_i^a)^2 \int_0^\infty dp \frac{1}{p^2} \]  
(from terms not depending on R)

\((T_1^a T_2^b) + (T_i^a)^2 = (-4/3) + (4/3) = 0\) for color-singlet

\((T_1^a T_2^b) + (T_i^a)^2 = (1/6) + (4/3) = 9/6 = 3/2\) for color-octet

\((T_1^a T_2^b) + (T_i^a)^2 = (-2/3) + (4/3) = 2/3\) for color-antitriplet

\((T_1^a T_2^b) + (T_i^a)^2 = (1/3) + (4/3) = 5/3\) for color-sextet

\(4:9:10\) for \(3^*, 8\) and \(6\)
**Volume effect (divergence of color flux)**

- **Singlet channel** has little volume dependence.
- **Color non-singlet channels** have large volume dependence; namely, the color-flux of those diverges.
- **Color-non-singlet channels** require any partner to make color-singlet state.
Summary

1. We have tried to study the long-distance behavior of the color-dependent two quark potentials in the lattice QCD simulations.

2. Color-singlet and color anti-triplet (di-quark) channels yield the linearly rising potentials for large quark separations.

3. The instantaneous potentials seem to depend on the color (Casimir) factors qualitatively.

Future works

✓ We obtain clear signals of the potential including the vacuum polarizations? Effect of vacuum polarization?

✓ Color-dependent three quark potentials? In particular, di-quark potential in 3 quark state?

✓ Maybe, we need any smearing techniques, in particular, for a calculation of full-length polyakov line correlator.
Divergence part of color-Coulomb instantaneous potential

- Hamiltonian in the Coulomb gauge

\[ H = \frac{1}{2} \int d^3x (E_i^2 + B_i^2) + \frac{1}{2} \int d^3x d^3y (\rho(x)D(x,y)\rho(y)) \]

- Color-Coulomb instantaneous potential

\[ V_{\text{inst}}(r) = \langle D \rangle = \left\langle \frac{1}{M} \left(-\partial_i^2\right) \frac{1}{M} \right\rangle \]

- Color charge density

\[ \rho_a \sim T_1^a \delta(x - x_0) + T_2^a \delta(x - y_0) \]